ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 3, September 2012

Lead Compensators from Universal Design Method

R. Kiranmayi

Associate Professor, Department of EEE, JNTUHCEH, Hyderabad, India

ABSTRACT: Compensators are corrective sub systems to force the chosen plant to meet the given specifications. Their purpose is to compensate for the deficiency in the performance of the plant. The Universal chart facilitates accurate compensators design and also satisfies the system specifications in frequency domain and steady state error. This paper shows the derivation of lead compensator from universal design which is useful for different time domains. The design is carried out with the approach of frequency domain specification.

KEYWORDS: Lead compensators, Universal design, Universal design chart, Bode plots, Continuous systems.

I. INTRODUCTION

Control system design via the frequency domain and specifically using the Bode plots has been well established. Various Bode design charts and formulae were developed for the most common continuous-time and discrete-time compensators by Yeung. This paper enhances, emphasis and shows the possibility to derive lead compensators from a single universal design which also facilitates the design of many of other conventional compensators. In the sequel, first the essence of the method is presented, and then formulae for the compensators are given which are used in conjunction with the universal design.

II. DESIGN METHOD PRINCIPLE

To illustrate the basic idea of the compensator design, consider a plant with the frequency response $G_p(j\omega)$. A compensator $G_c(j\omega)$ is to be inserted in series with the plant so that a desired value for the phase margin (*PM*) or gain margin (*GM*) is obtained.

By the definitions of *PM* and GM, for the case of PM

$$|G_c(j\omega_g)G_p(j\omega_g)|_{db} = 0$$
$$\angle(G_c(j\omega_g)G_p(j\omega_g)) = PM - 180^o \qquad (1)$$

and for the case of GM

$$|G_c(j\omega_{ph})G_p(j\omega_{ph})|_{db} = -GM$$
$$\angle(G_c(j\omega_{ph})G_p(j\omega_{ph})) = -180^o \qquad (2)$$

Where $|\ |_{db}$ denotes the 20log decibels of the magnitude, |_ denotes the phase angle. ω_g is the gain crossover frequency and ω_{ph} is the phase crossover frequency.
Equations (1) and (2), respectively, can be rewritten as

$$|G_c(j\omega_g)|_{db} = -|G_p(j\omega_g)|_{db}$$
$$\angle(G_c(j\omega_g)) = -\angle G_p(j\omega_g) + PM - 180^o \qquad (3)$$

and

$$|G_c(j\omega_{ph})|db = -|G_p(j\omega_{ph})|_{db} = -GM$$
$$\angle(G_c(j\omega_{ph})) = -\angle G_p(j\omega_{ph}) - 180^o \qquad (4)$$

The left-hand sides of (3) and (4) depend only on the compensator. It can also be shown that for all the compensators frequency response can be normalized into the standard form.

$$G_c(A, B) = \frac{(1 + jB)}{(1 + jA)} \qquad (5)$$

ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 3, September 2012

where A, B and C are in general functions of the frequency and compensator parameters. Let

$$G_c(A, B) = \frac{(1 + jB)}{(1 + jA)} \quad (6)$$

so that

$$G_c(j\omega) = CG_c(A, B) \quad (7)$$

Substitution of (7) into (3) at $\omega = \omega_g$ gives

$$|G_c(A, B)|_{db} = -|CG_p(j\omega)|_{db}$$
$$\angle G_c(A, B) = -\angle(CG_p(j\omega)) + PM - 180^o \quad (8)$$

Substitution of (7) into (4) at $\omega = \omega_{ph}$ gives

$$|G_c(A, B)|_{db} = -|CG_p(j\omega)|_{db} - GM$$
$$\angle G_c(A, B) = -\angle(CG_p(j\omega)) - 180^o \quad (9)$$

If C is known, then the right-hand sides of (8) or (9) can be plotted as a plant curve. Intersection of plant curve with the appropriate curve on the universal design chart will yield the A and B values which in turn will give the compensator parameter values.

III. PLOTTING OF UNIVERSAL DESIGN CHART

Standard form of compensator in frequency domain is

$$G_c(A, B) = \frac{(1 + jB)}{(1 + jA)}$$

The above equation in rectangular form is given as

$$G_c(j\omega) = C\frac{(1 + jB)}{(1 + jA)}$$
$$= C\frac{(1 + AB)}{(1 + A^2)} + jC\frac{(B - A)}{(1 + A^2)}$$

This is the basic equation to plot the universal design chart. The plotting of universal design chart is done in two steps.

1. Plotting of the A – curves.
2. Plotting of the B – curves.

PLOTTING OF A CURVES:

A curves are plotted using basic equation by varying the values of A for the fixed value of B. The A curve is drawn with the magnitude on Y-axis and the phase angle on X- axis. This curve is known as A curve. To draw more A curves above procedure can be repeated for different values of A.

PLOTTING OF B CURVES:

B curves are plotted using basic equation by varying the values of B for the fixed value of A. The magnitude is taken on Y- axis and the phase angle is taken on X- axis. This curve is known as B curve. Many values of A gives many B curves. To draw more B curves above procedure can be repeated for different B values.
The universal design indicates the plotting of A and B curves on same plane which results in universal design chart.

