



Modelling of Wind Turbine by Aerodynamic Efficiency in a Wind Energy Conversion System

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ABSTRACT: The world is growing in terms of population and industrialization, it needs additional energy to meet the electrical demand. Wind energy is a free resource to generate electrical energy among all other available renewable energy sources. In the present electricity market, wind energy can compete with conventional fossil fuels. For the better competition, the production cost of electrical energy should be minimum, can be maintained by effective designing of the wind turbine. A mathematical model of a wind turbine is crucial to understand the behaviour of the wind turbine. This paper addresses the most commonly used wind turbine models in terms of aerodynamic efficiency (or) power coefficients for the analysis of wind energy conversion systems (WECS). Different wind turbine models are helpful to wind turbine researchers. This information is useful to optimize the design of wind turbines and minimize generation cost.

KEYWORDS: Wind Energy Conversion System (WECS), Aerodynamic Efficiency.

I.INTRODUCTION

Currently, wind energy is the hot research topic in various countries of the world and different organizations like Global Wind Energy Council (GWEC), National Renewable Energy Laboratory (NREL) and Ministry of New and Renewable Energy (MNRE) are associated with this research. Different conventional fuels like coal, gas, and oil are used to generate electrical energy. They are gradually decreasing, so exploration of alternate fuel. Wind energy is one of the most cost-effective, environmentally friendly, more advantageous compared to conventional fuels, and it does not produce any hazardous waste. Wind energy is the fastest growing technology in India. Under the Paris Agreement (COP21), India is steadily moving towards climate change commitments. India's wind power industry has the capability to meet the country's climate and energy security goals. India had a record year in 2017, adding 4.15 GW, the first time the country has broken 4 GW in a single year.

Installed Wind Capacity in MW

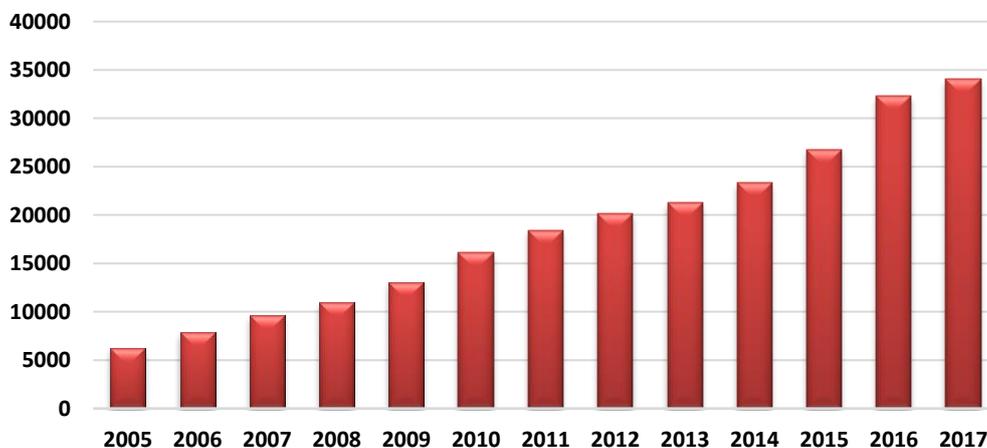


Fig.1. Installed Wind Capacity in India upto 2017

At present, India is the fourth largest wind market globally. According to the GWEC survey, the cumulative installed capacity up to 2017 is 32,848 MW in India. This total has risen to 34,046 MW at the end of March 2018. In future, the government of India has a commitment to install 60 GW of wind power by 2022 [1]. Fig.1 shows the installed wind capacity in India since 2005 to 2017.

The kinetic energy in the wind is converted into mechanical energy by means of shaft and gearbox arrangement. This energy then turned into electrical energy in the generators. The electricity generated can be used to home power grids or utility companies. The main basic components of wind energy conversion system (WECS) for modelling, can be grouped as follows: (i) the wind model (ii) the turbine model (iii) the gearbox model (iv) the generator model (v) the controller model [2] as shown in Fig.2.

