



# **A Low Complexity Based Spectrum Monitoring in Broad Band Applications**

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**ABSTRACT:** A low complexity based spectrum monitoring in broad band applications consists of two modules. In the first module presents a spectrum monitoring algorithm for Orthogonal Frequency Division Multiplexing (OFDM) based cognitive radios by which the primary user reappearance can be detected during the secondary user transmission. The proposed technique reduces the frequency with which spectrum sensing must be performed and greatly decreases the elapsed time between the start of a primary transmission and its detection by the secondary network. In the second module, the statistical covariance of the received signal and noise are usually different, they can be used to differentiate the case where the primary user's signal is present from the case where there is only noise. Spectrum-monitoring algorithms are proposed based on the sample covariance matrix calculated from a limited number of received signal samples. Two test statistics are then extracted from the sample covariance matrix. A decision on the signal presence is made by comparing the two test statistics. Theoretical analysis for the proposed algorithms is given. Detection probability and the associated threshold are found based on the statistical theory. The methods do not need any information about the signal, channel, and noise power. In addition, no synchronization is needed. Simulations based on  $P_d$ , SNR with respect to noise, Multiple antenna received signals, Comparison between ED and CAV are presented to verify the method.

**KEYWORDS:** Cognitive Radio, Spectrum Sensing, Efficient Communication, primary transmission, secondary network, spectrum-monitoring, sample covariance matrix.

## **I.INTRODUCTION**

There has been tremendous interest in the field of cognitive radio (CR), which has been introduced. CR is an enabling technology that allows unlicensed (secondary) users to operate in the licensed spectrum bands. This can help to overcome the lack of available spectrum in wireless communications, and achieve significant improvements over services offered by current wireless networks. It is designed to sense the changes in its surroundings, thus learns from its environment and performs functions that best serve its users. This is a very crucial feature of CR networks which allow users to operate in licensed bands without a license. To achieve this goal, spectrum sensing is an indispensable part in cognitive radio.

There are three fundamental requirements for spectrum sensing. In the first place, the unlicensed (secondary) users can use the licensed spectrum as long as the licensed (primary) user is absent at some particular time slot and some specific geographic location. However, when the primary user comes back into operation, the secondary users should vacate the spectrum instantly to avoid interference with the primary user. Hence, a first requirement of cognitive radio is that the continuous spectrum sensing is needed to monitor the existence of the primary user. Also, since cognitive radios are considered as lower priority and they are secondary users of the spectrum allocated to a primary user, the second fundamental requirement is to avoid the interference to potential primary users in their vicinity. Furthermore, primary user networks have no requirement to change their infrastructure for spectrum sharing with cognitive radios. Therefore, the third requirement is for secondary users to be able to independently detect the presence of primary users. Taking those three requirements into consideration, such spectrum sensing can be conducted non-cooperatively (individually), in which each secondary user conducts radio detection and makes decision by itself. However, the sensing performance for one cognitive user will be degraded when the sensing channel experiences fading and shadowing. In order to



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improve spectrum sensing, several authors have recently proposed collaboration among secondary users, which means a group of secondary users perform spectrum sensing by collaboration. As the result, it shows that collaboration may enhance secondary spectrum access significantly. 3G cellular systems operating in 2 GHz band has promised data rates of at least 384 kbps for mobile and 2 Mbps for indoor applications. 4G systems are to yield about 20-40 Mbps. An IP based 4G systems which are considered to be an integration of 3G systems and wireless LAN (WLAN) systems, has promised more advanced services like enhanced multimedia, smooth video streaming, universal access and portability across all types of devices. Future wireless broadband applications like video conferencing and virtual reality would require data rates of hundreds of Mbps. The universal goal in all approaches towards 4G for achieving high data rates is increasing spectral efficiency using MIMO techniques.

