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Pitch Control of Aircraft Using LQR & LQG Control

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ABSTRACT: The paper presents the control of aircraft in the longitudinal plane during cruising stage using Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) Control. For the design of aircraft system, the linearized longitudinal dynamics is used. Simulation results for controlling the pitch angle is presented in time domain. The performance areanalyzed and investigated based on common criteria of step response in order to identify which control strategy delivers better performance for a desired pitch angle in the presence and absence of noise.

KEYWORDS: Aircraft, Longitudinal dynamics, Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG) Control.

I.INTRODUCTION

The rapid advancement of many technologies has contributed to the development of aircraft design from the very limited capabilities of the Wright brothers, first successful airplane to today’s high performance military, commercial andgeneral aviation aircraft. Development has made in its aerodynamics, structures, materials used, propulsion system and flight control systems. Modern aircraft have a variety of automatic control system [1], [2] that helps the flight crew in navigation, augmenting the stability characteristicand flight management of the airplane. In the early days of aviation, in order to fly safely aircraft requires the continuous attention of a pilot. As aircraft range increases allowing many hours of flight, the constant attention of pilot may lead to serious fatigue. After many years of advancing technology aircraft soon adapt the concept of autopilot and it is designed to perform some tasks of the pilot [9]. The first aircraft autopilot was developed by Sperry Corporation in 1912. The autopilot is also called as a pilot assistant, it assist the pilot during long journey flight. It permits the aircraft to fly straight and on a level course without a pilot's attention, thereby it greatly reduces the pilot's workload.

The paper focuses on the design of an autopilot that controls the pitch of aircraft,which can be used by the flight crew to lessen their workload during cruising stage. In the longitudinal plane, elevator controls the pitch of aircraft. Elevators are also called longitudinal controls. By moving the elevator control backwards, the rear of elevator deflects upwards and tail is given a negative camber resulting in a downward force which causes pitch down of the tail and pitch up of the airplane. Similarly, by moving the elevator control forwards, the rear of elevator deflects downwards and tail is given a positive camber resulting in an upward force which causes pitch up of the tail and pitch down of the airplane. In early days of aviation classical controllers were used but the main disadvantage of using classical controllers is that they have limited capability of disturbance handling [9]. The paper presents the design of an autopilot that controls the pitch of the aircraft using LQR and LQG control, during cruising stage. The performance is analyzed based on common criteria of step response for a desired pitch angle. Performance of both LQR and LQG control strategy with respect to the pitch angle and pitch rate are examined in the presence and absence of disturbance.

II.SYSTEM MODEL AND ASSUMPTIONS

The forces and moments acting on the aircraft are shown in Fig.1. The applied forces and moments on the aircraft and the resulting response of the aircraft are described by a set of equations known as equations of motion [1], [7]. The forces acting on the airplane includes gravitational force, thrust forces and aerodynamic forces [9]. The gravitational force acts through CG and therefore does not contribute any moment about CG. The thrust force and aerodynamic force also can be

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assumed to act at CG and hence the moment contribution is taken as zero. In order to reduce the complexity of analysis, the six equations of motion can be decoupled into two sets of three equations each, namely the longitudinal and lateral equations. For controlling the pitch of an aircraft, it is necessary to use only the longitudinal equations of motion. Longitudinal motion consists of those movements where the aircraft would only move within the x-z plane that is, translation in x and z directions and rotation about y axis. The three longitudinal equations of motion consist of X-force, Z-force and Y- moment equations respectively.

$$X - mg\ sin\Theta = m(\dot{u} - rv + qw \quad (1)$$
$$M = I_y \dot{q} + I_{xz}(p^2 - r^2) + rp(I_x - I_z) \quad (2)$$
$$Z + mg\ cos\Theta\ cos\phi = m(\dot{w} + pv - qu) \quad (3)$$

