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# On Certain Results on Extended Beta Function Associated to Multiple Generating Functions

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**ABSTRACT**: In this paper certain results involving extended Beta function $B(\alpha,\beta;A)$ in the form of series integral representation are established. A multiple generating function $\psi(x_1,...,x_r;t_1,...,t_r)$ is considered to established the results for the more general form of extended beta function. Many new results involving the product of simpler functions are obtained here. Some known results are also shown to be obtained.

**KEYWORDS:** Extended Beta function, Generating functions, Laguerre polynomials, Humbert's confluent hypergeometric function.

## I. INTRODUCTION

The well-known Euler integrals for Beta and Gamma functions are defined and represented by the following [9]

$$B(\alpha,\beta)=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx; \quad \mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0 \tag{1.1}$$

$$\Gamma\alpha=\int_0^\infty x^{\alpha-1}e^{-x}dx; \quad \mathrm{Re}(\alpha)>0 \tag{1.2}$$

The relation between these two functions is as follows [9]

$$B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}; \quad \mathrm{Re}(\alpha)>0, \mathrm{Re}(\beta)>0 \tag{1.3}$$

These Euler functions defined in (1.1) and (1.2) have many applications in pure and applied mathematics and widely in the field of science and engineering. In the literature these functions are represented in many different forms of finite and infinite integrals. Due to the usefulness of these functions Chaudhary and Zubair [2] and Chaudhary et al. [1] studied these functions by inserting the regularization factor $e^{-A/x}$ and $e^{[-A/x(1-x)]}$ in the integrands. They extended the definition of Beta and Gamma functions in the following forms. The extended Beta function is represented as follows [4, p.1994, eq.(1.6)]:

$$B(\alpha,\beta;A)=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\exp\left[\frac{-A}{x(1-x)}\right]dx; \quad \mathrm{Re}(A)>0 \tag{1.4}$$

and the extended Gamma function is represented in the following manner [1, p.20, eq.(1.2)]:

$$\Gamma_A(\alpha)=\int_0^\infty x^{\alpha-1}\exp\left[-x-\frac{A}{x}\right]dx; \quad \mathrm{Re}(A)>0 \tag{1.5}$$

For $A=0$ the extended Beta function $B(\alpha,\beta;A)$ and extended Gamma function $\Gamma_A(\alpha)$ reduce to the Beta and Gamma functions given respectively in (1.1) and (1.2).

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The generalized hypergeometric function is defined and represented as follows [12, p.42, eq. (1)]:

$$ {}_pF_q\left[\begin{matrix} a_1,...,a_p\,; \\ b_1,...,b_q\,; \end{matrix} x\right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{p}(a_j)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{x^n}{n!}; \qquad |x|<1 \tag{1.6}$$

If $p=1,\ q=0$ it reduce into the following binomial series [12, p.44, eq.(8)]

$$ {}_1F_0\left[\begin{matrix} a\,; \\ -; \end{matrix} x\right] = \sum_{n=0}^{\infty} (a)_n \frac{x^n}{n!} = (1-x)^{-a} \tag{1.7}$$

The following four generating functions are also required in the sequel,

If $L_n^{(\delta)}(x)$ is the Laguerre polynomials then we have [12, p.170, eq.(9(ii))]

$$ (1+t)^{\delta} \exp(-xt) = \sum_{n=0}^{\infty} L_n^{(\delta-n)}(x)\, t^n \tag{1.8}$$

If $Y_n(x,a,b)$ is the generalized Bessel Polynomials then we have [12, p.170, eq.(9(iii))]

$$ \left(1-\frac{xt}{b}\right)^{(t-a)} \exp(t) = \sum_{n=0}^{\infty} Y_n(x,a-n,b)\frac{t^n}{n!} \tag{1.9}$$

For the Humbert's confluent hypergeometric function $\phi_2$, we have [8, p.409, eq.(12)]

$$ \phi_2[a,a;b;x,t] = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n}\, {}_1F_1\left[\begin{matrix} a\ \ ; \\ b+n; \end{matrix} x\right] \frac{t^n}{n!} \tag{1.10}$$

Also for the Hermite polynomials $H_n(x)$ the following relation [12, p.83, eq. (11)]

$$ \exp\left(2xt-t^2\right) = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}; \qquad |x|,|t|<\infty \tag{1.11}$$