Receiver diversity is used in present cellular mobile systems such as GSM, IS-136, etc. to gain certain benefits like improving quality and range of uplink. Though it is hard to locate more than two antennas in a small mobile handheld unit, it has been shown that transmit diversity can increase the channel capacity considerably. Error control coding can be combined with transmit diversity to achieve improved error performance of multiple antenna transmission systems and thus leads to coding gain advantage in addition to diversity benefit, at the cost of bandwidth expansion due to code redundancy. A joint design of error control coding, modulation and transmit diversity as a single block needs to use space-time codes, then it is possible to achieve coding gain as well as diversity benefit without bandwidth expansion. The combination of space-time codes with received diversity can further enhance the performance of multi-antenna system by minimizing multipath fading effect and help achieve the capacity of MIMO systems. Recent research in MIMO systems are shown that large capacity gains over wireless channels are possible using multiple antennas at both ends of the wireless channel.

## II.SYSTEM MODEL

Cognitive radio, which was first proposed in, is a promising technology for exploiting the under-utilized spectrum in an opportunistic manner. One application of cognitive radio is spectral reuse, which allows secondary networks/users to use the spectrum allocated/licensed to the primary users when they are not active. To do so, the secondary users are required to frequently perform spectrum sensing, i.e., detecting the presence of the primary users. If the primary users are detected to be inactive, the secondary users can use the spectrum for communications. On the other hand, whenever the primary users become active, the secondary users have to detect the presence of those users in high probability and vacate the channel within a certain amount of time.

One communication system using the spectrum reuse concept is the IEEE 802.22 wireless regional area networks, which operate on the very high-frequency/ultrahigh-frequency bands that are currently allocated for TV broadcasting services and other services, such as wireless microphones. Cognitive radio is also an emerging technology for vehicular devices. For example, cognitive radio is proposed for underwater vehicles to fully use the limited underwater acoustic bandwidth, and in, it is used for autonomous vehicular communications. spectrum sensing is a fundamental task for cognitive radio. However, there are several factors that make spectrum sensing practically challenging. First, the signal-to-noise ratio (SNR) of the primary users may be very low.

For example, the wire-less microphones operating in TV bands only transmit signals with a power of about 50 mW and a bandwidth of 200 kHz. If the secondary users are several hundred meters away from the microphone devices, the received SNR may be well below -20 dB. Second, multipath fading and time dispersion of the wireless channels make the sensing problem more difficult. Multipath fading may cause signal power fluctuation of as large as 20—30 dB. On the other hand, coherent detection may not be possible when the time-dispersed channel is un-known, particularly when the primary users are legacy systems, which do not cooperate with the secondary users. Third, the noise/interference level may change with time, which yields noise uncertainty.

There are two types of noise uncertainty: 1) receiver device noise uncertainty and 2) environment noise uncertainty. The receiver device noise uncertainty comes from the nonlinearity of components and the time-varying thermal noise in the components. The environment noise uncertainty may be caused by the transmissions of other users, either unintentionally or intentionally. Because of noise uncertainty, in practice, it is very difficult to obtain accurate noise power.



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There have been several sensing methods, including the likelihood ratio test (LRT), energy detection, matched filtering (MF)-based method, and cyclo-stationary detection method, each of which has different requirements and advantages/disadvantages. Although LRT is proven to be optimal, it is very difficult to use, because it requires exact channel information and distributions of the source signal and noise. To use LRT for detection, it needs to obtain the channel, signal and noise distributions first, which are practically intractable.

The MF-based method requires perfect knowledge of the channel responses from the primary user to the receiver and accurate synchronization (otherwise, its performance will dramatically be reduced). As mentioned earlier, this may not be possible if the primary users do not cooperate with the secondary users. The cyclo-stationary detection method needs to know the cyclic frequencies of the primary users, which may not be realistic for many spectrum reuse applications. Further-more, this method demands excessive analog-to-digital (A/D) converter requirements and signal processing capabilities.

Energy detection, unlike the two other methods, does not need any information of the signal to be detected and is robust to unknown dispersed channels and fading. However, energy detection requires perfect knowledge of the noise power. Wrong estimation of the noise power leads to an SNR wall and high probability of false alarm. As pointed out earlier, the estimated noise power could be quite inaccurate due to noise uncertainty. Thus, the main drawback for the energy detection is its sensitiveness to noise uncertainty. Furthermore, while energy detection is optimal for detecting an independent and identically distributed (i.i.d.) signal, it is not optimal for detecting a correlated signal, which is the case for most practical applications.