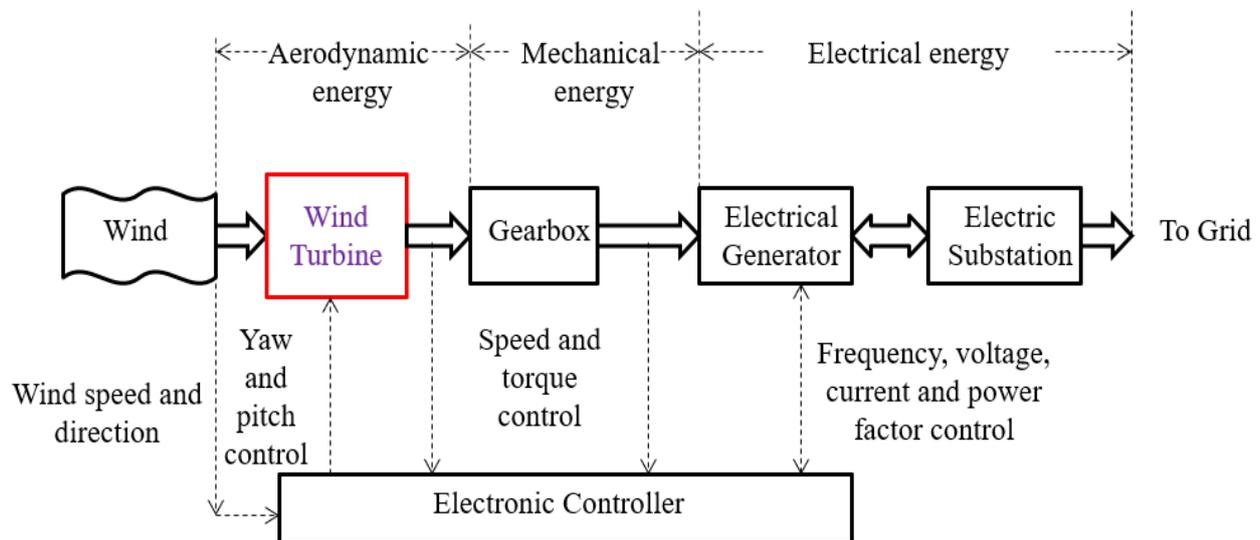


Fig.2. Wind energy conversion system

A typical wind turbine involves a set of rotor blades, a nacelle and the tower. The nacelle is a large house, which houses a gearbox and a generator. The tower supports the rotor and the nacelle. In this paper, the mathematical models of a wind turbine in terms of power coefficients are presented.

II. TYPES OF WIND TURBINE

The main basic components of wind turbine system are rotor blades, a hub, a gearbox and a generator. The wind turbines are classified [3] based on various aspects like axis of rotation, output power scale, location of installation, and rotational speed. They are described as follows.

1. Based on axes

The vertical axis wind turbine (VAWT) and horizontal axis wind turbine (HAWT) are the two wind turbine types based on axis of rotation. A brief description is as follows.

(i) Vertical axis wind turbine (VAWT): The generator shaft is positioned vertically with the blades rotating around it. They are normally of small height. An example is given in Fig.3.

(ii) Horizontal axis wind turbine (HAWT): In this type of turbine, the generator shaft is positioned parallel to the ground and the blades are positioned on top of a long tower. The HAWT is taller and this type is popular for its high output power. A HAWT is shown in Fig.3. Compared to vertical axis wind turbines, horizontal axis wind turbines are more advantageous such that they operate in all wind directions and thus need no yaw adjustment.

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Website: www.ijareeie.com

Vol. 7, Issue 6, June 2018

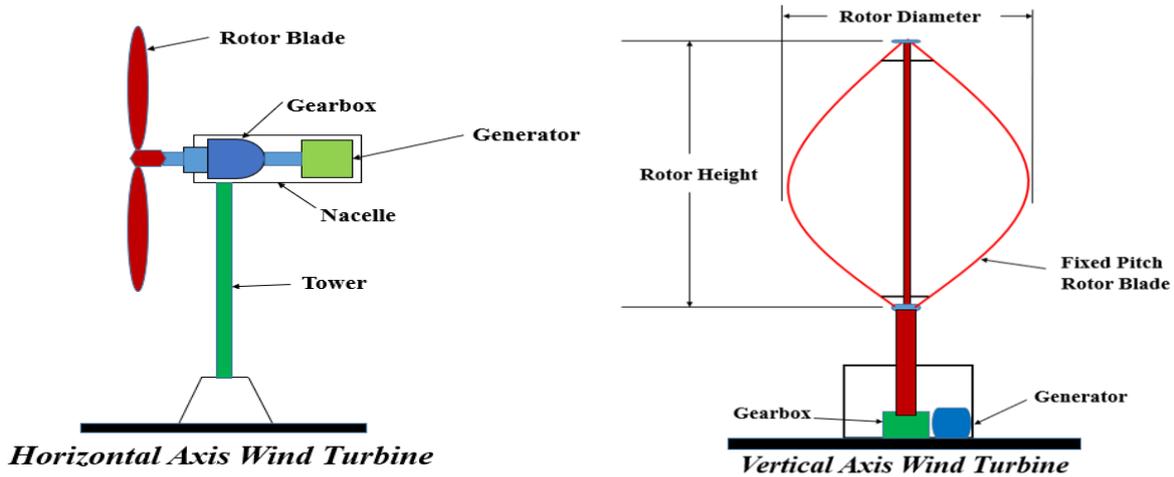


Fig.3. Types of wind turbines based on axes

2. Based on output power scale

Day-by-day output electrical power capacity of the wind turbine is increasing. According to the output power capacity, they can be categorized into different classes. Small and medium scale turbines are utilized in the remote standalone system, and large scales wind turbines are connected to micro-grids and power grids.

