



# Stabilization of Inverted Pendulum System using Fuzzy logic Controller

Vivek Kumar Pathak<sup>1</sup>, Sankalp Paliwal<sup>2</sup>

PG Student [PS&C], Dept. of EE, Babu Banarasi Das University, Lucknow, Uttar Pradesh, India<sup>1</sup>

Assistant Professor, Dept. of EE, Babu Banarasi Das University, Lucknow, Uttar Pradesh, India<sup>2</sup>

**ABSTRACT:** The inverted pendulum problem is one of the most important problems in control theory and has been studied excessively in control literatures. A proportional integral derivative (PID) controller is the most commonly used controller in controlling industrial loops .

Tuning a PID controller is an important task This Paper present the intelligent methods based on fuzzy logic for tuning PID controller. Simulation results reveals that intelligent methods provide better performance than the conventional methods.

**KEYWORDS:** Inverted pendulum, fuzzy PID, intelligent control.

## I.INTRODUCTION

The inverted pendulum system has been considered as well as prototype system of representing nonlinear systems for testing control algorithms. A single input to the inverted pendulum system has to control both the pendulum angle and the cart position simultaneously It is well established benchmark problem that provides many challenging problems to control design. The system is nonlinear, unstable, non minimum phase and underactuated. Because of their nonlinear nature pendulums have maintained their usefulness and they are now used to illustrate many of the ideas emerging in the field of nonlinear control [1]. The challenges of control made the inverted pendulum system classic tool in control laboratories.

According to control purposes of inverted pendulum, the control of inverted pendulum can be divided into three aspects. The first aspect that is widely researched is the swing-up control of inverted pendulum [2,3]. The second aspect is the stabilization of the inverted pendulum [4–6]. The third aspect is tracking control of the inverted pendulum [7,8]. In practice, stabilization and tracking control is more useful for application. It is rather surprising that virtually almost all the technical literature refers to the inverted pendulum with one freedom. Only recently there are a few references dealing with the inverted pendulum with two or three degrees of freedom [7–10].

Ref. [13] applied interval type-2 fuzzy sliding-mode controller in the inverted pendulum. But the model of the inverted pendulum did not consider the dynamic of the cart of the inverted pendulum. Ref. [14] applied coupled sliding-mode control to orbital stabilization of inverted pendulum systems. In Ref. [15], the proposed methodology, which performs swing up and control simultaneously, uses elements from input–output linearization, energy control, and singular perturbation theory.

The organization of this paper is as follows. Section 2 will introduce the structure and models of x inverted pendulum. .In Section 3, we will give the design procedure of the PID controllers for stabilizing inverted pendulum. Simulation results for inverted pendulum controlled by PID controllers are discussed in section 4. Section 5 gives the conclusions of the paper.

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

## II. STRUCTURE AND MATHEMATICAL MODEL OF INVERTED PENDULUM

The x inverted pendulum on a pivot driven by horizontal control force is shown in fig 1(a) the control based on the horizontal displacements  $x$  inverted pendulum are the total kinetic energy and potential energy .  $K=1/2MX_2+1/2(X_p^2+Z^2)$ ,  $Z= mgz_p$  Let  $l$  is the distance from the pivot to the mass center of the pendulum  $M, m$  are the pivot and the pendulum  $(x, z)$  respectively  $(x_p, z_p)$  is the position of the pivot in the  $xoz$  coordinate is the speed in the  $xoz$  coordinate  $(x_p, z_p)$  and angle  $\theta$  and  $F_x$  is the horizontal control force.