### III. COVARIANCE ABSOLUTE VALUE (CAV) DETECTION ALGORITHM

#### Covariance-Based Detections:

Let  $x_c(t) = s_c(t) + \eta_c(t)$  be the continuous-time received signal, where  $s_c(t)$  is the possible primary users signal and  $\eta_c(t)$  is the noise.  $\eta_c(t)$  is assumed to be a stationary process satisfying  $E(\eta_c(t)) = 0$ , and  $E(\eta_c^2(t)) = \sigma_\eta^2$ , and  $E(\eta_c(t)\eta_c(t+\tau)) = 0$  for any  $\tau \neq 0$ . Assume that it is interested in the frequency band with central frequency  $f_c$  and bandwidth  $W$  and sample the received signal at a sampling rate  $f_s$  where  $f_s \geq W$ . Let  $T_s = \frac{1}{f_s}$  be the sampling period. For notation simplicity,  $x(n) \triangleq x_c(nT_s)$ ,  $s(n) \triangleq s_c(nT_s)$ , and  $\eta(n) \triangleq \eta_c(nT_s)$  is defined. There are two hypotheses:

- 1)  $H_0$  i.e., the signal does not exist, and
- 2)  $H_1$  i.e., the signal exists. The received signal samples under the two hypotheses are given by

$$H_0 : x(n) = \eta(n) \tag{3.1}$$

$$H_1 : x(n) = s(n) + \eta(n) \tag{3.2}$$

respectively, where  $s(n)$  is the transmitted signal samples that passed through a wireless channel consisting of path loss, multipath fading, and time dispersion effects, and  $\eta(n)$  is the white noise, is which Independent and Identically

Distributed (i.i.d) having mean zero and variance  $\sigma_\eta^2$ . Note that

$$\hat{R}_x(N_s) = \begin{bmatrix} \lambda(0) & \lambda(1) & \dots & \lambda(L-1) \\ \lambda(1) & \lambda(0) & \dots & \lambda(L-2) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(L-1) & \lambda(L-2) & \dots & \lambda(0) \end{bmatrix} \tag{3.3}$$

can be the superposition of the received signals from multiple primary users. No synchronization is needed here.



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Two probabilities are of interest for spectrum sensing: 1) probability of detection  $P_d$  which defines, at hypothesis  $H_1$ , the probability of the sensing algorithm having detected the presence of the primary signal, and 2) probability of false alarm  $P_{fa}$  which defines, at hypothesis  $H_0$ , the probability of the sensing algorithm claiming the presence of the primary signal.

### CAV Detection:

Let us consider L consecutive samples and define the following vectors

$$x(n) = [x(n) \quad x(n-1) \quad \dots \quad x(n-L+1)]^T \quad (3.4)$$

$$s(n) = [s(n) \quad s(n-1) \quad \dots \quad s(n-L+1)]^T \quad (3.5)$$

$$\eta(n) = [\eta(n) \quad \eta(n-1) \quad \dots \quad \eta(n-L+1)]^T \quad (3.6)$$

Parameter L is called the smoothing factor in the following. Considering the statistical covariance matrices of the signal and noise defined as

$$R_x = E[x(n)x^T(n)] \quad (3.7)$$

$$R_s = E[s(n)s^T(n)] \quad (3.8)$$

$$\text{It can be verified by } R_x = R_s + \sigma_\eta^2 I_L \quad (3.9)$$

If signal  $s(n)$  is not present,  $R_s = 0$ . Hence, the off-diagonal elements of  $R_x$  are all zeros. If there is a signal and the signal samples are correlated,  $R_s$  is not a diagonal matrix. Hence, some of the off-diagonal elements of  $R_x$  should be non-zeros. Denote  $r_{nm}$  as the element of matrix  $R_x$  at the  $n^{\text{th}}$  row and  $m^{\text{th}}$  column, and let

$$T_1 = \frac{1}{L} \sum_{n=1}^L \sum_{m=1}^L |r_{nm}| \quad (3.10)$$

$$T_2 = \frac{1}{L} \sum_{n=1}^L |r_{nn}| \quad (3.11)$$

Then, if there is no signal,  $T_1 / T_2 = 1$ . If the signal is present,  $T_1 / T_2 > 1$ . Hence, ratio  $T_1 / T_2$  can be used to detect the presence of the signal.