- (i) Small-scale wind turbine: They can operate at relatively low annual wind speed ranging from 2.5 m/s to 4 m/s with a fairly low capacity of 0.025 kW to 10 kW.
- (ii) Medium-scale wind turbine: They give output power from 10 kW to 100 kW and operate fairly at an annual wind speed of 4 m/s to 5 m/s.
- (iii) Large-scale wind turbine: Any wind turbine with output power capacity greater than 100kW lies in this range, and they need average annual speed over 5 m/s.

3. Based on installation location

Traditionally, the wind turbines are installed on land. But in 1991, the first wind turbine was installed on the shallow sea near the coast. They are termed as an offshore wind turbine. To differentiate an offshore wind turbine, wind turbines placed on land are termed as onshore wind turbines.

- (i) Onshore wind turbine: This is the old concept of wind turbine placement. They provide good performance at low installation along with low maintenance cost but gives low efficiency. Fig. 4 shows onshore wind turbine.



Fig.4. Onshore wind turbine



Fig.5. Offshore wind turbine

- (ii) Offshore wind turbine: Offshore turbine as shown in Fig.5 provides higher efficiency as the wind speed at offshore location is higher and more consistent. They are gaining popularity and large capacity wind turbines that are built at present days are offshore type.

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Nevertheless, onshore wind turbines also have some advantages over offshore wind turbines:

- Foundations are cheaper.
- Integration with electric power grid network is cheaper.
- Installation and access of construction phase are cheaper.
- Operation and maintenance are cheaper and easier.

III. MODELLING OF WIND TURBINE

In the modelling of a wind turbine, a number of geometrical and physical features are involved. A mathematical model of wind turbine is crucial in the understanding of the behaviour of the wind turbine. A wind turbine converts the kinetic energy of moving air into mechanical energy that can be converted into electrical energy by means of a generator.

The kinetic energy (E), in a packet of air of mass (m), flowing at speed (v) in the x -direction [3] can be expressed as

$$E = \frac{1}{2} mv^2 = \frac{1}{2}(\rho Ax)v^2 \quad \text{Joules} \quad (1)$$

Where ρ is the air density (Kg/m^3), A is the cross-sectional area in sq. meter, x is the thickness of the packet in meter.

From Fig.6, with side x with speed v and the opposite side fixed at the origin, the mass is increasing uniformly then the kinetic energy also increasing uniformly with x .

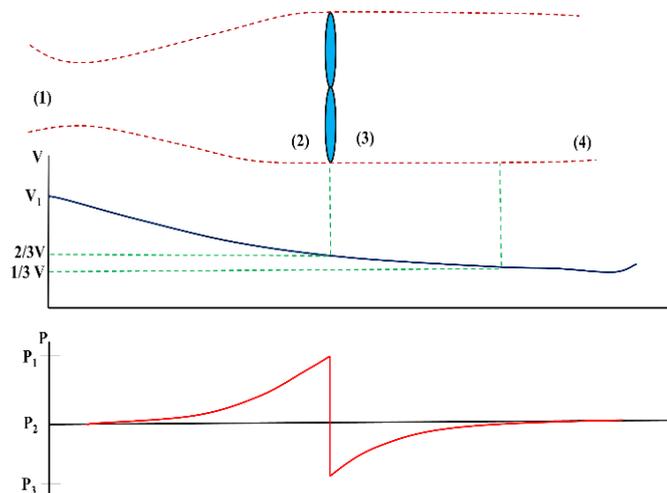
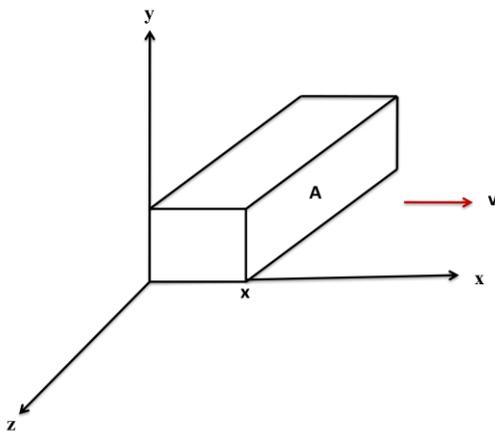


Fig.6. Packet of air moving with speed(v)

Fig.7. Circular tube of air flowing through ideal wind turbine

The power in the wind turbine p_w is the time derivative of the kinetic energy (E)

$$p_w = \frac{dE}{dt} = \frac{1}{2} * \rho * A * v^2 * \frac{dx}{dt} = \frac{1}{2} \rho A v^3 \quad \text{Watts} \quad (2)$$

From side x , a wind turbine will extract power and "equation (2)" represents the total power available at this surface for possible extraction.