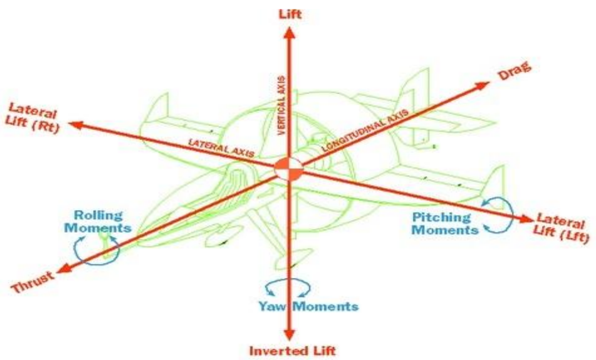

Figure 1. Forces and moments on aircraft

$X$and$Z$ are the total aerodynamic and propulsive forces acting in the x and z directions. u, v, w-Linear velocities along x, y and z axes and p, q, r- components of angular velocity along x, y and z axes respectively i.e. roll rate, pitch rate and yaw rate respectively, $w = pi + qj + rk$. The pitch control system is shown in Fig.2, where X_b, Y_b and $Z_b$represent the aerodynamics force components. Θ, ϕ andδ_e represent the orientation of aircraft (pitch angle), roll angle in the earth-axis system and elevator deflection angle.

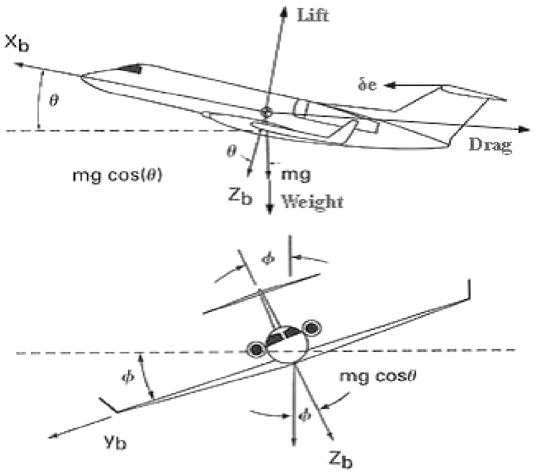

Figure 2. Pitch control system of aircraft

The equations of motion in (1),(2) and (3) can be linearized using small disturbance theory, assuming the motion of the airplane consists of deviations about a steady flight condition. All the variables in the equations of motion are replaced by a reference value plus a disturbance as shown below.

$$u = u_o + \Delta u \qquad v = v_o + \Delta v \qquad w = w_o + \Delta w p = p_o + \Delta p \qquad q = q_o + \Delta q \qquad r = r_o + \Delta r$$
$$X = X_o + \Delta X \qquad M = M_o + \Delta M \qquad Z = Z_o + \Delta Z \delta_e = \delta_{e_o} + \Delta \delta_e$$

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The equations for perturbations can be obtained by substituting the values of perturbed variables in the governing equations. Assuming that the reference flight condition is to be symmetric and the propulsive forces remain constant. This assumption implies that, $v_o = p_o = r_o = \phi_o = \psi_o = w_o = 0$. After linearization the equations (4), (5) and (6) are obtained.

$$\left(\frac{d}{dt} - X_u\right)\Delta u - X_w \Delta_w + (g\ cos\Theta_o)\Delta\Theta = X_{\delta_e}\Delta\delta_e \quad (4)$$

$$-Z_u\Delta u + \left[(1 - Z_w)\frac{d}{dt} - Z_w\right]\Delta w - \left[(u_o + Z_q)\frac{d}{dt} - g\ sin\Theta_o\right]\Delta\Theta = Z_{\delta_e}\Delta\delta_e \quad (5)$$

$$-M_u\Delta u - \left(M_w\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\Theta = M_{\delta_e}\Delta\delta_e \quad (6)$$

From the above equations (4), (5) and (6) transfer function for the change in the pitch rate to the change in elevator deflection angle and the change in pitch angle to the change in elevator deflection are obtained as equation (7) and (8).

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{-\left(M_{\delta_e} + \frac{M_{\dot\alpha}Z_{\delta_e}}{u_o}\right)s - \left(\frac{M_\alpha Z_{\delta_e}}{u_o} - \frac{M_{\delta_e}Z_\alpha}{u_o}\right)}{s^2 - \left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{u_o}\right)s + \left(\frac{Z_\alpha M_q}{u_o} - M_\alpha\right)} \quad (7)$$

$$\frac{\Delta\Theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}.\frac{-\left(M_{\delta_e} + \frac{M_{\dot\alpha}Z_{\delta_e}}{u_o}\right)s - \left(\frac{M_\alpha Z_{\delta_e}}{u_o} - \frac{M_{\delta_e}Z_\alpha}{u_o}\right)}{s^2 - \left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{u_o}\right)s + \left(\frac{Z_\alpha M_q}{u_o} - M_\alpha\right)} \quad (8)$$