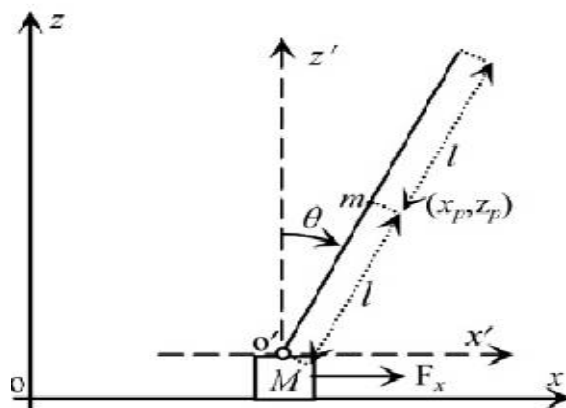


Figure -1

Inverted pendulum system free-body diagrams of the two elements

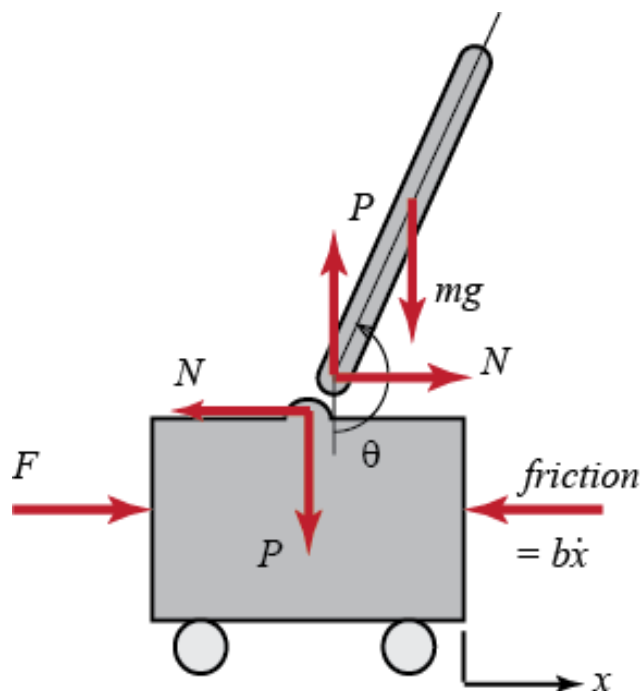


Figure -2



# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

free-body diagram of the cart in the horizontal direction Summing the forces you get the following equation of motion

$$Mx + bx + N = F \quad (1)$$

forces in the free-body diagram of the pendulum in the horizontal direction, you get the following expression for the reaction force  $N$ .

$$N = mx + ml\theta\cos\theta - ml\theta^2\sin\theta \quad (2)$$

One of the two governing equations for this system is obtained if this equation is substituted into the first equation

$$(M+m)x + bx + ml\theta\cos\theta - ml\theta^2\sin\theta = F \quad (3)$$

To get the second equation of motion for this system, sum the forces perpendicular to the pendulum

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml^2\ddot{\theta} + mx\cos\theta \quad (4)$$

To get rid of the  $P$  and  $N$  terms in the equation above, sum the moments about the centroid of the pendulum to get the following equation

$$-P\sin\theta - N\cos\theta = I\ddot{\theta} \quad (5)$$

Combining these last two expressions, the second governing equation is obtained

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -mlx\cos\theta \quad (6)$$

Let  $\phi$  represent the deviation of the pendulum's position from equilibrium, that is,  $\theta = \pi + \phi$ . Again presuming a small deviation ( $\phi$ ) from equilibrium, we can use the following small angle non linear function

$$\cos\theta = \cos(\pi + \phi) = -1 \quad (7)$$

$$\sin\theta = \sin(\pi + \phi) = -\phi \quad (8)$$

$$\theta^2 = \phi^2 = 0 \quad (9)$$

After substituting the above approximations into our nonlinear governing equations, the two linearized equations of

Motion are obtained. Note  $u$  has been substituted for the input  $F$ .

$$(I + ml^2)\ddot{\phi} - mgl\sin\phi = -mlx \quad (10)$$

$$(M+m)x + bx - ml\phi = u \quad (11)$$

### III. CONTROLLER DESIGN

A proportional–integral–derivative controller (PID controller) is a control loop feedback mechanism (controller) commonly used in industrial control systems. A PID controller continuously calculates an error value  $e(t)$  (difference between a desired set point and measured process variable) and applies a correction based on proportional, integral and derivative terms (sometimes denoted P, I, and D respectively) which give their name to the controller type.

$$u(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{de(t)}{dt} \quad (12)$$

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

In this paper two tuning methods for PID controller have been used.