In practice, the statistical covariance matrix can only be calculated using a limited number of signal samples. Define the sample autocorrelations of the received signal as

$$\lambda(l) = \frac{1}{N_s} \sum_{m=0}^{N_s-1-l} x(m)x(m-l) \quad l = 0, 1, \dots, L-1 \quad (3.12)$$

Where  $N_s$  is the number of available samples. Statistical covariance matrix  $R_x$  can be approximated by the sample covariance matrix defined as

$$\hat{R}_x(N_s) = \begin{bmatrix} \lambda(0) & \lambda(1) & \dots & \lambda(L-1) \\ \lambda(1) & \lambda(0) & \dots & \lambda(L-2) \\ \vdots & \vdots & & \vdots \\ \lambda(L-1) & \lambda(L-2) & \dots & \lambda(0) \end{bmatrix} \quad (3.13)$$

Note that the sample covariance matrix is symmetric. Based on the sample covariance matrix, signal detection method is proposed.



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**Algorithm:** Covariance Absolute Value (CAV) Detection Algorithm

Step 1) Sample the received signal, as previously described.

Step 2) Choose a smoothing factor  $L$  and a threshold  $\gamma_1$ , where  $\gamma_1$  should be Chosen to meet the requirement for the probability of false alarm.

Step 3) Compute the autocorrelations of the received signal  $\lambda(l), l = 0, 1, \dots, L - 1$ , and form the sample covariance matrix.

Step 4) compute

$$T_1(N_s) = \frac{1}{L} \sum_{n=1}^L \sum_{m=1}^L |r_{nm}(N_s)| \quad (3.14)$$

$$T_2(N_s) = \frac{1}{L} \sum_{n=1}^L |r_{nm}(N_s)| \quad (3.15)$$

Where  $r_{nm}(N_s)$  are the elements of the sample covariance matrix.

Step 5) Determine the presence of the signal based on  $T_1(N_s)$ ,  $T_2(N_s)$  and threshold  $\gamma_1$ . That is, if

$T_1(N_s) / T_2(N_s) > \gamma_1$ , the signal exists; otherwise, the signal does not exist.

## IV. SPECTRUM MONITORING USING MULTIPLE ANTENNAS

Multiple-antenna systems have widely been used to increase the channel capacity or improve the transmission reliability in wireless communications. In the following, it can be assumed that there are  $M > 1$  antennas at the receiver and exploit the received signals from the antennas for spectrum sensing.

In this case, the received signal at antenna  $i$  is given by

$$H_0 : x_i(n) = n_i(n) \quad (4.1)$$

$$H_1 : x_i(n) = S_i(n) + n_i(n) \quad (4.2)$$

$$X(n) = [x_1(n) \quad \dots \quad x_M(n) \quad x_1(n-1) \quad \dots \quad x_M(n-1) \quad \dots \quad x_1(n-L+1) \quad \dots \quad x_M(n-L+1)]^T \quad (4.3)$$

$$S(n) = [s_1(n) \quad \dots \quad s_M(n) \quad s_1(n-1) \quad \dots \quad s_M(n-1) \quad \dots \quad s_1(n-L+1) \quad \dots \quad s_M(n-L+1)]^T \quad (4.4)$$

$$\eta(n) = [\eta_1(n) \quad \dots \quad \eta_M(n) \quad \eta_1(n-1) \quad \dots \quad \eta_M(n-1) \quad \dots \quad \eta_1(n-L+1) \quad \dots \quad \eta_M(n-L+1)] \quad (4.5)$$

In hypothesis  $H_1$ ,  $S_i(n)$  is the signal component received by antenna  $i$ . Since all  $S_i(n)$ 's is generated from the

same source signal, the  $S_i(n)$ 's are correlated for  $i$ . It is assumed that the  $n_i(n)$ 's are Independent and Identically

