

International Journal of Advanced Research

in Electrical, Electronics and Instrumentation Engineering

Volume 12, Issue 12, December 2023

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6381 907 438

9940 572 462

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Impact Factor: 8.317

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 | e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com **| Impact Factor: 8.317|| A Monthly Peer Reviewed & Referred Journal |**

||Volume 12, Issue 12, December 2023||

|DOI:10.15662/IJAREEIE.2022.1212007 |

Mathematical Domain or Analysis

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ABSTRACT: I[n mathematical analysis,](https://en.wikipedia.org/wiki/Mathematical_analysis) a domain or region is a [non-empty,](https://en.wikipedia.org/wiki/Empty_set) [connected,](https://en.wikipedia.org/wiki/Connected_(topology)) an[d open set i](https://en.wikipedia.org/wiki/Open_set)n a [topological](https://en.wikipedia.org/wiki/Topological_space) [space,](https://en.wikipedia.org/wiki/Topological_space) in particular any non-empty connected open subset of the [real coordinate space](https://en.wikipedia.org/wiki/Real_coordinate_space) $Rⁿ$ or the complex coordinate [space](https://en.wikipedia.org/wiki/Complex_coordinate_space) $Cⁿ$. A connected open subset of coordinate space is frequently used for the [domain of a function,](https://en.wikipedia.org/wiki/Domain_of_a_function) but in general, [functions](https://en.wikipedia.org/wiki/Function_(mathematics)) may be defined on sets that are not topological spaces.

The basic idea of a connected subset of a space dates from the 19th century, but precise definitions vary slightly from generation to generation, author to author, and edition to edition, as concepts developed and terms were translated between German, French, and English works. In English, some authors use the term domain,^{[\[1\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-1)} some use the term region,^{[\[2\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-2)} some use both terms interchangeably,^{[\[3\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-3)} and some define the two terms slightly differently;^{[\[4\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-4)} some avoid ambiguity by sticking with a phrase such as non-empty connected open subset.

KEYWORDS: mathematical, domain, analysis, subset, coordinate, function

I. INTRODUCTION

One common convention is to define a domain as a connected open set but a region as the [union](https://en.wikipedia.org/wiki/Union_(set_theory)) of a domain with none, some, or all of its [limit points.](https://en.wikipedia.org/wiki/Limit_point)^{[\[6\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-6)} A closed region or closed domain is the union of a domain and all of its limit points.

Various degrees of smoothness of the [boundary](https://en.wikipedia.org/wiki/Boundary_(mathematics)) of the domain are required for various properties of functions defined on the domain to hold, such as integral theorems [\(Green's theorem,](https://en.wikipedia.org/wiki/Green%27s_theorem) [Stokes theorem\)](https://en.wikipedia.org/wiki/Stokes_theorem), properties of [Sobolev spaces,](https://en.wikipedia.org/wiki/Sobolev_space) and to define [measures o](https://en.wikipedia.org/wiki/Measure_(mathematics))n the boundary and spaces of [traces](https://en.wikipedia.org/wiki/Trace_operator) (generalized functions defined on the boundary). Commonly considered types of domains are domains with [continuous](https://en.wikipedia.org/wiki/Continuous_function) boundary, [Lipschitz boundary,](https://en.wikipedia.org/wiki/Lipschitz_domain) $C¹$ boundary, and so forth.[1,2,3]

A bounded domain is a domain that is [bounded,](https://en.wikipedia.org/wiki/Bounded_set) i.e., contained in some ball. Bounded region is defined similarly. An exterior domain or external domain is a domain whose [complement](https://en.wikipedia.org/wiki/Complement_(set_theory)) is bounded; sometimes smoothness conditions are imposed on its boundary.

In [complex analysis,](https://en.wikipedia.org/wiki/Complex_analysis) a complex domain (or simply domain) is any connected open subset of the [complex plane](https://en.wikipedia.org/wiki/Complex_plane) C. For example, the entire complex plane is a domain, as is the open [unit disk,](https://en.wikipedia.org/wiki/Unit_disk) the open [upper half-plane,](https://en.wikipedia.org/wiki/Upper_half-plane) and so forth. Often, a complex domain serves as the [domain of definition](https://en.wikipedia.org/wiki/Domain_of_a_function) for a [holomorphic function.](https://en.wikipedia.org/wiki/Holomorphic_function) In the study of [several complex](https://en.wikipedia.org/wiki/Function_of_several_complex_variables) [variables,](https://en.wikipedia.org/wiki/Function_of_several_complex_variables) the definition of a domain is extended to include any connected open subset of $Cⁿ$.