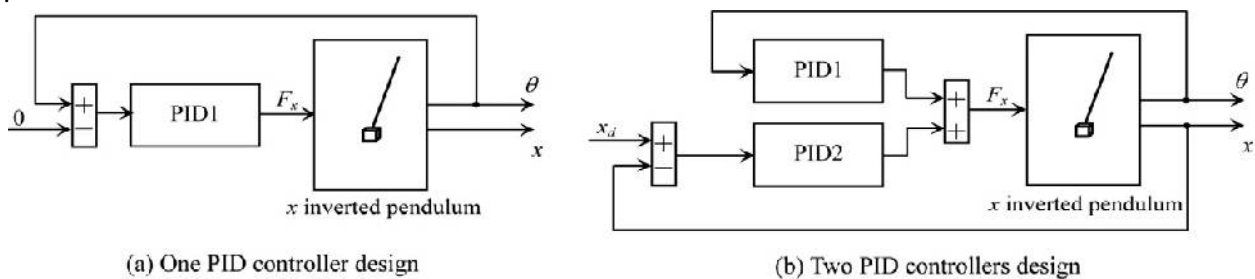
**4.1. TRAIL ERROR METHOD:** - In the first step, we design one PID controller of the  $x$  inverted pendulum for The angle control the pivot position control problem. There are lots of control applied to the inverted pendulum angle control without consider the dynamics of the pivot. In this step, the goal of the control PID1 controller design is to stabilize the angle of the  $x$  inverted pendulum. The parameters of PID1 controller of the inverted pendulum are given as following

$$\text{PID1: } P_1 = 85, I_1 = 80, D_1 = 20$$

**4.1.1 The second step with two PID controllers design:**

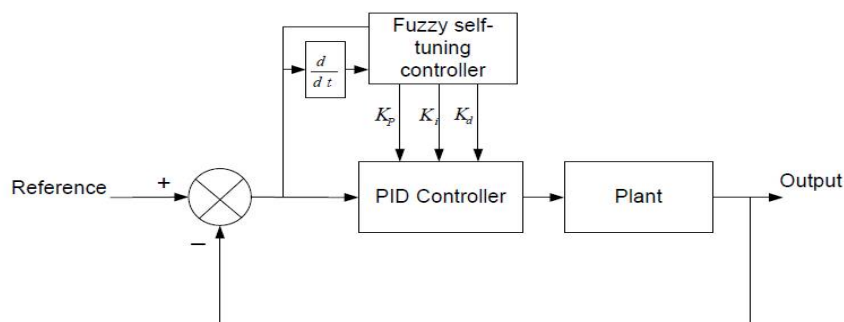
Based on the PID1 controller designed in the first step, we add PID2 controller to control the position of the pivot. The control structure is given in the Fig. 3(b). In this step, PID1 need not change any more. We can adjust the parameters of PID2. The parameters of PID2 controller of the inverted pendulum are given as follows

$$\text{PID2 : } P_2 = -19, I_2 = -19, D_2 = -10$$



**Figure-3**  
**Structure of  $x$  inverted pendulum.[10]**

**4.2. Fuzzy PID CONTROLLER:** The fuzzy PID controller has been implemented using fuzzy logic. It is a closed loop system the reference signal is taken, fuzzy self tuning controller is governed by two inputs & three out puts. Inputs used are error & derivative of error .outputs are parameters of PID is  $K_p$ ,  $k_i$ , &  $k_d$  these parameter are given to PID controller and further controlling of system is done. Fuzzy self tuning controller if we do not take an account of fuzzy self tuning controller then.(1) our output will show non linearity .(2) It will be affected by time lagging or time delay.



