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PID Controllers from Universal Design Method

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ABSTRACT: Controllers are corrective sub systems to force the chosen plant to meet the given specifications. Their purpose is to compensate for the deficiency in the performance of the plant. The Universal chart facilitates accurate controller design and also satisfies the system specifications in frequency domain and steady state error. This paper shows the derivation of PID controller from universal design which is useful for different time domains. The design is carried out with the approach of frequency domain specification.

KEYWORDS: PID controller, Universal design, Universal design chart, Bode plots, Continuous systems.

I. INTRODUCTION

Control system design via the frequency domain and specifically using the Bode plots has been well established. Various Bode design charts and formulae were developed for the most common continuous-time and discrete-time compensators by Yeung. This paper enhances, emphasis and shows the possibility to derive PID controllers from a single universal design which also facilitates the design of many of other conventional compensators. In the sequel, first the essence of the method is presented, and then formulae for the controller are given which are used in conjunction with the universal design.

II. DESIGN METHOD PRINCIPLE

To illustrate the basic idea of the controller design, consider a plant with the frequency response $G_p(j\omega)$. A compensator $G_c(j\omega)$ is to be inserted in series with the plant so that a desired value for the phase margin (*PM*) or gain margin (*GM*) is obtained.

By the definitions of *PM* and GM, for the case of PM

$$|G_c(j\omega_g)G_p(j\omega_g)|_{db} = 0$$
$$\angle(G_c(j\omega_g)G_p(j\omega_g)) = PM - 180^o \qquad (1)$$

and for the case of GM

$$|G_c(j\omega_{ph})G_p(j\omega_{ph})|_{db} = -GM$$
$$\angle(G_c(j\omega_{ph})G_p(j\omega_{ph})) = -180^o \qquad (2)$$

Where $|\ |_{db}$ denotes the 20log decibels of the magnitude, $|\!\_$ denotes the phase angle. ω_g is the gain crossover frequency and ω_{ph} is the phase crossover frequency.
Equations (1) and (2), respectively, can be rewritten as

$$|G_c(j\omega_g)|_{db} = -|G_p(j\omega_g)|_{db}$$
$$\angle(G_c(j\omega_g) = -\angle G_p(j\omega_g) + PM - 180^o \qquad (3)$$

and

$$|G_c(j\omega_{ph})|db = -|G_p(j\omega_{ph})|_{db} = -GM$$
$$\angle(G_c(j\omega_{ph}) = -\angle G_p(j\omega_{ph}) - 180^o \qquad (4)$$

The left-hand sides of (3) and (4) depend only on the controller. It can also be shown that for the controllers frequency response can be normalized into the standard form.

(1 + jB)

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$$G_c(A, B) = \frac{}{(1 + jA)} \qquad (5)$$

where A, B and C are in general functions of the frequency and controller parameters. Let

$$G_c(A, B) = \frac{(1 + jB)}{(1 + jA)} \qquad (6)$$

so that

$$G_c(j\omega) = CG_c(A, B) \qquad (7)$$

Substitution of (7) into (3) at $\omega = \omega_g$ gives

$$|G_c(A, B)|_{db} = -|CG_p(j\omega)|_{db}$$
$$\angle G_c(A, B) = -\angle(CG_p(j\omega)) + PM - 180^o \qquad (8)$$

Substitution of (7) into (4) at $\omega = \omega_{ph}$ gives

$$|G_c(A, B)|_{db} = -|CG_p(j\omega)|_{db} - GM$$
$$\angle G_c(A, B) = -\angle(CG_p(j\omega)) - 180^o \qquad (9)$$

If C is known, then the right-hand sides of (8) or (9) can be plotted as a plant curve. Intersection of plant curve with the appropriate curve on the universal design chart will yield the A and B values which in turn will give the controller parameter values.

III. PLOTTING OF UNIVERSAL DESIGN CHART

Standard form of controller in frequency domain is

$$G_c(A, B) = \frac{(1 + jB)}{(1 + jA)}$$

The above equation in rectangular form is given as

$$G_c(j\omega) = C\frac{(1 + jB)}{(1 + jA)}$$
$$= C\frac{(1 + AB)}{(1 + A^2)} + jC\frac{(B - A)}{(1 + A^2)}$$

This is the basic equation to plot the universal design chart. The plotting of universal design chart is done in two steps.

1. Plotting of the A – curves.
2. Plotting of the B – curves.

PLOTTING OF A CURVES:

A curves are plotted using basic equation by varying the values of A for the fixed value of B. The A curve is drawn with the magnitude on Y-axis and the phase angle on X- axis. This curve is known as A curve. To draw more A curves above procedure can be repeated for different values of A.