In [Euclidean spaces,](https://en.wikipedia.org/wiki/Euclidean_space) the extent of one-, two-, and three-dimensional regions are called, respectively, [length,](https://en.wikipedia.org/wiki/Arc_length) [area,](https://en.wikipedia.org/wiki/Area) and [volume.](https://en.wikipedia.org/wiki/Volume)

Historical notes

[\[edit\]](https://en.wikipedia.org/w/index.php?title=Domain_(mathematical_analysis)&action=edit§ion=2)

Definition. An open set is connected if it cannot be expressed as the sum of two open sets. An open connected set is called a domain.

[German:](https://en.wikipedia.org/wiki/German_language) Eine offene Punktmenge heißt zusammenhängend, wenn man sie nicht als Summe von zwei offenen Punktmengen darstellen kann. Eine offene zusammenhängende Punktmenge heißt ein Gebiet.

— [Constantin Carathéodory,](https://en.wikipedia.org/wiki/Constantin_Carath%C3%A9odory) [\(Carathéodory 1918,](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#CITEREFCarath%C3%A9odory1918) p. 222)

IJAREEIE

 | e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com **| Impact Factor: 8.317|| A Monthly Peer Reviewed & Referred Journal |**

||Volume 12, Issue 12, December 2023||

|DOI:10.15662/IJAREEIE.2022.1212007 |

According to [Hans Hahn,](https://en.wikipedia.org/wiki/Hans_Hahn_(mathematician))^{[\[7\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-7)} the concept of a domain as an open connected set was introduced by Constantin [Carathéodory](https://en.wikipedia.org/wiki/Constantin_Carath%C3%A9odory) in his famous book [\(Carathéodory 1918\)](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#CITEREFCarath%C3%A9odory1918). In this definition, Carathéodory considers obviously [non](https://en.wikipedia.org/wiki/Empty_set)[empty](https://en.wikipedia.org/wiki/Empty_set) [disjoint](https://en.wikipedia.org/wiki/Disjoint_sets) sets. Hahn also remarks that the word "Gebiet" ("Domain") was occasionally previously used as [a synonym](https://en.wikipedia.org/wiki/Synonym) o[f open set.](https://en.wikipedia.org/wiki/Open_set)^{[\[8\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-8)} The rough concept is older. In the 19th and early 20th century, the terms domain and region were often used informally (sometimes interchangeably) without explicit definition.^{[\[9\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-9)}

However, the term "domain" was occasionally used to identify closely related but slightly different concepts. For example, in his influential [monographs](https://en.wikipedia.org/wiki/Monograph) on [elliptic partial differential equations,](https://en.wikipedia.org/wiki/Elliptic_partial_differential_equation) [Carlo Miranda](https://en.wikipedia.org/wiki/Carlo_Miranda) uses the term "region" to identify an open connected set,^{[\[10\]\[11\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-M&Mp1-10)} and reserves the term "domain" to identify an internally connected,^{[\[12\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-12)} perfect [set,](https://en.wikipedia.org/wiki/Perfect_set) each point of which is an accumulation point of interior points,^{[\[10\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-M&Mp1-10)} following his former master Mauro [Picone:](https://en.wikipedia.org/wiki/Mauro_Picone)^{[\[13\]](https://en.wikipedia.org/wiki/Domain_(mathematical_analysis)#cite_note-13)} according to this convention, if a set A is a region then its [closure](https://en.wikipedia.org/wiki/Closure_(topology)) A is a domain.

II. DISCUSSION

In order to respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. Describing these skills plays a crucial role in the development of an assessment like TIMSS 2019, because they are vital in ensuring that the survey covers the appropriate range of cognitive skills across the content domains already outlined.

The first domain, knowing, covers the facts, concepts, and procedures students need to know, while the second, applying, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems.

Knowing, applying, and reasoning are exercised in varying degrees when students display their mathematical competency, which goes beyond content knowledge. These TIMSS cognitive domains encompass the competencies of problem solving, providing a mathematical argument to support a strategy or solution, representing a situation mathematically (e.g., using symbols and graphs), creating mathematical models of a problem situation, and using tools such as a ruler or a calculator to help solve problems.