We have established here the following two main results for the extended Beta function defined in (1.4) associated to multivariable generating function defined formally by [12, p.81, eq.(12)]

$$ \psi(x_1,...,x_r;t_1,...,t_r) = \sum_{n_1,...,n_r=0}^{\infty} C(n_1,...,n_r) f_{n_1,...,n_r}(x_1,...,x_r) t_1^{n_1}...t_r^{n_r} \tag{1.12}$$

where each member of the generated set

$\{f_{n_1,...,n_r}(x_1,...,x_r)\}_{n_i=0}^{\infty} \quad \forall\ i \in 1,2,...,r$ is independent of $t_1,...,t_r$ and the coefficients set

$\{C(\mathrm{n}_1,...,\mathrm{n}_r)\}_{n_i=0}^{\infty} \quad \forall\ i=1,2,...,r$ may contain the parameters of the set

$\{f_{n_1,...,n_r}(x_1,...,x_r)\}_{n_i=0}^{\infty} \quad \forall\ i \in 1,2,...,r$ but is independent of $x_1,...,x_r$ and $t_1,...,t_r$.

Motivated by the work on series integral representation of Hurwitz zeta function and Hurwitz-Lerch zeta function [5,6,7,8], the following series integral representation are obtained here.

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## II. MAIN RESULTS

If the generating function defined in (1.12) be such that $\psi(x_1,\dots,x_r;t_1u^{\mu_1}(1-u)^{\sigma_1},\dots,t_ru^{\mu_r}(1-u)^{\sigma_r})$ remains uniformly convergent for $u\in(0,1)$ then we have the following results in the form of two theorems.

**Theorem-2.1**

Let $\lambda\in C,\rho\geq 0,\ \mu_i,\sigma_i\geq 0$ and $\mu_i+\sigma_i\geq 0\ \forall i\in\{1,\dots,r\}$ then

$$\sum_{k=0}^{\infty}\frac{(\lambda)_k}{k!}\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\psi\left[x_1,\dots,x_r;t_1u^{u_1}(1-u)^{\sigma_1},\dots,t_ru^{u_r}(1-u)^{\sigma_r}\right]\exp\left[\frac{-(A+kp)}{u(1-u)}\right]$$

$$\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho}du$$

$$=\sum_{n_1,\dots,n_r,k=0}^{\infty}\frac{(\lambda-\rho)_k}{k!}C(n_1,\dots,n_r)f_{n_1,\dots,n_r}(x_1,\dots,x_r).$$

$$B\left(\alpha+\sum_{i=1}^{r}(\mu_in_i),\beta+\sum_{i=1}^{r}(\sigma_in_i);(A+kp)\right)t_1^{n_1}\dots t_r^{n_r}\tag{2.1}$$

$\min\{\mathrm{Re}(\alpha,\beta,A,p)\}>0$

Where the series involved are absolutely convergent and the results in (2.1) exists.

**Theorem-2.2**

Let $\lambda\in C,\ \rho\geq 0,\ \mu_i,\sigma_i\geq 0;\mu_i+\sigma_i>0\ \forall\ i\in\{1,\dots,s\}$ .Then for $\delta_i>0$ and $\max\{|y_1|,\dots,|y_r|\}<1$, then we have

$$\sum_{k=0}^{\infty}\frac{(\lambda)_k}{k!}\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\prod_{i=1}^{r}\left[\left(1-y_iu\right)^{-\delta_i}\right]\psi\left[x_1,\dots,x_s;t_1u^{\mu_1}(1-u)^{\sigma_1},\dots,t_su^{\mu_s}(1-u)^{\sigma_s}\right].$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho}du$$

$$=\sum_{k,m_1,\dots,m_r,n_1,\dots,n_s=0}^{\infty}\frac{(\lambda-\rho)_k}{k!}(\delta_1)_{m_1},\dots,(\delta_r)_{m_r}m_r\frac{y_1^{m_1}}{m_1!},\dots,\frac{y_r^{m_r}}{m_r!}.$$

$$C(n_1,\dots,n_s)f_{n_1,\dots,n_s}(x_1,\dots,x_s)t_1^{n_1}\dots t_s^{n_s}B\left(\alpha+\sum_{i=1}^{r}m_i+\sum_{i=1}^{s}n_i\mu_i,B+\sum_{i=1}^{s}n_i\sigma_i;(A+pk)\right)\tag{2.2}$$

$\min\{\mathrm{Re}(\alpha,\beta,A,p)\}>0$ Where the series involved are absolutely convergent and the results in (2.2) exists.