PLOTTING OF B CURVES:

B curves are plotted using basic equation by varying the values of B for the fixed value of A. The magnitude is taken on Y- axis and the phase angle is taken on X- axis. This curve is known as B curve. Many values of A gives many B curves. To draw more B curves above procedure can be repeated for different B values.
The universal design indicates the plotting of A and B curves on same plane which results in universal design chart.

IV. TRANSFORMATION OF PID CONTROLLER INTO STANDARD FORM

The frequency response of PID controller is equated to the standard form

$$G_c(j\omega) = C\frac{(1 + jB)}{(1 + jA)} = K_p + (K_i / j\omega) + K_d\, j\omega \qquad (10)$$

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yielding
$$A = \frac{K_i - \omega K_p}{K_i - \omega^2 K_d} \tag{11}$$

$$B = \frac{K_i^2 + \omega^2 K_p^2 + \omega^4 K_d^2 - \omega K_p K_i - 2\omega^2 K_i K_d}{\omega^2 K_i K_d - K_i^2} \tag{12}$$

$$C = K_i / \omega \tag{13}$$

C depends only upon K_i and ω. K_i is determined from the steady-state accuracy specification and ω can be chosen either as the gain crossover frequency, ω_g or as the phase crossover frequency ω_{ph}, depending upon whether the phase margin PM or the gain margin GM is specified. Using inverse relations, the values for the compensator parameters are given by

$$K_p = \frac{K_i (1 + AB)}{\omega(1 + A^2)}$$

$$K_d = (K_i / \omega^2)(1 + (B - A)/(1 + A^2))$$

$$K_i = C\,\omega$$

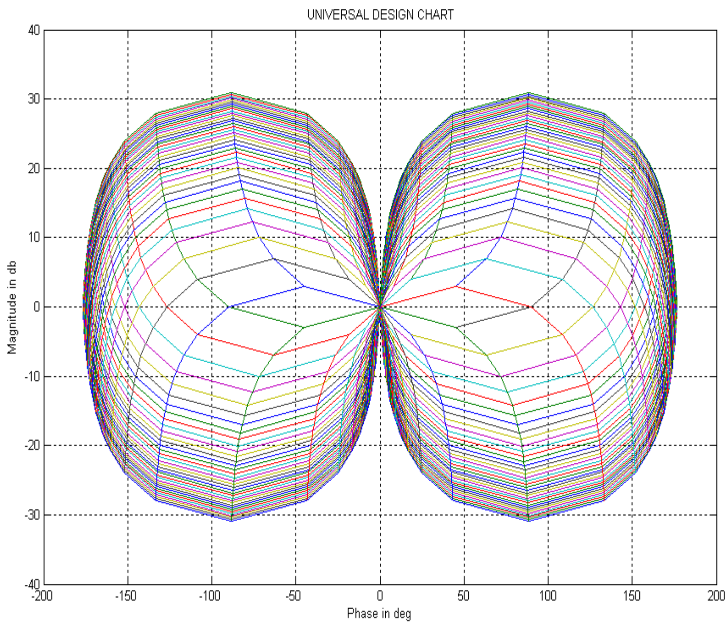

Fig. 1 General Universal Design Chart

V. ILLUSTRATIVE EXAMPLE

Following illustrates the designing of PID controller to satisfy the following specifications:

(i) The phase margin of the system = 48 degree
(ii) Acceleration constant Ka = 2
(iii) The gain crossover frequency must be 2.5 rad / sec for the system $G_p(S) = 3/ S(S^2 + 4S + 5)$, H(S) = 1.

Method: The given plant transfer function is

$$G_p(S) = \frac{3}{S(S^2 + 4S + 5)}$$

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The transfer function of PID controller is

$$G_c(S) = \frac{K_d S^2 + K_p S + K_i}{S}$$

Transformation of controller into standard form yields

$$C = K_i / \omega \text{ (K is determined from steady state requirements)}$$
$$K_p = (K_i / \omega)(1+AB)/(1+A^2)$$
$$K_d = (K_i/\omega^2)(1 + (B-A)/(1 + A^2)\, A^2$$