The three cognitive domains are used for both grades, but the balance of testing time differs, reflecting the difference in age and experience of students in the two grades. For the fourth and eighth grades, each content domain will include items developed to address each of the three cognitive domains. For example, the number domain will include knowing, applying, and reasoning items as will the other content domains.

Facility in applying mathematics, or reasoning about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she understands, the greater the potential for engaging in a wide range of problem solving situations.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the knowledge that provides the basic language of mathematics, as well as the essential mathematical concepts and properties that form the foundation for mathematical thought.[4,5,6]

Procedures form a bridge between more basic knowledge and the use of mathematics for solving problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

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|DOI:10.15662/IJAREEIE.2022.1212007 |

Applying

The applying domain involves the application of mathematics in a range of contexts. In this domain, the facts, concepts, and procedures as well as the problems should be familiar to the student. In some items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Problem solving is central to the applying domain, with an emphasis on more familiar and routine tasks. Problems may be set in real life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets.

Reasoning

Reasoning mathematically involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations. Such problems may be purely mathematical or may have real life settings. Both types of items involve transferring knowledge and skills to new situations; and interactions among reasoning skills usually are a feature of such items.

Even though many of the cognitive skills listed in the reasoning domain may be drawn on when thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

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The domain of mathematics is extremely complex, including such distinct areas as arithmetic, algebra, and geometry, with each of these areas consisting of many subdomains and encompassing many cognitive processes. For elementary school children, standardized achievement tests assess a wide range of arithmetic skills, including such diverse cognitive processes as learning, remembering, and retrieving arithmetic facts; executing calculation procedures; using problem-solving strategies that involve executive function and working memory [\(Geary, 2004;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R5) [Landerl, Bevan, &](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R13) [Butterworth, 2004\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R13); and implementing general semantic memory and visuospatial processes [\(Geary, 1993;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R4) Mazzocco & [Myers, 2003\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R16). Although standardized achievement tests often assess diverse mathematical skills, this information frequently is aggregated into a global score of mathematical achievement that essentially averages information about children's patterns of strengths and weaknesses.

Although relatively little is known about the phenotypic (observed) relationships among mathematics skills, even less is known about the genetic and environmental origins of these relationships. The few univariate twin and adoption studies of mathematics performance have reported a wide range of heritabilities, from .20 to .90 (reviewed in [Oliver et al.,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R28) [2004\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R28), using global assessments of mathematics. In a recent report, three different aspects of mathematics ability and disability in 7-year-olds were assessed by teachers using U.K. national curriculum criteria during the 2nd year of elementary school [\(Oliver et al., 2004\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R28). The results showed substantial genetic influence (.69–.74) for the three measures. However, it is important to note that the average phenotypic correlation among the three measures was very high (.81) in this study, which might be partly because of the teachers' bias toward rating the three aspects of mathematics as if they were more similar than they are. Although several multivariate genetic studies addressed the etiology of covariation between mathematics and other cognitive skills[7,8,9] (reviewed in [Kovas, Harlaar, Petrill, &](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11) [Plomin, 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11), we could find no published reports of multivariate analyses that addressed the heterogeneity within the domain of mathematics.

The purpose of the present study is to provide the first multivariate genetic analysis of the etiological relationship among five aspects of mathematics that are part of the U.K. national curriculum. Our focus is on individual differences in mathematics ability for a representative sample of children rather than on mathematics disability, for which much more has been written about possible subtypes (e.g., [Geary, 2004;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R5) [Geary, Hamson, & Hoard, 2000;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R6) [Landerl et al.,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R13) [2004;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R13) [Mazzocco & Myers, 2003;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R16) [Miranda Casas & García Castellar, 2004;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R18) [Temple & Sherwood, 2002\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R37). Nonetheless, genetic research suggests that individual differences in mathematical ability are relevant to mathematical disability because, rather than having a unique etiology, common disability appears to be the quantitative extreme of the same genetic and environmental factors responsible for typical variation in ability [\(Oliver et al., 2004;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R28) [Plomin & Kovas,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33) [2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33). For example, for each of many genes associated with a specific ability, a "good" and a "bad" variant may exist, and the relative number of good and bad variants (together with other relevant factors) influences a person's position on a continuum of ability. In other words, when genes are found that are associated with mathematics ability, the same genes are expected to be associated with mathematics disability.