Distributed (i.i.d) for  $n$  and  $i$ . Let us combine all the signals from the  $M$  antennas and define the vector  $\sin$  (4.3)–(4.5). a special case ( $M = 1$ ) of the preceding equations. Defining the statistical covariance matrices in the same way as those in (3.7) and (3.8), it can be form as

$$R_x = R_s + \sigma_n^2 I_{ML} \quad (4.6)$$

Except for the different matrix dimensions, the preceding equation is the same as (3.9). Hence, the CAV algorithm and generalized covariance-based method previously described can directly be used for the multiple-antenna case. Let

$S_o(n)$  be the source signal. The received signal at antenna  $i$  is

$$S_i(n) = \sum_{k=0}^{N_i} h_i(k) S_o(n-k) + n_i(n), \quad i = 1, 2, \dots, M \quad (4.7)$$

Where  $h_i(k)$  is the channel responses from the source user to antenna  $i$  at the receiver.  $h(n)$  is Defined as



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$$h(n) = [h_1(n), h_2(n), \dots, h_M(n)]^T \quad (4.8)$$

$$H = \begin{bmatrix} h(0) & \dots & \dots & h(n) & \dots & o \\ & \ddots & & & \ddots & \\ o & \dots & h(o) & \dots & \dots & h(N) \end{bmatrix} \quad (4.9)$$

Where  $N = \max_i(N_i)$ , and  $h_i(n)$  is zero padded if  $N_i < N$ . Note that the dimension of H is  $ML \times (N + L)$  and  $R_s$  taken as

$$R_s = HR_{s_o}H^T \quad (4.10)$$

Where  $R_{s_o} = E(\hat{S}\hat{S}^T)$  is the statistical co-variant matrix of the source signal, where

$$\hat{S}_o = [S_o(n) \quad S_o(n-1) \quad \dots \quad S_o(n-N-L+1)]^T \quad (4.11)$$

Note that the received signals at different antennas are correlated. Hence, using multiple antennas, increase the correlations among the signal samples at the receiver and make the algorithms valid at all cases. In fact, at worst case, when all the channels are flat fading, i.e.,  $N_1 = N_2 = \dots = N_M = 0$ , and the source signal sample  $S_o(n)$  is Independent and Identically Distributed (i.i.d) and the  $R_s$  taken as  $R_s = \sigma_s^2 HH^T$ , where H is an  $ML \times L$  matrix, as previously defined. Obviously,  $R_s$  are not a diagonal matrix and the algorithms can work.

## V. RESULT AND DISCUS ION

In this paper, the simulation results are given for three situations  $P_d$ , SNR with respect to noise, Multiple antenna received signals, Comparison between ED and CAV. The probabilities of false alarm  $P_{fa}$  is simulated, because  $P_{fa}$  is not related to the signal. (At  $H_0$ , there is no signal at all.) The target is set to  $P_{fa} = 0.1$  and choose  $L = 10$  and  $N_s = 50000$ . Then the thresholds obtained based on  $P_{fa}$ ,  $L$ , and  $N_s$ . The  $P_{fa}$ 's for the proposed method and energy detection without noise uncertainty meet the target, but the Pfa for the energy detection with noise uncertainty (even as low as 0.5 dB) far exceeds the limit. This means that the energy detection is very unreliable in practical situations with noise uncertainty.