In this study the data from General Aviation Airplane is used in system analysis and modeling. The parameter include in dimensional derivatives are;

$Q = 36.8\,lb/ft^2$ $\quad QS = 6771lb$ $\quad QS\bar{c} = 38596ft.lb(\bar{c}/2u_o) = 0.016s$ $\quad X_u = -0.045$
$Z_u = -0.369$ $\quad M_u = 0$ $\quad X_w = 0.036$ $\quad Z_w = -2.02$ $\quad M_w = -0.05X_{\dot w} = Z_{\dot w} = 0$ $\quad M_{\dot w} = -0.051$
$X_{\dot\alpha} = X_\alpha = Z_\alpha = 0Z_\alpha = -355.42$ $\quad M_\alpha = -8.8$ $\quad M_{\dot\alpha} = -0.8976$ $\quad X_q = 0$ $\quad Z_q = 0$ $\quad M_q = -2.05$
$X_{\delta_e} = 0$ $\quad Z_{\delta_e} = -28.15$ $\quad M_{\delta_e} = -11.874$

Two assumptions made in this paper are: First, the aircraft is steady state cruise at constant altitude and velocity. Second, the change in pitch angle does not change the speed of an aircraft under any circumstance. Substituting the values of the stability derivatives in the equations (4), (5), (6), (7) and (8) transfer function and the state space form $\dot{x} = Ax + Bu$ is obtained as equation (9), (10) and (11).

$$\frac{\Delta\Theta(s)}{\Delta\delta_e(s)} = \frac{11.7304s + 22.578}{s^3 + 4.967s^2 + 12.941s} \quad (9)$$

$$\begin{bmatrix}\Delta\dot\alpha \\ \Delta\dot q \\ \Delta\dot\Theta\end{bmatrix} = \begin{bmatrix}-2.02 & 1 & 0 \\ -6.9868 & -2.9476 & 0 \\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha \\ \Delta q \\ \Delta\Theta\end{bmatrix} + \begin{bmatrix}0.16 \\ 11.7304 \\ 0\end{bmatrix}[\Delta\delta_e] \quad (10)$$

$$y = \begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta\alpha \\ \Delta q \\ \Delta\Theta\end{bmatrix} + [0] (11)$$

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III.METHODOLOGY

In this section two control schemes are proposed in detail which is the Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) Control. In this work the design considerations are rising time to be less than 3 second, settling time to be less than 5 second, percentage of overshot to be less than 12% and steady state error to be less than 2% for controlling the pitch angle of 0.2 radian (11.5 degree).

A.LQR CONTROLLER

LQR is a modern control technique that usesstate-space approach to analyze a system. It is relatively simple to work with a multi-output system using state space approach. Using full-state feedback the system can be stabilized. In the Linear Quadratic Regulator (LQR) design method it focuses on the selection of Performance Index (PI) weighting matrices Q and R which are the design parameters. By the modifications of Performance Index a wide range of performance objectives can be obtained. Block diagram of LQR with full-state feedback is shown in Fig.3

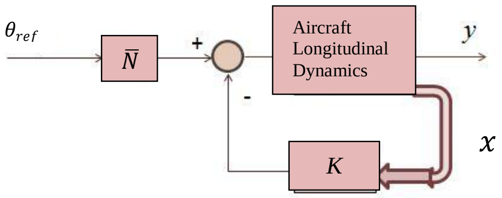

Figure 3. Pitch control system using LQR

Consider a state- space system given as;

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \qquad (12)$$

A feedback control law;
$u = -Kx$(13)
have to be designed in such a way that it minimizes the performance measure given by,

$J = \int_0^\infty (x^T Qx + u^T Ru)dt$(14)

where Q and R are positive semi definite & positive definite respectively. $x^T Qx$ minimize the deviation of final position from desired position and $u^T Ru$ minimize the expenditure of control effort. The control input can drive the states to their equilibrium point and by considering the Riccati equation given by;

$A^T P + PA - PBR^{-1}B^T P + Q = 0$ (15)
From the solution of Riccati equation P is obtained and the optimal gain K_r is given by;

$$K = R^{-1}B^T P(16)$$

A pre-compensator is also added to scale the reference input so that the output will be equal to the reference input in steady state condition. LQR control is designed in MATLAB and in order to determine the value of gain matrix K it is necessary to choose the values of R and Q given as;

R = 1;
Q = $C^T Cx$ *=[0 0 0;0 0 0;0 0 x];*

LQR gain is given as;

K= lqr [A, B, Q, R];

By trial and error we obtain the value of $x = 500$, which will satisfy the design requirements. Nbar which is a userdefined function and the value of pre-compensator can be obtained from m-file code; Nbar=rscale(A,B,C,D,K_r).
The value of K and pre-compensator (Nbar) are obtained from the controller design is given as;

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$$K= [-0.5704 \quad 1.6929 \quad 22.3607] \quad (17)$$

$$Nbar = 22.360 \quad (18)$$

The main disadvantage of LQR problem is that it cannot handle disturbance in the system.