Rather than focusing on genetic and environmental contributions to the variance of a single trait (univariate genetic analysis), multivariate genetic analysis investigates the genetic and environmental contributions to the covariance between traits [\(Martin & Eaves, 1977;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R15) [Plomin, DeFries, McClearn, & McGuffin, 2001\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R32), as described in detail in the Method section of the current investigation. Multivariate genetic analysis yields a statistic called the genetic correlation, which indexes the extent to which genetic effects on one trait correlate with genetic effects on another trait independently of the heritability of the two traits. The genetic correlation can be roughly interpreted as the extent to which the same genes affect the two traits; a genetic correlation of 1.00 indicates that the same genes affect both traits, and a genetic correlation of .00 signifies that different genes are involved.

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 | e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com **| Impact Factor: 8.317|| A Monthly Peer Reviewed & Referred Journal |**

||Volume 12, Issue 12, December 2023||

|DOI:10.15662/IJAREEIE.2022.1212007 |

Although no multivariate genetic research has addressed the issue of heterogeneity within the domain of mathematics, multivariate genetic research in other cognitive areas demonstrates a surprisingly high degree of genetic overlap among such diverse aspects of ability as grammar, vocabulary and phonology in language (average genetic correlations \sim .60), and reading of words and nonwords (genetic correlation \sim .80; reviewed in [Plomin & Kovas, 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33). Moreover, substantial genetic overlap has been found for whole cognitive areas, such as reading, language, and general cognitive ability [\(Plomin & Kovas, 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33), and a recent analysis included mathematics in this network of genetic links [\(Kovas et](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11) [al., 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11). Such multivariate genetic research has led to the hypothesis that a single set of genes is largely responsible for genetic effects on diverse cognitive and learning abilities and disabilities [\(Plomin & Kovas, 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33). This generalist genes hypothesis predicts that when genes are found that are associated with a particular cognitive ability, the same genes will also be associated with other cognitive abilities. The generalist genes hypothesis leads us to predict substantial genetic overlap within the domain of mathematics, a hypothesis that we can test directly using multivariate genetic analysis. In the present study, our prediction is that genetic correlations will be substantial among five aspects of mathematics ability.

A practical problem in conducting genetic research on mathematics ability is that large samples of twins must be assessed, but it is expensive to test such large samples in person, especially when their home residences are distributed over a wide area. Our previous research circumvented this problem by using teacher assessments based on U.K. national curriculum criteria [\(Kovas et al., 2005;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11) [Oliver et al., 2004\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R28). Although teacher assessments are a valuable source of information about children's progress, the criteria used were broad and not amenable to multivariate genetic analysis. Even if more specific ratings were obtained, it is possible that teachers would not be able to adequately discriminate a child's strengths and weaknesses in different aspects of mathematics.

In the current research, we address this problem by the use of Web-based tests, which make it possible to assess large samples efficiently and economically. To increase the relevance of the multivariate research to current educational practice, we decided to base our testing on the U.K. national curriculum, which focuses on five aspects of mathematics when children are 10 years of age: (a) using mathematics in a problem-solving situation; (b) understanding the numerical and algebraic process to be applied when solving problems; (c) retrieving and computing facts; (d) interpreting information from diagrams, graphs, tables, charts, and scales; and (e) understanding nonnumerical mathematical processes and concepts.

Phenotypic analyses

The data were first explored with descriptive statistics analyses in SPSS. Analysis of variance was performed to assess the effects of sex and zygosity on mathematical ability in our sample. Phenotypic relationships among the five categories were explored with Pearson correlation and principal-components factor analysis.

Before assessing genetic and environmental influences on variance and covariance among the five aspects of mathematics, we explored the phenotypic structure of interrelationships between them. We used a phenotypic Cholesky decomposition analysis to test the shared and unique influences on the five aspects of mathematics. The phenotypic Cholesky is analogous to a hierarchical multiple regression analysis in which the first four categories are entered sequentially and the fifth category is the dependent variable (see [Tiu, Wadsworth, Olson, & DeFries, 2004,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R38) for details of this procedure). We used a five-factor model in which the first factor represented shared variance for the five aspects of mathematics. The second factor represented the shared variance among the remaining four categories after we accounted for the variance in common with the first test. The third factor represented the shared variance among the remaining three categories after we accounted for the shared variance with the first and second tests. The fourth factor represented the proportion of variance in common for the remaining variables after we accounted for the shared variance with the first three tests. Finally, the fifth factor estimated the proportion of the variance that was unique to the fifth test. In other words, we used this procedure to test the independent effect of one variable (e.g., influences on Mathematical Interpretation) on another variable (e.g., Non-Numerical Processes) after controlling for influences that were also important for the preceding aspects[10,11,12] of mathematics (Mathematical Application, Understanding Number, and Computation and Knowledge).