**Figure-4**

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

## BLOCK DIAGRAM FUZZY PID CONTROLLER [11]

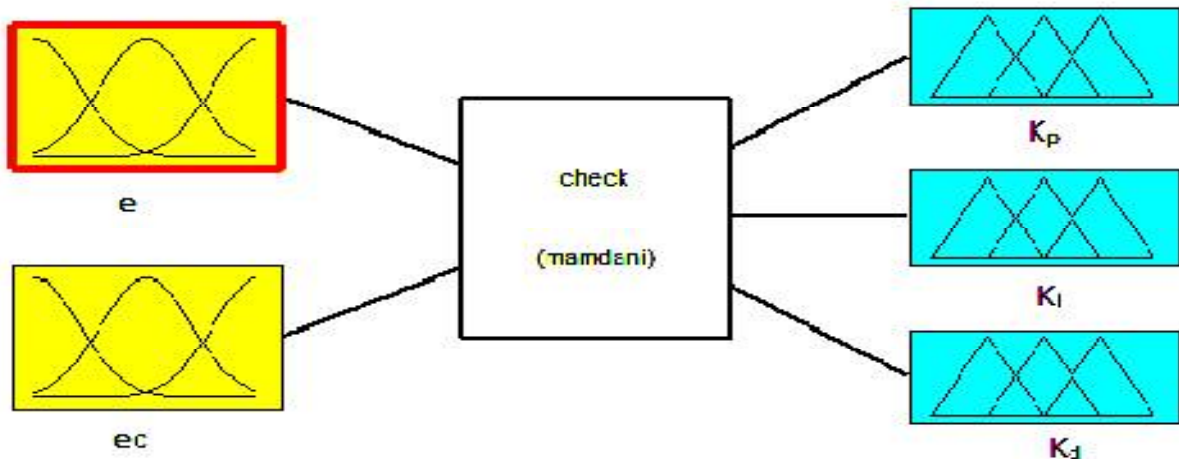


Figure- 5  
Two input three output FLC structure

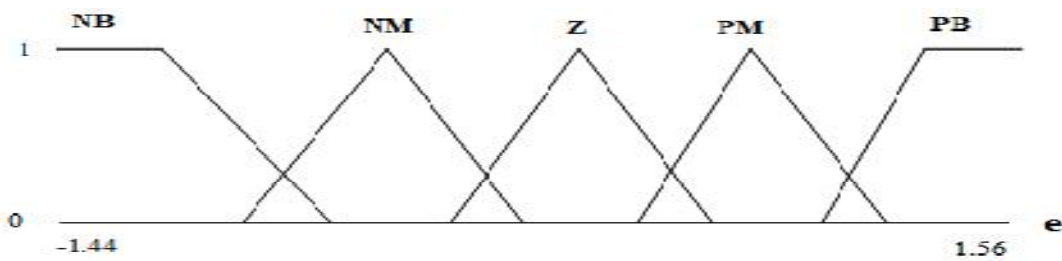


Figure- 6 (A)

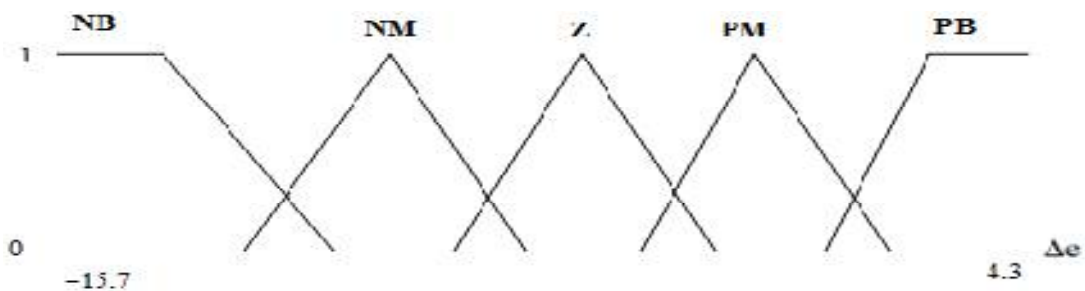


Figure- 6 (B)

Input fuzzy sets (a) error (e) (b) Change of error ( $\Delta e$ )

The output membership functions are shown in Figure 4. For the output fuzzy sets the scaling of range has been done corresponding to the formulas

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

$$K'_p = \frac{K_p - K_{pmin}}{K_{pmax} - K_{pmin}} \quad (13)$$

$$K'_i = \frac{K_i - K_{imin}}{K_{imax} - K_{imin}} \quad (14)$$

$$K'_d = \frac{K_d - K_{dmin}}{K_{dmax} - K_{dmin}} \quad (15)$$

The minimum and maximum values of various gain have been obtained by analysing the step response using trail error method.