Fig.7 is drawn for a conventional horizontal axis propeller type turbine. The physical reality of a wind turbine in a huge moving air mass modifies the local air and pressure as shown in Fig.7.

Consider a circular tube of moving air with an undisturbed diameter (d_1), speed (v_1), and pressure, (p_1), as it comes close to the turbine. In front of the turbine, the air pressure will increase to a maximum value and behind the wind turbine, the air pressure will drop below atmospheric pressure. Some of the kinetic energy (K.E) in the air is converted to potential energy (P.E) to generate this increase in pressure. Still, more amount of K.E will be converted to P.E after the turbine to increase the air pressure back to the atmosphere. This causes the wind speed to continue to decrease in



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anticipation of the pressure is in equilibrium. Once the low point of wind speed is attained, the speed of the tube of air will rise back to $v_4 = v_1$ as it is given kinetic energy from the surrounding air.

From Fig.7, the following relationships hold good under optimum conditions, when maximum power is being transferred from the tube of air to the wind turbine.

$$v_2 = v_3 = \frac{2}{3} * v_1 \quad (3)$$

$$v_4 = \frac{1}{3} * v_1 \quad (4)$$

$$A_2 = A_3 = \frac{3}{2} * A_1 \quad (5)$$

$$A_4 = 3 * A_1 \quad (6)$$

The mechanical power harnessed from the wind is the difference between the input and output power, can be written as follows:

$$P_m = P_1 - P_4 = \frac{1}{2} * \rho * (A_1 * v_1^3 - A_4 * v_4^3) \quad (7)$$

$$P_m = \frac{1}{2} * \rho * \frac{8}{9} * A_1 * v_1^3 \quad (8)$$

$$P_m = \frac{1}{2} * \rho * \frac{8}{9} * \frac{2}{3} * A_2 * v_1^3 \quad (9)$$

$$P_m = \frac{1}{2} * \rho * \frac{16}{27} * A_2 * v_1^3 \quad (10)$$

According to Betz, the maximum wind turbine power output,

$$p_m = \frac{1}{2} \rho \frac{16}{27} A v^3 \quad \text{Watts} \quad (11)$$

$$p_m = \frac{1}{2} \rho c_p A v^3 \quad \text{Watts} \quad (12)$$

The factor $16/27=0.593$ is called the Betz coefficient or power coefficient or aerodynamic efficiency. It is denoted as c_p . An actual turbine cannot extract more than 59.3% of the power in an undisturbed tube of air of the same area. Because of mechanical imperfections, the fraction of power extracted will always be less in practice. Under optimum conditions, a good fraction is about 35–40% of the air in the wind.

Air density (ρ) is another flow input quantity in the rotor system and it depends upon both absolute pressure (P) and absolute temperature (T). When absolute pressure increases air density (ρ) increases. When absolute temperature decreases ρ increases. The density of air can be expressed as,

$$P = \rho * R * T \quad (13)$$

$$\rho = \frac{P}{R * T} \quad (14)$$

Where R is the specific gas constant.

According to International Standard Atmosphere (ISA), At sea level and at 15 °C air has a density of approximately 1.225 kg/m³.

IV.MATHEMATICAL MODELLING OF WIND TURBINE BY AERODYNAMIC EFFICIENCY ANALYSIS

The aerodynamic efficiency (or) power coefficient of a wind power plant is the instantaneous efficiency of the conversion of the wind energy into mechanical energy at the shaft. It is also defined as the ratio of actual power extracted by the rotor to the total power available in the wind. Equation (12) represents the power extracted by the wind turbine. The aerodynamic efficiency or power coefficient (c_p) is the most important parameter in terms of power regulation. It is a non-linear function and each wind turbine manufacturer provides c_p value with the help of lookup tables. Other than look up tables, power coefficient has been developed. The most commonly used wind turbine models in terms of aerodynamic efficiency (or) power coefficients are described as follows:



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A. Model 1

The commonly used equation for the mechanical power (P_m), captured by a wind turbine can be expressed as follows

$$P_m = 0.5 * \rho * \pi R^2 * V_w^3 * C_p(\lambda, \beta) \quad (15)$$

Where ρ is the air density in kg/m^3 , R is the wind turbine blade radius in meter, v_w is the wind velocity in m/s, and $C_p(\lambda, \beta)$ is the power coefficient, which is a function of tip speed ratio (λ) and blade pitch angle (β). According to Betz's law, theoretically 59.3% power can be extracted from the wind. The tip speed ratio is expressed as

$$\lambda = \frac{R * \omega_r}{V_w} \quad (16)$$

Where ω_r is the angular mechanical speed in rad/sec. The power coefficient $C_p(\lambda, \beta)$ equation [4] is given by,

$$C_p(\lambda, \beta) = c_1 \left(\frac{c_2}{\lambda_i} - c_3 \beta - c_4 \right) e^{-\frac{c_5}{\lambda_i}} + c_6 \lambda \quad (17)$$

Where

$$\lambda_i = \left[\frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \right]^{-1} \quad (18)$$

For a particular turbine type, the coefficients from c_1 to c_6 are $c_1 = 0.5176$, $c_2 = 116$, $c_3 = 0.4$, $c_4 = 5$, $c_5 = 21$ and $c_6 = 0.0068$.

$$C_p(\lambda, \beta) = 0.5176 \left(\frac{116}{\lambda_i} - 0.4\beta - 5 \right) e^{-\frac{21}{\lambda_i}} + 0.0068\lambda \quad (19)$$

The C_p - λ characteristics for the different values of blade pitch angle (β) as shown in the Fig.8. For a fixed blade pitch angle ($\beta=0^\circ$), a maximum C_p ($C_{p\max}=0.49$) is achieved when the tip speed ratio is at the optimum value ($\lambda_{\text{opt}}=8$).

B. Model 2

Anderson and Bose suggested the $C_p(\lambda, \beta)$ equation [5] expressed as

$$C_p(\lambda, \beta) = 0.5(\lambda - 0.022\beta^2 - 5.6)e^{-0.17\lambda} \quad (20)$$

Where

$$\lambda = \frac{v_w}{w_r} \quad (21)$$

Where w_r is the angular mechanical speed in rad/sec. Fig.9 depicts the C_p - λ characteristics for different pitch angles.

The maximum value of C_p ($C_{p\max}=0.43$) is achieved for a fixed blade pitch angle ($\beta=0^\circ$) and optimum tip speed ratio ($\lambda_{\text{opt}}=9.7$).

C. Model 3

Based on [6], the $C_p(\lambda, \beta)$ equation can be expressed as

$$C_p(\lambda, \beta) = 0.73 \left(\frac{151}{\lambda_i} - 0.58\beta - 0.002\beta^{2.14} - 13.2 \right) e^{-\frac{18.4}{\lambda_i}} \quad (22)$$

Where

$$\lambda_i = \left[\frac{1}{\lambda + 0.02\beta} - \frac{0.003}{\beta^3 + 1} \right]^{-1} \quad (23)$$

$$\lambda = \frac{w_r * R}{v_w} \quad (24)$$

Fig.10. depicts the C_p - λ characteristics for the different values of blade pitch angle (β). For a fixed blade pitch angle ($\beta=0^\circ$), a maximum C_p ($C_{p\max}=0.45$) is achieved when the tip speed ratio is at the optimum value ($\lambda_{\text{opt}}=5.8$).

D. Model 4

The aerodynamic torque (T_m) and the mechanical power (P_m) of a wind turbine can be expressed as:

$$T_m = \frac{1}{2} \rho c_t(\lambda) \pi R^3 V_w^2 \quad (25)$$



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$$P_m = \frac{1}{2} \rho c_p(\lambda, \beta) \pi R^2 V_w^3 \quad (26)$$

Where ρ is the air density, R is the radius of the wind turbine, V_w is the wind velocity, and $C_p(\lambda, \beta)$ is the power coefficient [7] given by

$$C_p(\lambda, \beta) = 0.5(\gamma - 0.022\beta^2 - 5.6)e^{-0.17\gamma} \quad (27)$$

Where

$$\gamma = \frac{R(3600)}{\lambda(1609)} \quad (28)$$

$$C_t(\lambda) = \frac{c_p(\lambda)}{\lambda} \quad (29)$$

$$\lambda = \frac{w_t * R}{v_w} \quad (30)$$

Where w_t is the rotational speed of the wind turbine, λ is the tip speed ratio. The relationship between C_p and λ is shown in Fig.11 for different blade pitch angles. For a fixed blade pitch angle ($\beta=0^\circ$), a maximum $C_p(C_{p\max}=0.42)$ is achieved when the tip speed ratio is at the optimum value ($\lambda_{\text{opt}}=9.6$).