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Genetic analyses

The twin method addresses the origins of individual differences by estimating the proportion of variance that can be attributed to genetic, shared environment, and nonshared environment factors [\(Plomin, DeFries, et al., 2001\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R32). In the case of complex traits that are likely to be influenced by multiple factors, the genetic component of variance refers to the influence of alleles at all gene loci that affect the trait. The similarity between twins for any particular trait can be due to a common set of genes. It may also be due to the shared environment, which refers to environmental influences that vary among families but not within families and that contribute to the similarity between cotwins. For example, twins experience similar conditions during gestation, have the same socioeconomic status, live in the same family, and usually go to the same school. These factors could reasonably be expected to increase similarity between cotwins. Nonshared environment refers to any aspect of environmental influence that makes cotwins different from each other, including measurement error. Such influences involve aspects of environment that are specific to an individual, such as traumas and diseases, idiosyncratic experiences, different peers, differential treatment by the parents and teachers, and different perceptions of such influences.

Genetic influence is estimated via comparison of the covariance between identical (MZ) twins, who are genetically identical, and fraternal (DZ) twins, who share 50% of the same genes, on average. If the MZ twin correlation exceeds the DZ twin correlation, then genetic influences (or heritability) are implicated. Shared environmental effects are implied to the extent that MZ and DZ correlations are similar beyond heritability. Nonshared environment is the extent to which the MZ twin correlation is not 1.00.

Structural equation model fitting is a comprehensive way of estimating variance components on the basis of these principles. For example, variations on the so-called ACE model can be used for analyses of individual differences; this model apportions the phenotypic variance into genetic (A), shared environmental (C), and nonshared environmental (E) components, assuming no effects of nonadditive genetics or nonrandom mating. The researcher can estimate the ACE parameters and their confidence intervals by fitting the models to variance–covariance matrices using the model-fitting program Mx [\(Neale, 1997;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R20) [Neale, Boker, Xie, & Maes, 2002\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R21).

These principles can be extended to investigate the etiology of the covariance between traits. Multivariate genetic analysis assesses the extent to which genetic and environmental factors are responsible for the phenotypic correlation between two traits. For twin studies, multivariate genetic analysis is based on cross-trait twin correlations for two or more traits. For example, one twin's Computation and Knowledge score is correlated with the cotwin's Non-Numerical Processes score. Similar to the univariate case described above, the phenotypic covariance between two traits is attributed to genetic overlap to the extent that the MZ cross-trait twin correlation exceeds the DZ cross-trait twin correlation. Shared environmental influences are indicated to the extent that the DZ cross-twin correlation is more than half of the MZ correlation.

As with univariate analysis, structural equation modeling represents a more comprehensive way of estimating the proportion of covariance. In particular, we fitted a multivariate Cholesky decomposition model (with five variables) to the variance–covariance matrices derived from the twin data to test for common and independent genetic and environmental effects on variance in the five aspects of mathematics. The Cholesky procedure is similar to hierarchical regression analyses in nongenetic studies, whereby the independent contribution of a predictor variable is assessed after its shared variance with other predictor variables is accounted for (see [Tiu et al., 2004,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R38) an[d Neale et al., 2002,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R21) for more detail). The order in which the variables were entered in the analysis was determined by the logical assumption that Mathematical Application is the most general ability of the five and is therefore likely to have shared etiology with the other abilities to a large extent. Thus, Mathematical Application was entered in the analysis first, followed by increasingly more specific categories (se[e Loehlin, 1996,](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R14) for a discussion of the Cholesky procedure). This order, however, is by no means the only possible logical order. For example, because one could argue that Mathematical Application is a higher order skill, it may not be an optimum base factor. Instead, the Computation and Knowledge category may be a better candidate, as children might have a better grounding in this aspect and it may underpin more of the other categories. However, if the generalist genes hypothesis is true for the five aspects of mathematics, the order of entry into the analysis should not make a difference. We tested this hypothesis by rerunning this analysis with the five tests in different orders.

