



# Set Point Tracking Capability Analysis of Various Controllers Designed For a First Order plus Dead Time Process

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**ABSTRACT:** It is very common to find an industrial process that can be modelled using first order plus dead time (FOPDT) model such as, a blending process. This process is selected for controller design based on different well established and relatively newer controller tuning methods and compared for set point tracking capability of the controllers. Based on comparisons of set point tracking capability of the controller and dynamic and steady state characteristics of step responses, best controller tuning technique is determined for the selected process.

**KEYWORDS:** FOPDT, PID controller, blending process, controller tuning, set-point tracking, step response.

## I. INTRODUCTION

In industries most of the processes are composed of many dynamic elements which are usually of first order. This leads the overall process to have a linear model of a very high order. Although these higher order models are very precise, they are not to be used for the control purposes. Instead of using high order model, behaviour of the process is simply modelled as a linear first order system with the dead time element, in most of the cases [1]. A time delay is generally present in the system which is actually a delay because of transport lag. The dead time may be because of many reasons, especially due to the distant sensor location [2, 3]. It is generally believed that PID controller and its variations (P, PI and PD) is the most commonly used controller in the process control application. Because they can compensate the effect of both the delayed and non-delayed process and ease of implementation, these controllers are used in industrial application [4], and more than 90% of existing control loop involve PID controller [5]. Numerous methods have been projected for tuning these controllers, but every method has some constraint [4]. As a result, the design of PID controller still remains a challenge before researchers and engineers. A PID controller has the following transfer function:

$$G(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (1)$$

In the PID controller tuning we find out PID parameter ( $K_c$ ,  $T_i$ , and  $T_d$ ) to meet a given set of a closed loop system performance [6].

The process selected in this analysis is a simple blending process. In Blending operation, control objective is to mix or blend two input inlet stream and make a final control output to ensure that the final product meet customer specification. A stirred- tank blending process is shown in fig. 1. Stream 1 is a mixture of a two chemical species, A and B such that its mass flow rate  $w_1$  is constant, but the mass fraction of A is  $x_1$ , varies with time. Stream 2 consist of a pure A and thus  $x_2=1$ .

The mass fraction of A in the exit stream is denoted by  $x$  and the desired value (set point) by  $X_{sp}$ . Thus for this control problem, the controlled variable is  $x$ , the manipulated variable is  $w_2$ , and the disturbance variable is  $x_1$  [6].

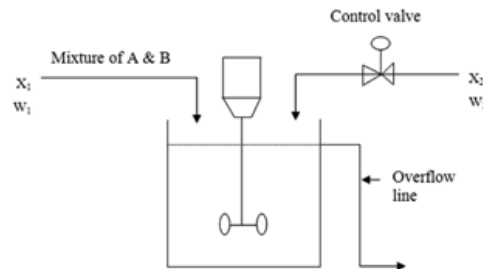


Fig. 1. Stirred Tank Blending System[6]

A large number of industrial processes can approximately be modelled by a FPODT transfer function as:

$$G(s) = \frac{k}{Ts+1} e^{-s\tau} \quad (2)$$

Where  $k$  is process static gain,  $\tau$  is the dead time and  $T$  is the time constant [10].

## II. LITERATURE REVIEW

There are many researchers who have contributed toward finding the controller tuning formula and claiming it to be better than many existing formulae.

According to Aidan O' Dwyer [2], the classification of techniques for the compensation of time delayed processes. Of the two parts of this paper is to provide a framework against which the literature may be viewed; Part 1 of the paper considers the use of parameter optimized controllers for the compensation problem, with Part 2 of the paper considering the use of structurally optimized compensators. And conclude that Iterative methods for controller design provide a first approximation to desirable controller parameters, direct synthesis tuning rules result in a controller that facilitates a specified closed loop response. These methods include pole placement strategies and frequency domain techniques, such as gain margin and/or phase margin specification and Analytical methods are suitable for the design of PI/PID controllers for non-dominant delay processes where there are well-defined performance requirements to be achieved.

Saeed Tavakoli & Mahdi Tavakoli [7] proposed in his paper an optimal technique for tuning PID parameters for FOPTD system. Dimensional analysis and numerical optimization methods were used to simplify the procedure of obtaining optimal relations. In addition, robustness studies proved the robustness of proposed method in comparison with Ziegler- Nichols and Cohen-Coon is better. Their future research is targeted at obtaining optimal formulas for tuning PID controllers for a second order plus time delay model.