B.LQG CONTROLLER

Linear Quadratic Gaussian (LQG) also falls under the category of modern controller. LQG control is the combination of Linear Quadratic Regulator (LQR) and Linear Quadratic Estimator (LQE). It uses separation principle for the design of LQR and LQE. The block diagram of LQG control is shown in Fig.4.

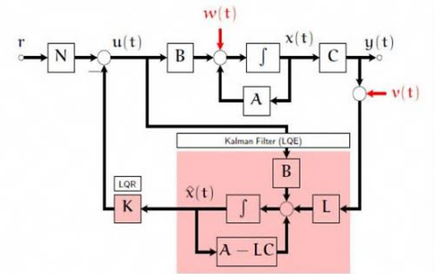

Figure 4. Pitch control system using LQG control

In the design of LQR controller it is assumed that all states are measurable but it cannot be true always so, in order to measure the unmeasurable states we use an estimator called Kalman Filter. Kalman Filter is an optimal estimator and provide best estimate of states of the plant which is affected by noise. Consider the system affected by noise;

$$\dot{x} = Ax + Bu + w$$
$$y = Cx + v \quad (19)$$

where w and v are zero-mean white noise with covariances $E(ww^T) = W$ and $E(vv^T) = V$. The optimal Kalman gain L is given as;

$$L = PC^T V^{-1} \quad (20)$$

where $P$is obtained from the solution of algebraic Riccati equation;

$$AP + PA^T - PC^T V^{-1} CP + W = 0 \quad (21)$$

The block diagram of plant with Linear Quadratic Estimator (LQE) is shown in Fig.5.

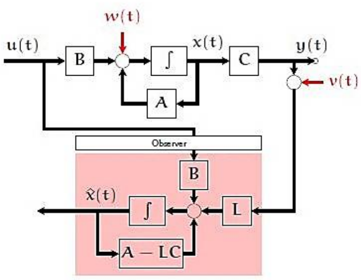

Figure 5. Block diagram of plant with LQE

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By combining Kalman Filter and LQR control, we are able to replace the equation (13) as;

$u = -K\hat{x} + r$ (22)

where $\hat{x}$ is the estimated states and r is the reference input to the system. This control input minimizes the performance measure given by;

$J = \lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T [x^T(t)Qx(t) + u^T(t)Ru(t)]dt\right\}$ (23)

The value of kalman gain obtained from the design is;

L=[11.5487 ; 73.8543 ; 12.1535]   (24)

IV.SIMULATION RESULTS

The proposed control schemes LQR and LQG control are successfully implemented in MATLAB/SIMULINK and the results are presented for an aviation aircraft based on common criteria of step response. The pitch control system using LQR and LQG control has been simulated in the presence and absence of disturbance and the response of the system for a given pitch angle is obtained. Simulink diagram of LQR is shown in Fig.6. The response of the system using LQR controller without disturbance is shown in Fig.7.

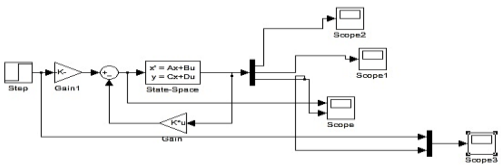

Figure 6. Simulation of system with LQR control

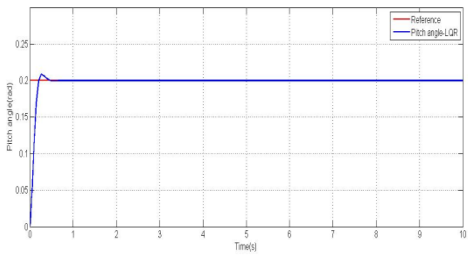

Figure 7.Performance of the system using LQR without disturbance.