**Proof of Theorem-2.1**

To prove the result of Theorem-2.1 we denote its left-hand side by $\Delta_1$ i.e.

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$$\Delta_1 = \sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \psi(x_1,\ldots,x_r; t_1 u^{u_1}(1-u)^{\sigma_1},\ldots,t_r u^{u_r}(1-u)^{\sigma_r}.$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho} du$$

Now using the definition of multivariable generating function give in (1.12) and on changing the order of integration and summation we obtain

$$\Delta_1 = \sum_{n_1,\ldots,n_r=0}^{\infty} C(n_1,\ldots,n_r) f_{n_1,\ldots,n_r}(x_1,\ldots,x_r) t_1^{n_1} \ldots t_r^{n_r} \int_0^1 u^{\alpha+\sum_{i=1}^{r} n_i\mu_i - 1} (1-u)^{\beta+\sum_{i=1}^{r}\sigma_i n_i - 1}.$$

$$\exp\left[\frac{-A}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{-(\lambda-\rho)} du$$

On expanding $\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{-(\lambda-\rho)}$ making the use of binomial relation (1.7) and in view of definition of extended Beta function (1.4), we atonce arrive at the desired result of Theorem-2.1

The proof of Theorem-2.2 can be developed similarly.

**3.Special cases**

In this section we have obtained the certain special cases and application of Theorem-2.1 and Theorem-2.2 in the form of corollaries.

If in Theorem-2.1 we set

$$\psi(x_1,\ldots,x_r; t_1 u^{\mu_1}(1-u)^{\sigma_1},\ldots,t_r u^{\mu_r}(1-u)^{\sigma_r}) = \prod_{i=1}^{r}\left[1 - x_i t_i u^{\mu_i}(1-u)^{\sigma_i}\right]^{-a_i} \quad \text{with}$$

$(x_i = 1 \; \forall \; i = 1,\ldots,r)$ then in view of the result in (1.7) the result in Theorem-2.1 reduce to the following series integral formula for extended Beta function.

**Corollary 3.1:**

Let $\lambda \in C, \rho \geq 0, \; \mu_i, \sigma_i \geq 0$ and $\mu_i + \sigma_i \geq 0 \;\; \forall i \in \{1,\ldots,r\}$ then for $a_i > 0 \;\; (i = 1,\ldots,r)$ we have

$$\sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \prod_{i=1}^{r}\left[1 - t_i u^{\mu_i}(1-u)^{\sigma_i}\right]^{-a_i} \exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho} du$$

$$= \sum_{n_1,\ldots,n_r,k=0}^{\infty} \frac{(\lambda-\rho)_k}{k!} \prod_{i=1}^{r}\left[\frac{(a_i)_{n_i}}{n_i!} t_i^{n_i}\right] B\left(\alpha + \sum_{i=1}^{r}\mu_i n_i, \beta + \sum_{i=1}^{r}\sigma_i n_i ; A+kp\right). \tag{3.1}$$

$$\min\{\text{Re}(\alpha,\beta,A,p)\} > 0$$

Where the series involved are absolutely convergent and the result in (3.1) exists.

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The result (3.1) in turn at $r=1$ reduces to the known result due to Jaimini and Sharma [3, pp.354-355, eq.(12)]
We remark here that for $r=1, \lambda=\rho, \beta=\gamma-\alpha$ and making the suitable changes therein the result (3.1) immediately reduce into the known result due to Khan et al. [4, pp.1997-99, eq.(3.5)]

Also for $r=1, A \to 0, \mu=\sigma=2, \delta=\lambda$ the result (3.1) reduces to an another known result [4, p.2000, eq.(4.10)].

If in the Theorem-2.1)we set

$$\psi\left(x_1,...,x_r;t_1u^{\mu_1}(1-u)^{\sigma_1},...,t_ru^{\mu_r}(1-u)^{\sigma_r}\right)$$

$$=\prod_{i=1}^{r}\left[\left[1-t_iu^{\mu_i}(1-u)^{\sigma_i}\right]^{\delta_i}.\exp\left[-x_it_iu^{\mu_i}(1-u)^{\sigma_i}\right]\right]$$

Then in the view of the result in (1.8) the result in Theorem-2.1 reduces to the following result for extended Beta function involving Laguerre Polynomial.