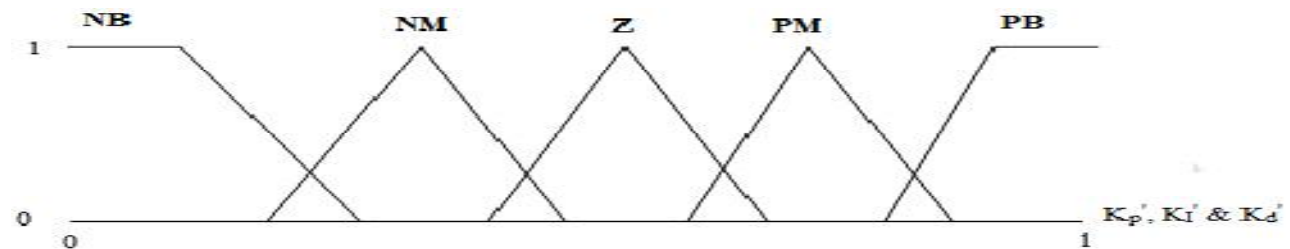


Figure 6(C)

Output fuzzy sets  $K_p, K_i, K_d$

The rule base for the fuzzy-PID controller is shown in Table1 which can be implemented for tuning the PID Controller.

Input fuzzy sets (a) error (e) (b) Change of error ( $\Delta e$ )

The rule base is defined as follows:

IF error e is NB and derivative of error de/dt is also NB then output variable i.e.  $K_p', K_i, K_d$  will be NB and if e is Z and derivative of error de/dt is NS then output will NS and so on.

e \ - $\Delta e$	NB	NM	Z	PM	PB
NB	NB	NB	NM	NM	Z
NM	NB	NM	NS	Z	PM
Z	NM	NM	Z	PM	PM
PM	NM	Z	PM	PM	PB
PB	Z	PM	PM	PB	PB

Table -1 Rule base for fuzzy-PID controller [11]

## IV. RESULT AND DISCUSSION

### 4.1 Result using trial and error technique

In trial and error two loop controller scheme is used for controlling x-inverted Pendulum. In this 1<sup>st</sup> design PID-1 for controlling the angle then we design position controller for tracking of inverted pendulum. In this method the PID parameters are obtained by hit and trial. First PID1 is designed for controlling the position  $x$  and then the second controller PID2 is designed for controlling the angle. In figure 7 results of angle controller for trial and error are shown.

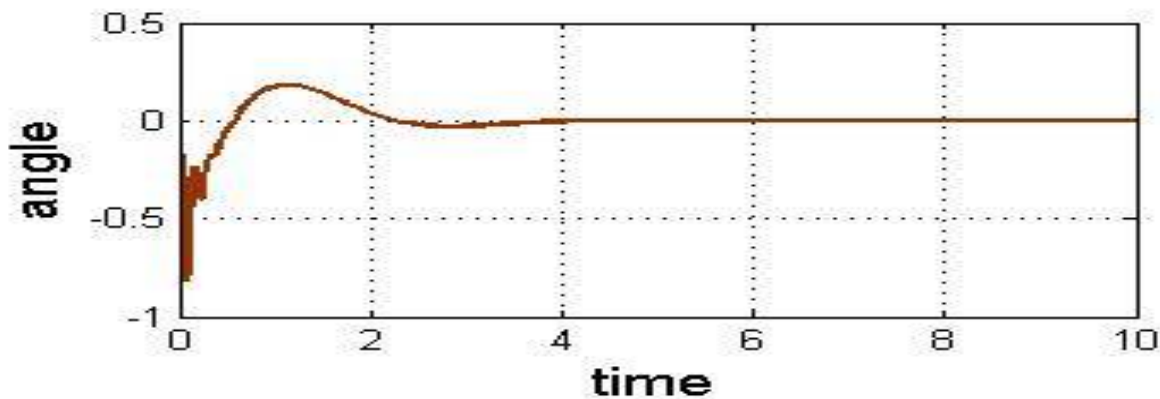


Figure 7 Angle controller for trial and error

In figure 8 results of position controller for trial and error are shown.