Wen Tan, Jizhen Liu, Tongwen Chen, Horacio J. Marquez [8] performed comparison of some well-known PID tuning formulas based on disturbance rejection and system robustness to assess the performance of PID controllers. A simple robustness measure is defined and the integral gains of the PID controllers are shown to be a good measure for disturbance rejection. An analysis of some well-known PID tuning formulas reveals that the robustness measure should lie between 3 and 5 to have a good compromise between performance and robustness.

Sipahi. R, Niculescu. S. Abdallah, C. & Michiels, W [13], give stability and stabilization of systems with time delay: limitations and opportunities. In this they give the performance result of a delayed process. Delay cause instability in the process, which cause delayed processing.

Pradeep Kumar Juneja, A K Ray and R Mitra [14] commented that many industrial processes are represented by first order plus dead time for tuning purposes. A PID controller can be used to control this type of process if the dead time is less than the process time constant. The regulatory control performance of the loop i.e. disturbance rejection, deteriorates rapidly when dead time exceeds time constant of the process model, even though the response to set point changes remains acceptable.



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Guillermo J. Silva, Aniruddha Datta, and S. P. Bhattacharyya [15] presented a procedure to determine the complete set of stabilizing PID controllers for a given first-order plant with dead-time. The procedure is based on first determining the range of proportional gain values for which a solution to the PID stabilization problem exists. Then, it is shown that for a fixed proportional gain value inside this range, the stabilizing integral and derivative gain values lies inside a region with known shape and boundaries. Furthermore, it has been demonstrated that this region can be characterized in a computationally tractable manner. By weeping over the entire range of allowable proportional gain values and determining the stabilizing regions in the pace of the integral and derivative gains, the complete set of stabilizing PID controllers can be determined.

### III. COMPARISON OF TUNING FORMULAS

Numerous examples are present in the literature which can be used to evaluate various PID design or tuning methods. Though, the specific method might be effective for a specific plant model or a process, so it is very difficult to draw general conclusion that which method is convenient or better for the selected process. We can bring to a close is that which method show better performance within the process. The performance can be calculated in terms of tuning parameter such as proportional gain constant  $K_p$ , integral gain constant  $T_i$  and derivative gain constant  $T_d$  and based on the time response characteristics such as rise time, setting time, overshoot (%), peak, gain and phase margin and closed loop stability.

1. The process model is first-order with dead time (FOPDT)

$$G(s) = \frac{k}{T_s + 1} e^{-s\tau} \quad (3)$$

2. The following PID tuning formulae are considered as shown in Table I:

- Ziegler-Nichols (Z-N) method has two version i.e. one depend on the reaction curve and the other, the ultimate gain and the ultimate period.
- Cohen-Coon (C-C) method which is based on reaction curve. A model with one tangent and point is derived first to tune the PID controller.
- Internal model control (IMC) method is proposed in Rivera, Morari and Skogested. The smaller it is the better performance the closed-loop system will have. Here the tuning parameter  $\lambda$  is chosen as  $0.25\tau$  of the delay, the smallest value suggested in reference [8].
- Saeed and Mahdi proposed formula for ITAE performance index using dimensional analysis and numerical optimization techniques, an optimal method for tuning PID controller for FOPDT model is presented [9].
- Optimum integral error for set point change (IAE-set point, ITAE-set point, ISE-set point, ISTE-set point) methods. There are many versions of the integral-error based methods. The original references can be found in papers and books written in the 1960's [11].
- Chien Hrones Reswick changed the step response method to give better damped closed loop system. They proposed to use “quickest response without overshoot” or “quickest response with 20% overshoot” as design criteria. They also made an important observation that tuning for set point response or load disturbance response is different. In set point response method, the controller parameter not only depend on  $a$  and  $\tau$  but also to the time constant  $T$  [12].
- Wang Juang Chan Method gives a set of controller tuning formulae for the proportional, integral and derivative gain [13].



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TABLE I  
PID TUNING FORMULAS