E. Model 5

Based on [8], the $C_p(\lambda, \beta)$ equation can be expressed as

$$C_p(\lambda, \beta) = (0.44 - 0.0167\beta) \sin \left[\frac{\pi(\lambda-3)}{(15-0.3\beta)} \right] - 0.00184(\lambda - 3) * \beta \quad (31)$$

Where

$$\lambda = \frac{w_{wt} * R}{v_w} \quad (32)$$

Fig.12 depicts the C_p - λ characteristics for the different values of blade pitch angle (β). For a fixed blade pitch angle ($\beta=0^\circ$), a maximum $C_p(C_{p\max}=0.44)$ is achieved when the tip speed ratio is at the optimum value ($\lambda_{\text{opt}}=10.4$).

F. Model 6

Based on [9], the $C_p(\lambda, \beta)$ equation can be expressed as

$$C_p(\lambda, \beta) = (0.5 - 0.0167(\beta - 2)) * \sin \left[\frac{\pi(\lambda+0.1)}{(18.5-0.3(\beta-2))} \right] - 0.00184(\lambda - 3) * (\beta - 2) \quad (33)$$

Where

$$\lambda = \frac{w_r * R}{v_w} \quad (34)$$

Where w_r is the angular mechanical speed in rad/sec. Fig.13 depicts the C_p - λ characteristics for different pitch angles. The maximum value of C_p ($C_{p\max}=0.56$) is achieved for blade pitch angle ($\beta=0^\circ$) and optimum tip speed ratio ($\lambda_{\text{opt}}=9.8$). The wind turbine parameters used in this work are given in Table 1. The type of wind turbine model depends on the type of manufacturers like Suzlon, Gamesa, Vestas, Regen, Inox, Enercon and GE.

Table 1. Wind Turbine Data

Turbine Type	Threeblade horizontal axis
Radius	46 m
Gear ratio	1:103
Rotor Speed	18 rpm
Air density	1.225 kg/m ³
Cut-in wind speed	4 m/sec
Rated wind speed	approximately 12 m/s
Tower height	about 100 m

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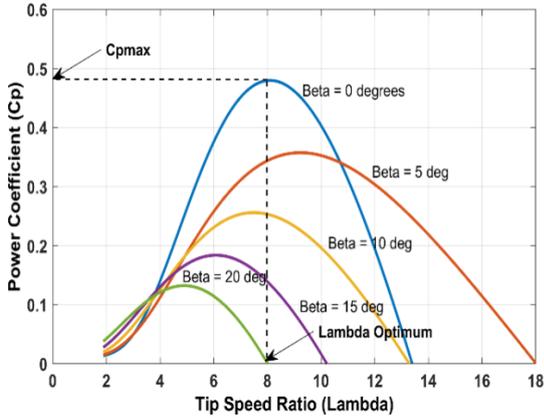