**Corollary 3.2**

$\lambda \in C, \rho \geq 0,\ \mu_i, \sigma_i \geq 0$ and $\mu_i+\sigma_i \geq 0\ \ \forall i \in \{1,...,r\}$ then for $a_i > 0\ \ (i=1,...,r)$ we have

$$\sum_{k=0}^{\infty}\frac{(\lambda)_k}{k!}\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\prod_{i=1}^{r}\left[\left[1-t_iu^{\mu_i}(1-u)^{\sigma_i}\right]^{\delta_i}\exp\left[-x_it_iu^{\mu_i}(1-u)^{\sigma_i}\right]\right]$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho}du$$

$$=\sum_{k,n_1,...,n_r=0}^{\infty}\frac{(\lambda-\rho)_k}{k!}\left(\prod_{i=1}^{r}L_{n_i}^{\delta_i-n_i}(x_i)t_i^{n_i}\right)B\left(\alpha+\sum_{i=1}^{r}\mu_in_i,\beta+\sum_{i=1}^{r}\sigma_in_i;A+kp\right) \quad (3.2)$$

$\min\{\mathrm{Re}(\alpha,\beta,A,p)\}>0$

Where the series involved are absolutely convergent and the result in (3.2) exists.
The result (3.2) in turn at $r=1$ reduce to the known result due to Jaimini and Sharma [3, p.355, eq.(13)].

We remark here that for $r=1, \lambda=\rho, \beta=\gamma-\alpha$ the result (3.2) immediately reduce into the known result due to Khan et al. [4, pp.1997-999, eq.(3.7)].

Also for $r=1, A \to 0, \mu=\sigma=1, \rho=\lambda$ the result in (3.2) reduce to an another known result [4, p.2000, eq.(4.11)].

If in Theorem-2.1 we set

$$\psi\left(x_1,...,x_r;t_1u^{\mu_1}(1-u)^{\sigma_1},...,t_ru^{\mu_r}(1-u)^{\sigma_r}\right)$$

$$=\prod_{i=1}^{r}\left[\left(1-\frac{x_it_iu^{\mu_i}(1-u)^{\sigma_i}}{b_i}\right)^{1-a_i}\exp\left(t_iu^{\mu_i}(1-u)^{\sigma_i}\right)\right]$$

then in view of the result (1.9) Theorem-2.1 reduces to the following result for extended Beta function involving generalized Bessel polynomial.

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**Corollary 3.3**

$\lambda > 0, \rho \geq 0,\ \mu_i, \sigma_i \geq 0$ and $a_i \leq 1,\ b_i \neq 0\ \forall i \in \{1,...,r\}$ then we have

$$\sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \prod_{i=1}^{r} \left[ \left(1 - \frac{x_i t_i u^{\mu_i}(1-u)^{\sigma_i}}{b_i}\right)^{1-a_i} \exp\left(t_i u^{\mu_i}(1-u)^{\sigma_i}\right) \right]$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho} du$$

$$= \sum_{k,n_1,...,n_r=0}^{\infty} \frac{(\lambda-\rho)_k}{k!} \prod_{i=1}^{r} y_{n_i}(x_i, a_i - n_i, b_i) \frac{t_1^{n_1}}{n_1!} ... \frac{t_r^{n_r}}{n_r!} B\left(\alpha + \sum_{i=1}^{r} \mu_i n_i, \beta + \sum_{i=1}^{r} \sigma_i n_i; A+kp\right) \tag{3.3}$$

$$\min\{\mathrm{Re}(\alpha,\beta,A,p)\}>0$$

Where the series involved are absolutely convergent and the result in (3.3) exists.
The result (3.3) in turn at $r=1$ i.e. $t_2 = t_3 = ... = t_r = 0$ reduce to the known result due to Jaimini and Sharma [3, pp.355-356, eq.(14)].

We remark here that for $r=1, \lambda=\rho, \beta=\gamma-\alpha$ the result (3.3) immediately reduce into the known result due to Khan et al. [4, pp.1997-1999, eq.(3.10)].

Also for $r=1,\ A \to 0,\ \mu=\sigma=1,\ \rho=\lambda$ the result in (3.3) reduce to an another known result [4, p.2000, eq.(4.12)].

If in Theorem-2.1 we set

$$\psi\left(x_1,...,x_r; t_1 u^{\mu_1}(1-u)^{\sigma_1},...,t_r u^{\mu_r}(1-u)^{\sigma_r}\right) = \prod_{i=1}^{r}\left\{\phi_2[a_i, a_i; b_i; x_i, t_i u^{\mu_i}(1-u)^{\sigma_i}]\right\}$$

Where $\phi_2$ is Humbert's confluent hypergeometric series in two variables
then in view of (1.10) Theorem-2.1 the result reduces to the following result.