Controller Tuning Methods	$K_c$	$T_i$	$T_d$
Z-N	$0.6 K_u$	$0.5 T_u$	$0.125 T_u$
C-C	$\frac{\frac{\tau}{4T} + \frac{4}{3}}{K \frac{\tau}{T}}$	$\frac{\frac{3}{4T} + \frac{4}{\tau}}{\frac{\tau}{T} + \frac{1}{8}}$	$\frac{2}{\frac{\tau}{T} + \frac{11}{2}}$
IMC	$\frac{2T + \tau}{2k(\lambda + \tau)}$	$T + \frac{\tau}{2}$	$\frac{4}{11 + 2\tau/T}$
S-M Proposed (ITAE criterion)	$\frac{0.8}{K(\frac{\tau}{T} + 0.1)}$	$\tau(0.3 + \frac{1}{\tau/T})$	$\tau(\frac{0.06}{\tau/T} + 0.04)$
ITAE	$\frac{1.12762}{K} (\frac{\tau}{T})^{-0.80368}$	$\frac{T}{0.99783 + 0.02860\tau/T}$	$0.42844T (\frac{\tau}{T})^{1.0081}$
ISTE	$\frac{1.042}{K} (\frac{\tau}{T})^{-0.897}$	$\frac{T}{0.987 - 0.238\tau/T}$	$0.385T (\frac{\tau}{T})^{0.906}$
IAE	$\frac{0.65}{K} (\frac{\tau}{T})^{-1.04432}$	$\frac{T}{0.9895 + 0.09593\tau/T}$	$0.50814T (\frac{\tau}{T})^{1.08433}$
T-L	$0.45 K_u$	$2.2 T_u$	$T_u / 6.3$
C-H-R	$0.95 / a$	$1.4T$	$0.47\tau$
W-J	$\frac{(0.7303 + 0.5307T/\tau)(T + 0.52)}{K(T + \tau)}$	$T + 0.5\tau$	$\frac{0.5\tau T}{T + 0.5\tau}$

Where  $\lambda \geq 0.25$  as suggested in Rivera et al and  $T_u$  is the ultimate period and  $a = \frac{K\tau}{T}$  [8, 10, 11, 12, 13]. The above table contain different tuning formulae to determine the controller parameters  $K_c$ ,  $T_i$  and  $T_d$ .

#### IV. METHODOLOGY

Blending operation is commonly used in many industrial to ensure that final product meet customer specification. The transfer function [9] is given as -

$$G(s) = \frac{1.54e^{-1.075s}}{5.93s + 1} \tag{4}$$

Using first order Pade's approximation of the delay term, the modified transfer function may be written as:



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$$G(s) = \frac{1.54(1-0.535s)}{(5.93s+1)(0.535s+1)} \quad (5)$$

Therefore, the ultimate gain can be found using,

$$1 + K_u G(s) = 0 \quad (6)$$

$$1 + K_u \frac{1.54(1-0.535s)}{(5.93s+1)(0.535s+1)} = 0 \quad (7)$$

$$(5.93s+1)(0.535s+1) + K_u 1.54(1-0.535s) = 0 \quad (8)$$

By Routh criterion we can find the value of ultimate gain  $K_u = 7.8133$

Now to find the value of ultimate period  $T_u$  make an auxiliary equation from the Routh criterion i.e.

$$3.1874s^2 + 1 + 1.54K_u = 0 \quad (9)$$

Solving above equation we get

$$s = W_u = 2.022$$

And finally the ultimate period  $T_u$  is

$$T_u = \frac{2\pi}{W_u} = 3.105$$

Therefore from above the value of ultimate gain  $K_u$  and ultimate period  $T_u$  are  $K_u = 7.8133$ ,  $T_u = 3.105$

TABLE II  
PID CONTROLLER PARAMETER

Controller Methods	$K_c$	$T_i$	$T_d$
Ziegler Nichols	4.68	0.33	1.77
Cohen-Coon	2.85	0.86	1.05
IMC	3.12	0.48	1.49
Saeed & Mahdi	1.84	0.29	0.53
Tyreus & Luyben	3.51	0.513	1.726
ITAE	3.94	0.666	1.788
ISTE	3.13	0.498	1.518
IAE	2.514	0.365	1.186
Chien Hrones	3.405	0.41	1.719
Wang Juang	2.192	0.338	1.089

The above table shows the controller parameter for different controller tuning formulae.

## V. RESULT AND DISCUSSION

Simulation is performed to analyse the set point tracking and the different unit step response characteristics i.e. rise time, settling time, peak, overshoot (%), and closed loop stability. Fig. 2 and Fig. 3 show the step responses for the comparison among the values of different controller tuning techniques i.e. Ziegler-Nichol, Cohen-Coon, Internal model control and the Saeed and Mahdi proposed formula, optimum integral error for set point change (IAE-set point, ITAE-set point, ISE-set point, ISTE-set point) methods, Chien Hrones Reswick, Wang Juang Chan.