Fig.8. C_p - λ characteristics for different pitch angles

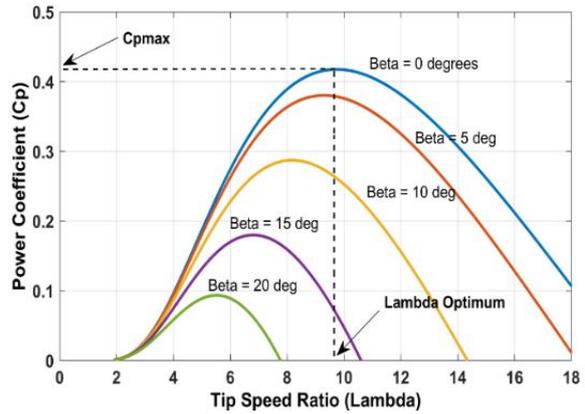


Fig.9. C_p - λ characteristics for different pitch angles

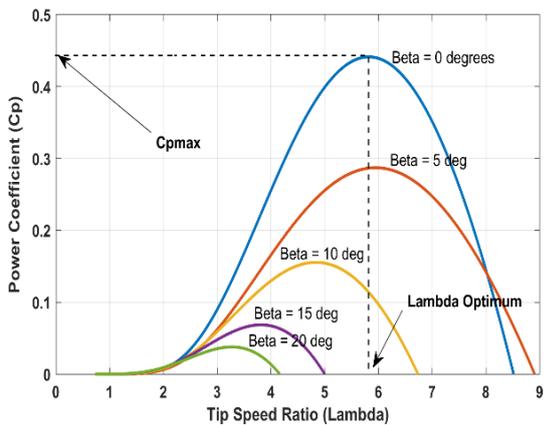


Fig.10. C_p - λ characteristics for different pitch angles

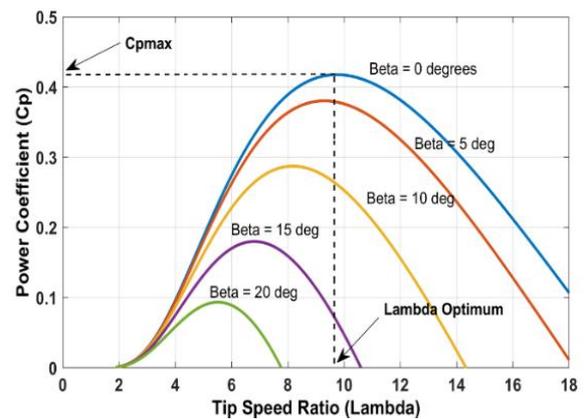


Fig.11. C_p - λ characteristics for different pitch angles

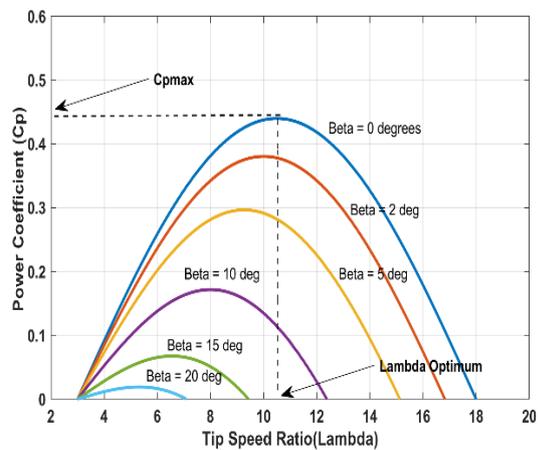


Fig.12. C_p - λ characteristics for different pitch angles

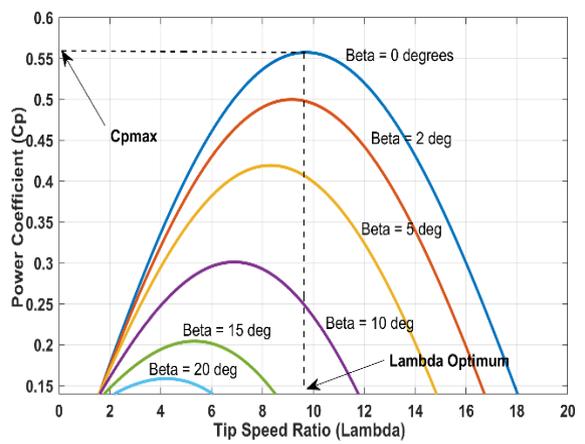


Fig.13. C_p - λ characteristics for different pitch angles

V. WIND TURBINE POWER OUTPUT AND ITS CONTROL

From “equation (12)”, the mechanical power of a wind turbine is determined by several factors. Among them are (i) Wind velocity, (ii) Blade radius (iii) Blade pitch angle (iv) Tip speed ratio (TSR), the tip speed ratio of a wind turbine is defined as the ratio of the blade tip speed to the speed of the undisturbed wind. There is only one TSR for the blade aerofoil under consideration for which the aerodynamic efficiency reaches its maximum value. Optimal TSR depends on the number of blades. (v) Angular mechanical speed (vi) Solidity, it describes the wind power plant rotor swept area that is filled with solid blades. Solidity is defined as the ratio of the projected blade area to the total swept area. (vii) Blade count, the determination of the number of blades involves design considerations of aerodynamic efficiency, component costs, system reliability, and aesthetics. A three-bladed wind turbine has become a more common choice of designers and investors (viii) Blade twist (ix) Blade taper, the higher tip speed ratio blades are generally possessed with a strong taper. To attain optimal usage of turbine output power, it is essential to select three design speeds: cut-in speed, rated speed and cut-out speed as shown in Fig.14.

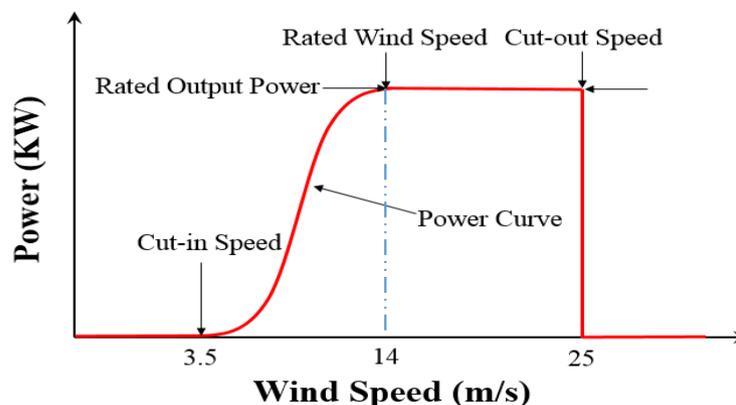


Fig.14. Typical wind turbine power curve showing three speeds

Cut-in speed: It is the minimum wind speed where wind turbine will generate utilizable power. This wind speed is normally between 3 m/s and 5 m/s for different turbines.