From the data , $K_a = 2$

$$K_a = \lim_{S \to 0} S^2 (G_p(S)\, G_c(S))$$

$$K_a = \lim_{S \to 0} S^2 \frac{(K_d S^2 + K_p S + K_i)\, 3}{S\, S(S^2 + 4S + 5)} = 2$$

$$K_a = 3 K_i / 5 = 2$$
$$K_i = 3.333$$

Also

$G_p(jw) = -12/(25 + 6\omega^2 + \omega^4) \;\; - j\,(15/(\omega - 3\omega))/(25+6\omega^2 + \omega^4))$

$x = -12 / (25 + 6\omega^2 + \omega^4)$

$y = (15 / (\omega - 3\omega)) / (25+6\omega^2 + \omega^4))$

$Z = x - j\,y$

$|G_p(j\omega)|\ db = 20 \log(abs(Z))$

$|G_c(j\omega)|\ db = -|CG_p(j\omega)|\ db$

$\angle G_p(j\omega) = -90 - \tan^{-1}(4\omega/(5 - \omega^2))$

or

$= -90 - (180 - \tan^{-1}(4\omega/(5 - \omega^2)))$

$\angle G_c(A,B) = -\angle \mathbf{C}G_p(j\omega) + PM - 180$

$= -(-90 -(\tan^{-1}(4\omega / (5 - \omega^2)))) + PM - 180$

$= 90 + \tan^{-1}(4\omega / (5 - \omega^2)) - 132$

$=90 + (180 + \tan^{-1}(4\omega / (5 - \omega^2))) + 132$

Using above equations plant curve is drawn on universal chart.

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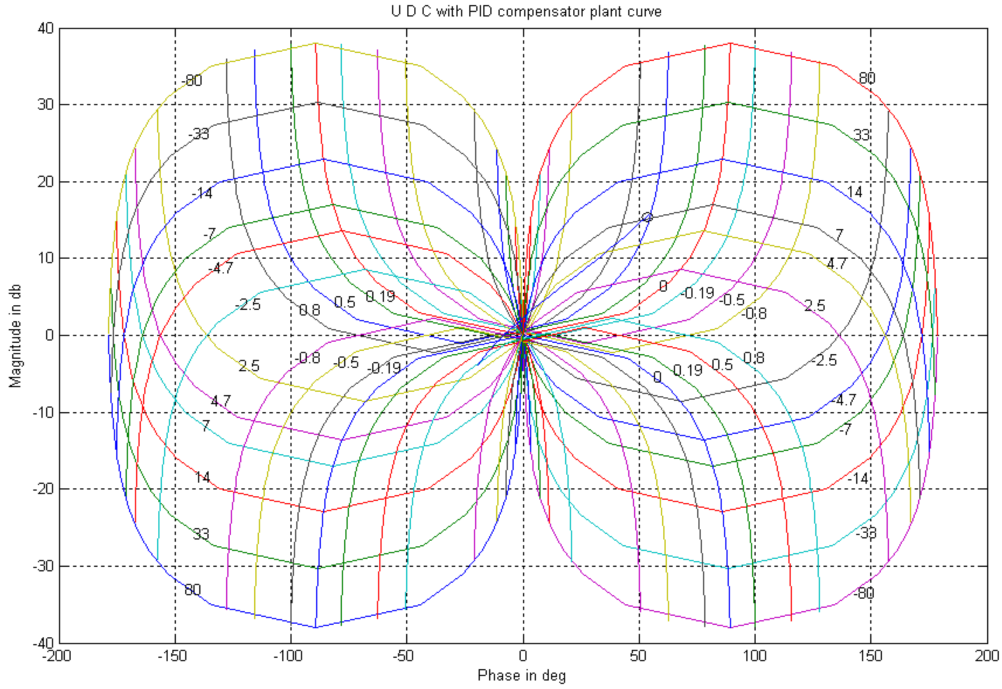

Fig. 2 Universal design chart with PID controller plant curve

From above figure A, B values are 0.5 and 7. Therefore

$K_p = 4.8$
$K_d = 3.3$
With these values the compensated system becomes

$$G_p(S)G_c(S) = (4.8 + (3.33/S) + 3.3S)\ \frac{3}{S\,(S^2 + 4S + 5)}$$

VI. CONCLUSIONS

This paper provides design of PID controllers using a universal chart which enables an efficient design of the most common compensators also. A method for PID controller design is presented. It is based on the idea of normalizing compensator parameters so that a design chart can be generated once for all. A design example is carried through to illustrate the use of this design chart. The coding is done in MATLAB. The method proves to take less time and iterations compared to conventional controller design. The method involves less complexity. Simultaneous fulfillment of the specifications of steady-state error, phase margin, gain margin, and gain crossover frequency can be achieved with accuracy.

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BIOGRAPHY

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