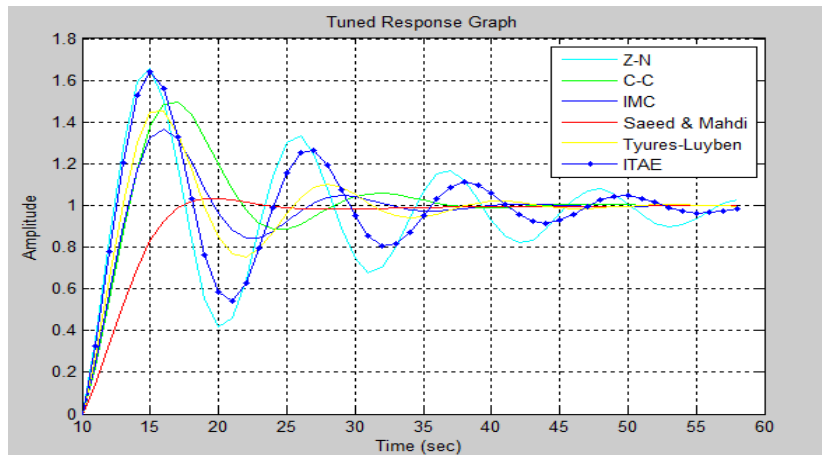


Fig 2: Combines response of Z-N, C-C, IMC, S-M, T-L, ITAE

The above graph shows the combine simulation response of the Ziegler-Nichol, Cohen-Coon, Internal Model Control, Saeed & Mahdi Proposed formula, Tyures- Luyben, ITAE- set point.

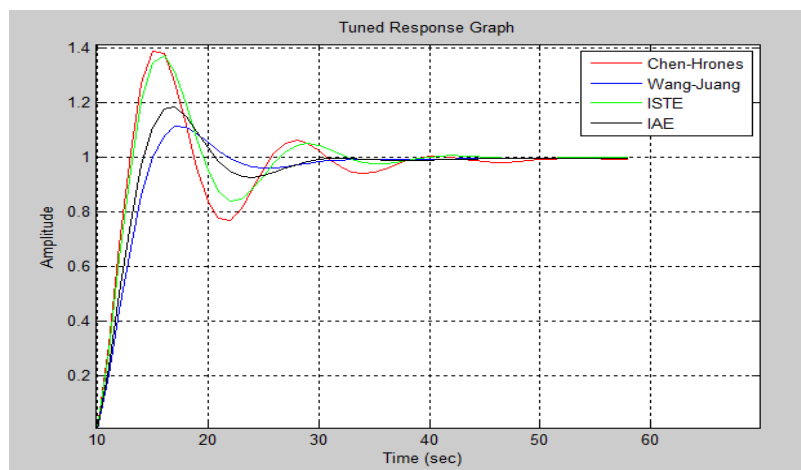


Fig 3: Combines response of C-H-R, W-J, ISTE, IAE

The below mention graph shows the simulation response of Chien-Hrones-Reswick, Wang-Juang Chan, ISTE-set point, IAE- set point,



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TABLE III  
COMPARISON OF TIME RESPONSE CHARACTERISTICS

Controller Methods	$K_c$	$T_i$	$T_d$	Rise Time(s)	Settling Time(s)	Overshoot %	Peak
Ziegler Nichols	4.68	0.33	1.77	0.66	31.9	68	1.68
Cohen-Coon	2.85	0.86	1.05	0.974	11	50.9	1.51
IMC	3.12	0.48	1.49	0.953	11.9	37.5	1.37
Saeed & Mahdi	1.84	0.29	0.53	1.94	5.48	3.24	1.03
Tyreus -Luyben	3.51	0.513	1.726	0.848	13.4	47.6	1.48
ITAE	3.94	0.666	1.788	0.748	19.8	63.3	1.68
ISTE	3.13	0.498	1.518	0.946	11.8	38.5	1.39
IAE	2.514	0.365	1.186	1.23	8.32	18.9	1.19
Chien Hrones	3.405	0.41	1.719	0.885	15.7	41	1.41
Wang Juang	2.192	0.338	1.089	1.46	8.76	11.5	1.12

Form the above table we can observe that the controller tuned by the Saeed and Mahdi proposed formula, Wang Juang and integral absolute error have small overshoot and less settling time with fairly good rise time. But the controller tune by the Ziegler Nichols, Cohen Coon, ITAE, and Tyreus & Lyuben have large overshoot i.e. 68, 50.9, 47.6, 63 which is undesirable as they lead to large settling time means process take large time get the set point value. The controller tuning parameter obtains from the ISTE, Chien Hrones, and shows large overshoot with large settling time. Among all the controller tuning methods, the best response is achieved in case of Saeed and Mahdi proposed tuning methods for the selected blending process modelled by FOPDT model.

## VI. CONCLUSION

A large number of PID controller tuning rules have been defined for the single input single output process with dead time. Here ten different types of controller tuning rules are selected and compared for the selected FOPDT process. The performance evaluation is based on the time response characteristics such as, rise time, settling time and overshoot. The comparison shows that the controller tuned by Saeed and Mahdi proposed method has the best response among all other selected tuning methods.

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## **BIOGRAPHY**

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