Rated wind speed: It is the minimum wind speed where wind turbine will generate rated power. It is in the range of 11 m/s to 16 m/s. At wind speeds between cut-in and rated, the output power from a wind turbine is proportional to the wind.

Cut-out speed: At high wind speeds between 17 m/s and 30 m/s, most wind turbines stop power generation and shut down. The wind speed at which shutdown occurs is called the cut-out speed. Operating at cut-out speed is a safety feature that guards the wind turbine against harm.

The turbine output power can be varied with the help of blade area and flow conditions. This forms the basis of the controller. Power coefficient (C_p) is achieved at a particular value of tip speed ratio (λ) which is definite to the design of the wind turbine. Hence, the simulation model [10] has been designed by using equation (2), power in the wind, equation (12), power harnessed from the wind, equation (19), the power coefficient and equation (16), the tip speed ratio of the turbine as shown in Fig.15.

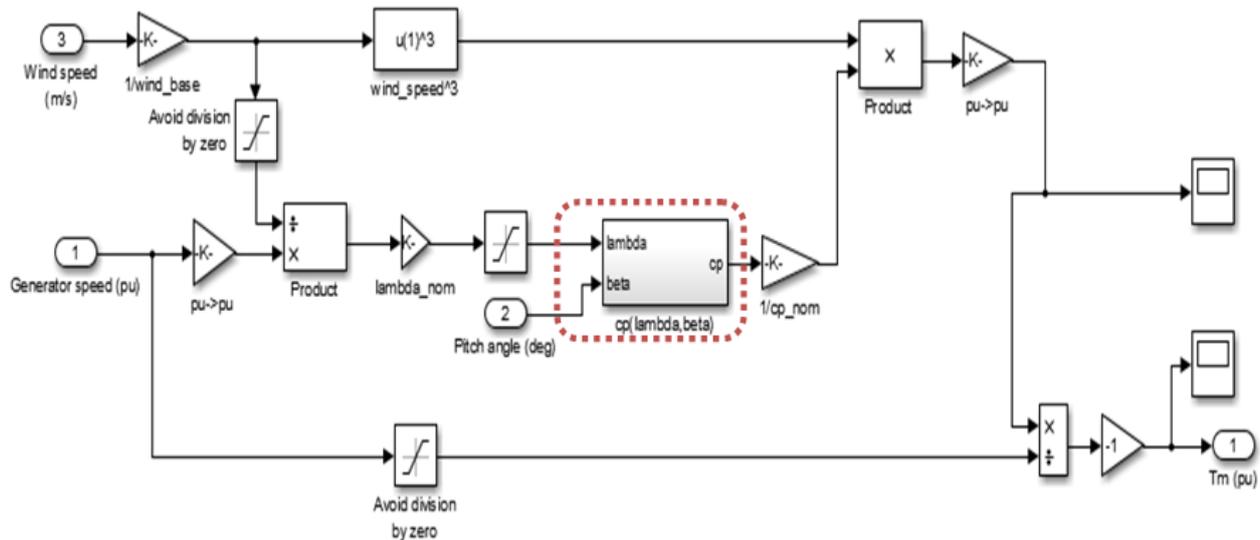


Fig.15 Simulation model for a wind turbine

Number of parameters lies in control of wind turbine output power. Fixed speed stall-regulated turbines have no options for control input. Variable speed wind turbines use generator torque to control and optimize power output. Pitch control and Stall control are two main technologies accomplished in controlling the power output from the blades of the turbine. Pitch control is based on the adjustment of the blades with the help of a control system. Stall control or Passive control is completely based on the aerodynamic properties of the blade. The usage of Pitch controller is that maintenance of rated output power of wind generator even wind speed is over the rated speed.

VI.CONCLUSION

In this paper, aerodynamic efficiency or power coefficient is used to describe the modelling of the wind turbine. The C_p - λ characteristics are useful to the wind turbine researchers and designers of new generation turbines. This effort helps the researchers to understand the mathematical modelling of the wind turbine and control of wind turbine's performance. Finally, the whole information is useful to optimize the design of wind turbine and decreases the cost of generation of wind energy.

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