Response of the system with LQR show that it has a delay time of 0.08s, rise time of 0.16s, settling time of 0.48s, percentage overshoot of 4.3% and steady state error that is zero.
Simulink diagram of LQG control is shown in Fig.8. The response of system using LQG control with disturbance is shown in Fig.9.

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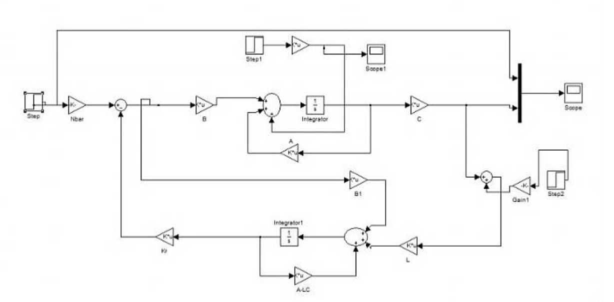

Figure 8.Simulation of plant using LQG control with disturbance.

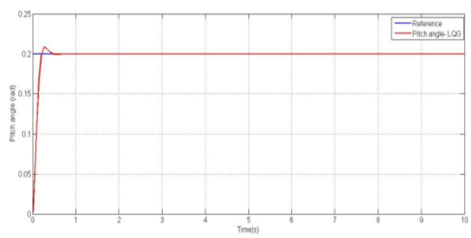

Figure 9. Performance of plant using LQG control with disturbance

The results show that LQR gives a good performance when the plant is not corrupted by disturbance. But when disturbance acts on the system, LQR gives a steady state error. It is found that LQR has a very small disturbance rejection capability of 0.004%. In order to overcome this disadvantage an LQG control is used which can handle process noise and measurement noise. LQG control has a good disturbance rejection capability; process noise up to 3.5% and measurement noise up to 1%.

V.CONCLUSION

Pitch control system requires a pitch controller to maintain the pitch angle at a desired value. Two controllers, LQR and LQG control are successfully designed and presented. Simulation results show that, LQR gives a better performance in the absence of noise where as in the presence of noise LQG control has the better capability in controlling the pitch angle of the aircraft system. For further research, effort can be devoted for the development of more advanced and robust control techniques that can control the pitch of aircraft. Besides, the proposed control algorithm for pitch control can be implemented to a real plant for the validation of theoretical results.

REFERENCES

[1] Nurbaiti Wahid, MohdFuaadRahmat, "Pitch Control System Using LQR and Fuzzy Logic Controller", *IEEE Symposium on Industrial Electronics and Applications*, vol. 3, no. 5, pp. 325-337, October 2010.

[2] A.B. Kisabo1, F. A. Agboola, C.A. Osheku1, M. A. L. Adetoro and A.A. Funmilay, "Pitch Control of an Aircraft Using Artificial Intelligence", *Journal of Scientific Research &Reports*,vol. 1, no. 1, pp. 1-16, November 2012.

[3] Romulus Lungu, MihaiLungu, Lucian and TeodorGrigorie, "Automatic Control of Aircraft in longitudinal plane during landing" ,*IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 2, pp. 1586-1603, April 2013.

[4] LabaneChrif ,ZemalacheMeguenniKadda, ''Aircraft Control System Using LQG and LQR Controller with Optimal Estimation-Kalman Filter Design'' *3rd International Symposium on Aircraft Airworthiness*, vol. 8, no. 6, pp. 245 – 257, June 2013.

[5] Maryam Malekzadeh, AbolghasemNaghash, H. A. Talebi, "A Robust Nonlinear Control Approach for Tip Position Tracking of Flexible Spacecraft", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 4, October 2011.

[6] W. Mackunis, P. M. Patre, M. K. Kaiser, and W. E. Dixon, "Asymptotic tracking for aircraft via robust and adaptive dynamic inversion methods", *IEEE Transactions on Control Systems Technology*, vol. 18, no. 6, pp. 342-356, November 2010.

[7] Yunjun XU, "Multi-Timescale nonlinear robust control for a miniature helicopter", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, no. 2, pp. 543-558, April 2010.

[8] J.G. Juang, K.C. Cheng, "Application of Neural Network to Disturbances Encountered Landing Control", *IEEE Transactions on Intelligent Transportation Systems*, vol. 7, no. 4, pp. 582 – 588, July 2006.

[9] JishaShaji, Aswin R.B, "Pitch Control of Flight System using Dynamic Inversion and PID Controller", *International Journal of Engineering Research and Technology*,vol.4, issue.07, July 2015.