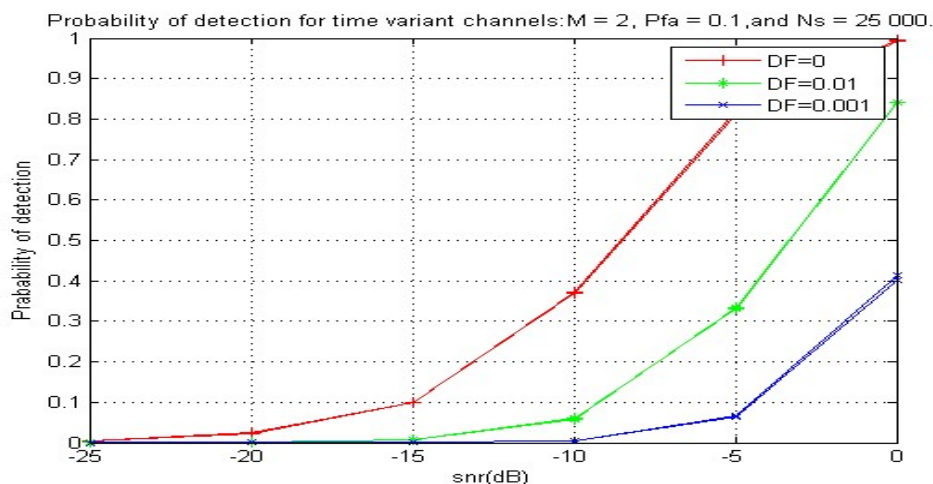


Figure 1:  $P_d$ , SNR with respect to noise

The relationship between probability of detection ( $P_d$ ) and signal to noise ratio with respect to variable DF which is nothing but noise with limited number of samples is observed in fig 1. In this noise considered as secondary signal. The

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main objective of this is to increase the transmitting or receiving capabilities of secondary user. It is shown in the fig 1. The probability of detection gives the information about primary signal i.e. primary user present in the spectrum or not. When noise increases the  $P_d$  decreases with respect to the SNR and it can increase the scope for the secondary user.

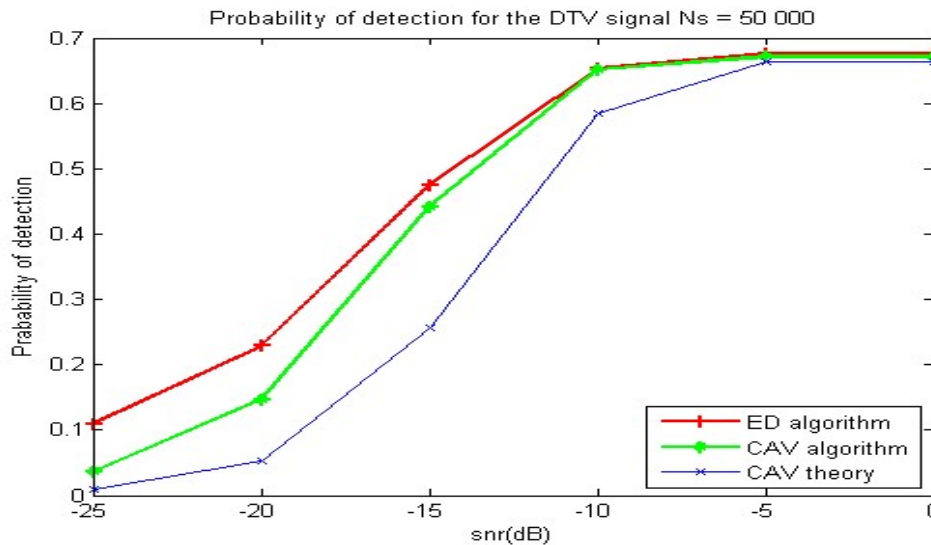


Figure 2: Comparison between ED and CAV

In fig 2 comparison between energy detection algorithm and covariance absolute value algorithm depending on the relationship between probability of detection ( $P_d$ ) and signal to noise ratio is observed. The difference in the probability of detection in both the algorithms are observed. Finally, and it can be concluded that the CAV algorithm is better than ED algorithm in both cases i.e. limited number of samples and increase scope of secondary user.