IV. TRANSFORMATION OF PHASE- LEAD COMPENSATOR INTO STANDARD FORM

The frequency response of the phase-lead compensator is equated to the standard form

$$G_c(j\omega) = C\frac{(1 + jB)}{(1 + jA)} = K\frac{(1 + j\alpha T\omega)}{(1 + jT\omega)} \quad (10)$$

yielding

$$C = K \quad (11)$$
$$\alpha = B/A \quad (12)$$
$$T = A/\omega \quad (13)$$

Frequency is to be specified as either ω_g or ω_{ph}.

ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 3, September 2012

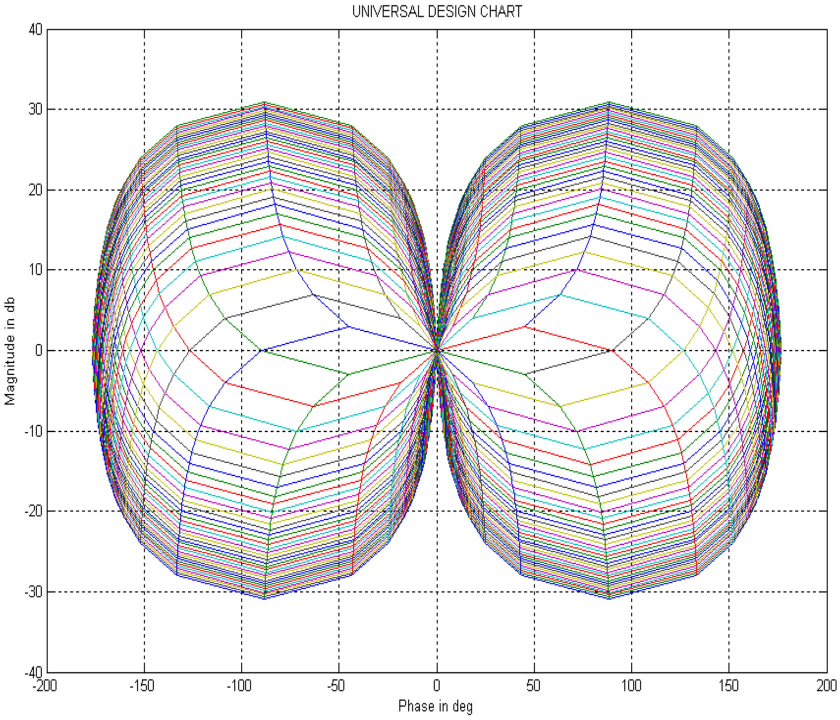

Fig. 1 General Universal Design Chart

V. ILLUSTRATIVE EXAMPLE

Following illustrates the designing of phase- lead compensator to satisfy the following specifications:

(i) The phase margin of the system = 47 degree
(ii) Steady state error for a unit ramp input = 15
(iii) The gain crossover frequency of the system must be less than 7.5 rad/sec for the system $G_p(S) = 1/ S(S+1)$, $H(S) = 1$.

Method: The given plant transfer function is

$$G_p(S) = \frac{1}{S(S+1)}$$

The transfer function of phase lead compensator is

$$G_c(S) = \frac{K\,(1+ S\,T\alpha)}{(1+ ST)}$$

Transformation of compensator into standard form yields

C = K (K is determined from steady state requirements)
$\alpha = B / A$
$T = A / \omega$

From the data , ess $=1/K_v=1/15$

$K_v = 15$

and

$$K_v = \lim_{S\to 0} S\,(G_p(S)\,G_c(S))$$

ISSN 2278 - 8875
International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering
Vol. 1, Issue 3, September 2012

$$K_v = S \lim_{S\to 0} \frac{1}{S(S+1)} \frac{K(1 + S\,T\,\alpha)}{(1 + ST)}$$

Therefore K =15 and C =15.

Also

$G_p(S) = 1/S(S+1)$
$G_p(jw) = -1/(1+\omega*\omega) - j/\omega(1+\omega*\omega)$
$X = -1/(1 + \omega * \omega)$
$Y = 1/\omega(1+ \omega * \omega)$
$Z = X - j\,Y$
$|G_p(j\omega)|\ db = 20\log(abs(Z))$
$|G_c(j\omega)|\ db = |C\,G_p(j\omega)|\ db$
$\angle \mathbf{C}\ G_p(j\omega) = -90 - \arctan(\omega)$
$\angle G_c(A,B) = -\angle C\,G_p(\omega) + PM - 180$
$= \arctan(\omega) - 43$

Using above equations plant curve is drawn on universal chart.