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|DOI:10.15662/IJAREEIE.2022.1212007 |

As with the phenotypic analysis, we used a five-factor model in which the first factor assessed genetic and shared and nonshared environmental influences on Mathematical Application, some of which also influenced the Understanding Number, Computation and Knowledge, Mathematical Interpretation, and Non-Numerical Processes categories. The second factor represented genetic and shared and nonshared environmental influences on Understanding Number that were not shared with Mathematical Application but were shared with the Computation and Knowledge, Mathematical Interpretation, and Non-Numerical Processes categories. The third factor represented genetic and shared and nonshared environmental influences on Computation and Knowledge that were not shared with Mathematical Application and Understanding Number but were shared with the Mathematical Interpretation and Non-Numerical Processes categories. The fourth factor represented genetic and shared and nonshared environmental influences on Mathematical Interpretation that were not shared with Mathematical Application, Understanding Number, and Computation and Knowledge but were shared with the Non-Numerical Processes category. Finally, the fifth factor estimated genetic and shared and nonshared environmental influences that were unique to the Non-Numerical Processes category. The model also allowed us to estimate the proportions of the total variance attributable to genetic and environmental factors for each of the categories (univariate estimates).[13,14,15]

In addition, we transformed the paths from the model to obtain estimates of genetic, shared environmental, and nonshared environmental correlations between each pair of factors. Genetic correlations index the extent to which the sum of genetic influences on one measure correlates with the sum of genetic influences on a second measure, regardless of the heritabilities of the traits. Put another way, it is the extent to which the heritabilities described are influenced by the same genetic factors. We also estimated the proportion of the phenotypic (observed) covariance among the math measures that could be attributed to genetic covariance between Trait 1 and Trait 2, which is bivariate heritability, the genetic correlation weighted by the product of the square roots of the heritabilities of the two traits and divided by the phenotypic correlation between them [\(Plomin & DeFries, 1979\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R31). Shared and nonshared environmental correlations, which index the extent to which the same shared and nonshared environmental influences are important for the two aspects of mathematics, and environmental contributions to the phenotypic correlation were also estimated. Details of the estimation of these statistics are provided elsewhere (e.g.[, Plomin & DeFries, 1979;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R31) [Posthuma et al., 2003;](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R35) [Tiu et](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R38) [al., 2004\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R38). In summary, we used a Cholesky decomposition to estimate the genetic, shared environmental, and nonshared environmental contributions to the variance of the measures and the covariance among the measures.[16,17,18]

IV. CONCLUSION

Another future direction for research is to investigate whether most of the variance in the diverse aspects of mathematics can be explained by factors that also influence reading and general cognitive ability. For example, one might be tempted to say that what is in common among these different aspects of mathematics is intelligence. However, our view is that this does not take us much farther in terms of understanding mechanisms, because we do not know what intelligence is any more than we know what causes the general factor that influences different aspects of mathematics. Although many brain and cognitive processes are likely to contribute to the phenotypic overlap among the subdomains of mathematics, the point of the present results is that the same set of genes is largely responsible for genetic influence in these domains (for more discussion on this issue, see [Plomin & Kovas, 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R33). We have collected data on reading and general cognitive ability in addition to mathematics as part of a large Web-based battery. The next step for our research is to include these variables in multivariate genetic analyses. From previous research (e.g., [Kovas](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11) [et al., 2005\)](https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2743325/#R11) and the present study, we predict substantial overlap among genetic influences on mathematics, reading, and general cognitive ability but also some unique genetic influences on mathematics.[19]

Finally, despite the large sample of this study, an even larger sample is needed to assess whether the small quantitative differences in etiology of the five aspects of mathematics found in this study are statistically significant. We are planning to investigate this issue when the data from the second TEDS cohort are available. Increasing the sample size will also allow us to investigate sex differences in the etiology of individual differences in mathematical ability.[20]

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IJAREEIE

 | e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com **| Impact Factor: 8.317|| A Monthly Peer Reviewed & Referred Journal |**

||Volume 12, Issue 12, December 2023||

|DOI:10.15662/IJAREEIE.2022.1212007 |

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