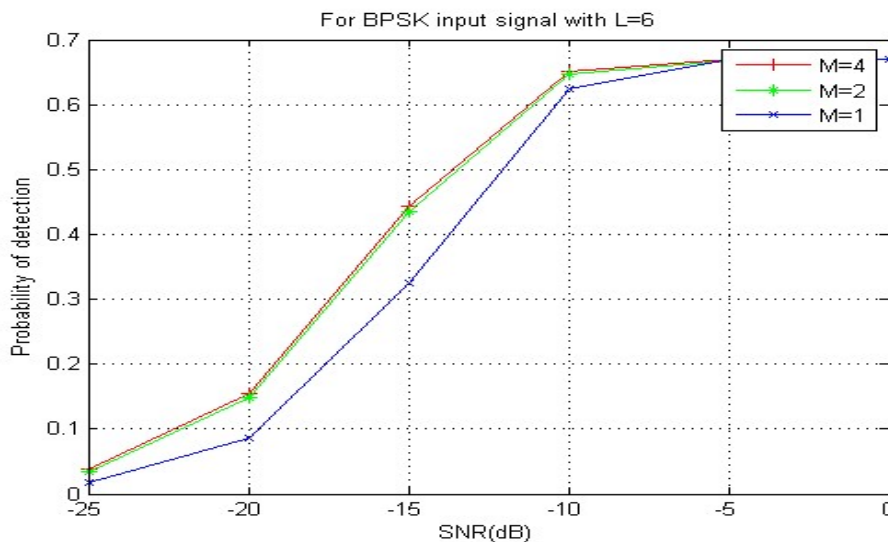


Figure 3: Multiple antenna received signals Probability of detection for time variant channels = 2,  $P_{fa} = 0.1$  and  $N_s = 25000$



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In fig 3 the effect in probability of detection because of increasing number of transmitting and receiving antennas are observed and observe the relationship between probability of detection ( $P_d$ ) and signal to noise ratio for different values of  $M$ .

CAV detection is better than ideal energy detection (without noise uncertainty), which verifies our assertion. The reason is that, the source signal is a narrow-band signal. Therefore, their samples are highly correlated. The  $P_d$  is not very sensitive to the smoothing factor for  $L \geq 8$  and a smaller  $L$  means lower complexity, in practice, it be can chose a relatively small  $L$ . However, it is very difficult to choose the best  $L$ , because it is related to the signal property (unknown). Note that energy detection is not affected by  $L$ . The multipath channel and SNR of the received signal are unknown. To use the signals for simulating the algorithms at a very low SNR, we need to add white noise to obtain various SNR levels.

## VI.CONCLUSION

Monitoring algorithms based on the sample covariance matrix of the received signal have been proposed. Statistical theories have been used to set the thresholds and obtain the probabilities of detection. The methods can be used for various signal detection applications without knowledge of the signal, channel, and noise power. Simulations based on the narrow-band signals, captured DTV signals, and multiple antenna signals have been carried out to evaluate the performance of the proposed methods. It is shown that the proposed methods are, in general, better than the energy detector when noise uncertainty is present. Furthermore, when the received signals are highly correlated, the proposed method is better than the energy detector, even if the noise power is perfectly known.

## REFERENCES

1. C. Han, J. Wang, S. Gong and S. Li, "Detection and Performance of OFDM-based TDCSI, International Conference on Communications, Circuits and Systems, Vol. 2, No. 6, pp. 1332-1336, 2006.
2. F.F. Digham, M.S. Alouini and M.K. Simon, "Energy Detection of unknown signals over fading channels", IEEE Transactions on Communications, Vol. 5, No.1, pp. 21-24, 2007.
3. V. I. Kostylev, "Energy detection of a signal with random amplitude", IEEE International Conference on Communications, Vol. 3, No. 4, pp. 1606-1610, 2002.
4. T. S. Shehata and M. T. L. Tanany, "A Novel Adaptive Structure of The Energy Detector Applied to the Cognitive Radio Networks", 11th Canadian Workshop on information Theory, No. 5, pp.95-98, 2009.
5. N. Reisi, M. Ahmadian and S. Salari, "Performance Analysis of Energy Detection based Spectrum Sensing over Fading Channels", 6th International Conference on Wireless Communications Networking and mobile Computing, No. 9, pp. 1-4, 2010.
6. S. Atapattu, C. Tellambura and H. Jiang, "Analysis of Area under ROC curve of energy detection", IEEE Transactions On Wireless Communications, Vol. 9, No.3, pp. 1216-1225, 2010.
7. R. Tandra and A. Sahai, "SNR Walls for Energy Detection", Vol. 2, No. 1, pp. 4-17, 2008