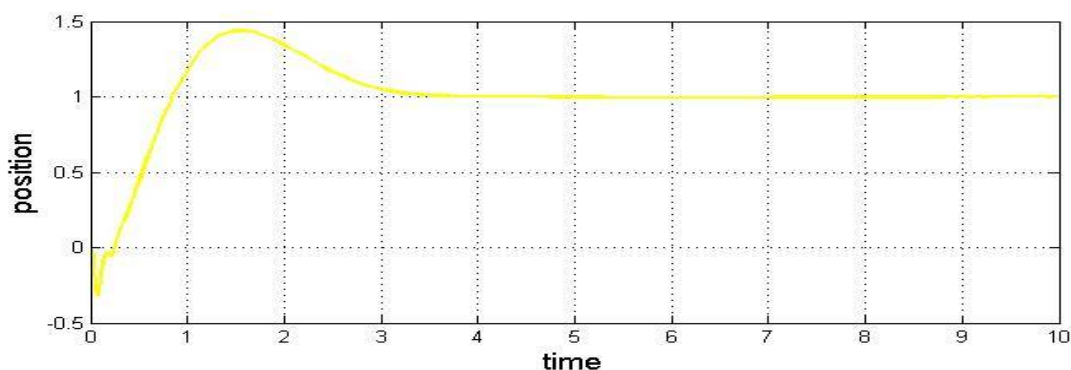


Figure 8 position controller

**5.2 Result using fuzzy PID control and technique:** The fuzzy PID controller has been implemented using fuzzy logic. It is a closed loop system the reference signal is taken, fuzzy self tuning controller is governed by two inputs & three out puts. Inputs used are error & derivative of error .outputs are parameters of PID is  $K_p$  ,  $k_i$  , &  $k_d$  these parameter are given to PID controller and further controlling of system is done. In figure 9 results of position controller for fuzzy PID are shown

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

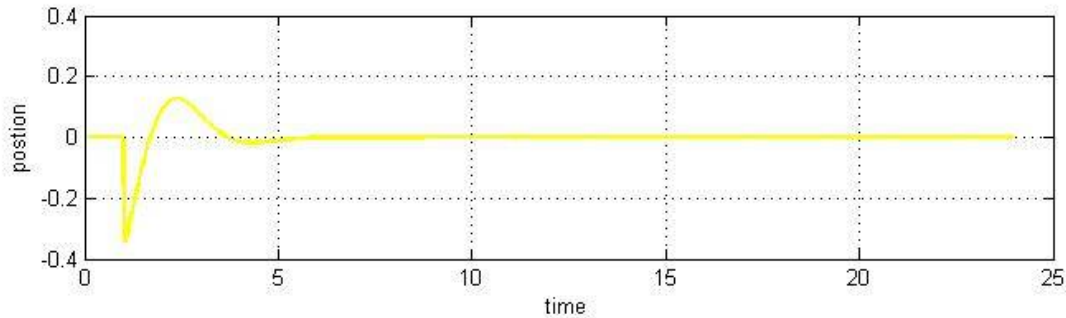


Figure (9) position controller

In figure 10 results of angle controller for fuzzy PID are shown

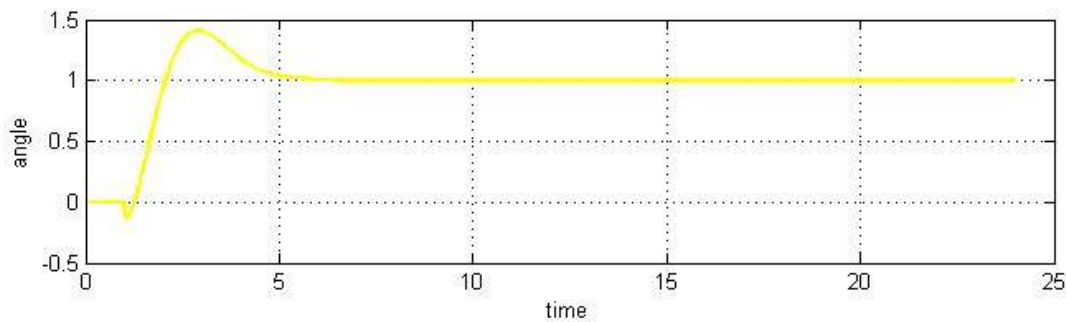


Figure (10) angle controller

In table (2) performance parameters of Position controller & angle controller of Fuzzy PID and trail and error are compared .