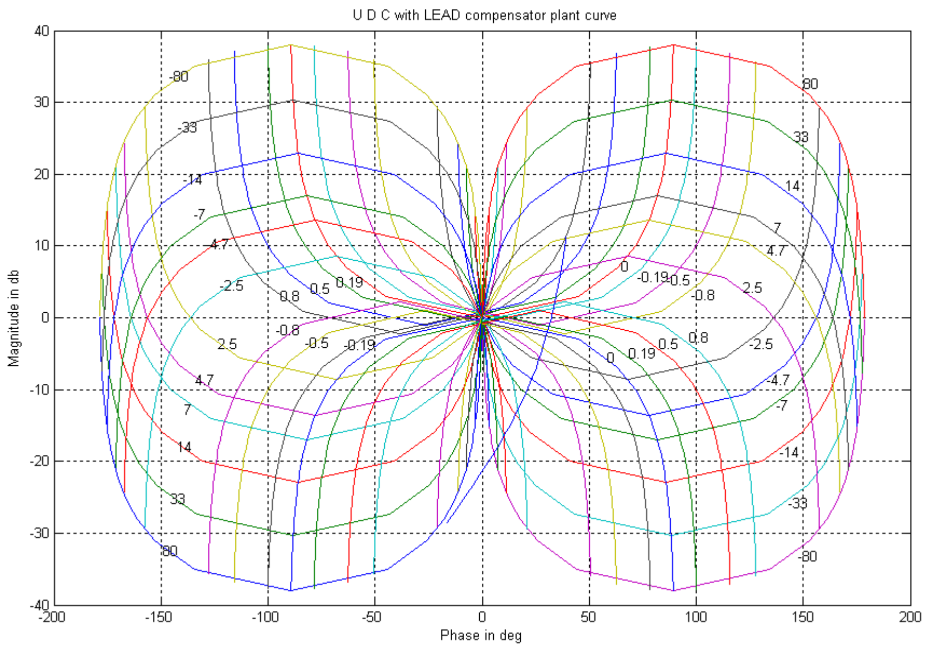

Fig. 2 Universal design chart with lead compensator plant curve

From above figure A, B values are 0.8 and 4.7. Therefore

$\alpha = B / A = 4.7 / 0.8 = 5.8$

$T = A / \omega = 0.8 / 0.106 = 0.106$

With these α and T values compensator transfer function is

$$G_c(S) = \frac{15\,(1 + 0.61\,S)}{(1 + 0.106\,S)}$$

Therefore the compensated system becomes

$$G_p(S)G_c(S) = \frac{15}{S\,(S+1)} \frac{(1+0.61\,S)}{(1+0.106\,S)}$$

VI. CONCLUSIONS

This paper provides design of lead compensators using a universal chart which enables an efficient design of the most common compensators also. A method for lead compensator design is presented. It is based on the idea of normalizing compensator parameters so that a design chart can be generated once for all. A design example is carried through to illustrate the use of this design chart. The coding is done in MATLAB. The method proves to take less time and iterations compared to conventional compensator design. The method involves less complexity. Simultaneous fulfillment of the specifications of steady-state error, phase margin, gain margin, and gain crossover frequency can be achieved with accuracy.

VII. ACKNOWLEDGEMENTS

I acknowledge the almighty for his blessings and my family for their constant support and also my student for his contribution towards this work.

REFERENCES

[1] Kai Shing Yeung, Kuo Hsum Lee, "A Universal Design Chart for Linear Time-Invariant Continuous-Time and Discrete-Time Compensators" IEEE Trans. on Educ., Vol.43, No.3, pp.309-315, Aug. 2000.
[2] W.R.Wakel, "Bode compensator design," IEEE Trans. Automat. Contr., Vol. AC-21, pp.771-773, Oct. 1976.
[3] J.R.Mitchell, "Comments on bode compensator design," IEEE Trans. Automat. Contri., vol. AC-22, pp.869-870, Oct.1977.
[4] K.Q.Chaid, "Bode compensator design," M.S. Thesis, Univ. Taxes, Arlingtion 1986.
[5] K.S.Yeung, K.Q.Chaid, "Bode design formulas for discrete compensators," Electron. Lett., vol.22, no.24, Nov. 21, 1986.
[6] K.S.Yeung, K.Q.Chaid, and T.X.Dinh, "Bode design chart for continuous-time and discrete-time compensators" IEEE Trans. Educ., vol.38, pp252-257, Aug.1995.
[7] K.S.Yeung, K.W.Wong, and K.L.Chen "A nontrial-and-error method for lag-lead compensator design" IEEE Trans. Educ., vol.41, pp.76-80, Feb.1998.
[8] Ogata K.,"Modern control Engineering" Practice Hall of India,Pvt. Ltd., New Delhi, 1995.
[9] D.R.Wilson, "Modern Practice in servo design" Pergmon Press, 1970.
[10] Benjamin C.Kuo, "Automatic control systems" Practice Hall of India, Pvt. Ltd ., New Delhi, 1979.

BIOGRAPHY

Mrs R.Kiranmayi, Graduated in Electrical and Electronics Engineering in the year 1993 from Jawaharlal Nehru Technological University, Anantapur and Post Graduated in Electrical Power Systems in the year 1995 from the same university. She is presently working as Associate Professor in Electrical Engineering on deputation at JNTUH college of Engineering, Hyderabad.