<b>Position controller &amp; angle controller</b>		
s.no	PID	FUZZY PID
Rise time $t_r$	1.29	1.84
Setling time $t_s$	3.45	5.2
Overshoot $M_p$	43%	30%
Under shoot up	32%	5%
s.no	PID	FUZZY PID
Rise time	.54	0.7 Sec
Setling time $t_s$	3.2	3.8sec
Over shoot $M_p$	18.2%	12%
Under shoot up	80%	18.2%

Table 2 Performance parameters for both tuning techniques





# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: [www.ijareeie.com](http://www.ijareeie.com)

Vol. 6, Issue 4, April 2017

## V.CONCLUSION

The stabilization of inverted pendulum system has done .using PID controllers tuned by trail error techniques are also by using fuzzy logic controllers . The result have been shown the conclusion which can be drawn from the results is that the parameter of fuzzy PID controller is better than PID controller tuned by trail error techniques for position tracking and angle stabilization in terms of overshoot and undershoot.

## REFERENCES

- [1] K.J. Astrom, K. Furuta, "Swinging up a pendulum by energy control", Automatica– 36 2,287-295, 2000
- [2] P. Mason, M. Broucke, B. Piccoli, "Time optimal swing-up of the planar pendulum", IEEE Transactions on Automatic Control 53 8 , 1876–1886, 2008
- [3] C.W. Tao, J.S. Taur, T.W. Hsieh, C.L. Tsai, "Design of a fuzzy controller with fuzzy swing-up and parallel distributed pole assignment schemes for an inverted pendulum and cart system", IEEE Transactions on Control Systems Technology, 16 6, 1277–1288, 2008
- [4] R. Shahnazi, T.M.R. Akbarzadeh, "PI adaptive fuzzy control with large and fast disturbance rejection for a class of uncertain nonlinear systems", IEEE Transactions on Fuzzy Systems 16 1, 187–197,2008
- [5] A.M. Bloch, N.E. Leonard, J.E. Marsden, "Controlled lagrangians and the stabilization of mechanical systems I: the first matching theorem", IEEE Transactions on Automatic Control 45 12, 2253–2270, 2000
- [6] N.A. Chaturvedi, N.H. McClamroch, D.S. "Bernstein, Stabilization of a 3D axially symmetric pendulum", Automatica 44 9, 2258–226, 2008
- [7] R.J. Wai, L.J. Chang, "Adaptive stabilizing and tracking control for a nonlinear inverted-pendulum system via sliding-mode technique", IEEE Transactions on Industrial Electronics 53 2,674–692, 2006
- [8] K. J. Astrom and T. Hagglund: "PID Controllers: Theory, Design and Tuning", Instrument Society of America, 2 edition, 1995
- [9] B. Wayne Bequette: "Process Control: Modeling, Design and Simulation", Prentice Hall, January 2003
- [10] J.-J. Wang "Simulation studies of inverted pendulum based on PID controllers", simulation modeling Practice and Theory 19, 440–449, 2011
- [11] Vikram Chopra, Sunil K. Singla, Lillie Dewan; "Comparative Analysis of Tuning a PID Controller using Intelligent Methods"; Acta Polytechnica Hungarica Vol. 11, No. 8, 236-249, 2014
- [12] K. Sinthipsomboon,W. Pongaen & P. Pratumswan: "A Hybrid of Fuzzy and Fuzzy self-tuning PID Controller for Servo Electro-hydraulic" System, 6th IEEE Conference on Industrial Electronics and Applications, , pp. 220-225, 2011
- [13] Zulfatman and M. F. Rahmat: "Application of Self Tuning Fuzzy PID Controller On Industrial Hydraulic Actuator Using System Identification Approach", International Journal On Smart Sensing and Intelligent Systems, 2. 2, 246-261, June 2009