**Corollary 3.4**

Let $\lambda > 0, \rho \geq 0,\ \mu_i, \sigma_i \geq 0$ and $\forall i \in \{1,...,r\}$ then we have

$$\sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \prod_{i=1}^{r} \{\phi_2(a_i, a_i; b_i; x_i, t_i u^{\mu_i}(1-u)^{\sigma_i})\}$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho} du$$

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$$= \sum_{k,n_1,\ldots,n_r=0}^{\infty} \frac{(\lambda-\rho)_k}{k!} \frac{(a_1)_{n_1}\ldots(a_r)_{n_r}}{(b_1)_{n_1}\ldots(b_r)_{n_r}} . \prod_{i=1}^{r} {}_1F_1\left[\begin{matrix} a_i \\ b_i+n_i \end{matrix} ; \quad x_i\right] \frac{t_1^{n_1}}{n_1!} \ldots \frac{t_r^{n_r}}{n_r!}$$

$$B\left(\alpha+\sum_{i=1}^{r}\mu_i n_i, \beta+\sum_{i=1}^{r}\sigma_i n_i; A+kp\right) \tag{3.4}$$

$$\min\{(\alpha,\beta,A,p)\}>0$$

Where the series involved are absolutely convergent and the result in (3.4) exists.

The result (3.4) in turn at $r=1$ reduces to the known result due to Jaimini and Sharma [3, p.356, eq.(15)].

We remark here at $r=1,\ \lambda=\rho,\ \beta=\gamma-\alpha$ the result (3.4) reduces into the known result due to Khan et al. [4, pp.1997-1999, eq.(3.12)].

Also for $r=1,\ A\to 0,\ \mu=\sigma=1,\ \rho=\lambda$ the result in (3.4) reduces to an another known result in [4, p.2000, eq.(4.13)].
If in Theorem-2.1 we set

$$\psi\left(x_1,\ldots,x_r;t_1u^{\mu_1}(1-u)^{\sigma_1},\ldots,t_ru^{\mu_r}(1-u)^{\sigma_r}\right)=\prod_{i=1}^{r}\exp\left[2x_it_iu^{\mu_i}(1-u)^{\sigma_i}-t_i^2u^{2\mu_i}(1-u)^{2\sigma_i}\right]$$

$$|x|,|t|<\infty$$

Then in view of (1.11) the result inTheorem-2.1 reduce to the following result for extended Beta function involving Hermite Polynomials.

**Corollary 3.5**

Let $\lambda > C, \rho \geq 0,\ \mu_i, \sigma_i \geq 0$ and $a_i \leq 1, \quad \forall i \in \{1,\ldots,r\}$ then we have

$$\sum_{k=0}^{\infty}\frac{(\lambda)_k}{k!}\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}\prod_{i=1}^{r}\exp\left[2x_it_iu^{\mu_i}(1-u)^{\sigma_i}-t_i^2u^{2\mu_i}(1-u)^{2\sigma_i}\right].$$

$$\exp\left[\frac{-(A+kp)}{u(1-u)}\right]\left[1-\exp\left(\frac{-p}{u(1-u)}\right)\right]^{\rho}du$$

$$= \sum_{k,n_1,\ldots,n_r=0}^{\infty}\frac{(\lambda-\rho)_k}{k!}\prod_{i=1}^{r}\left[H_{n_i}(x_i)\right]\frac{t_1^{n_1}}{n_1!}\ldots\frac{t_r^{n_r}}{n_r!}.$$

$$B\left(\alpha+\sum_{i=1}^{r}\mu_i n_i, \beta+\sum_{i=1}^{r}\sigma_i n_i; A+kp\right) \tag{3.5}$$

$$\mathrm{Re}(\alpha)>0;\ \mathrm{Re}(\beta)>0;\ \ \mu,\sigma\geq 0;\ \ |x|,|t|<\infty;$$

Where the series involved are absolutely convergent and the result in (3.5) exists.
The result (3.5) in turn at $r=1$ reduces into the known result due to Jaimini and Sharma [3, p.357, eq.(16)]

We remark here that for $r=1,\ \lambda=\rho$ and $\beta=\gamma-\alpha$ the result (3.5) reduces into the known result due to Khan et al. [4, pp.1997-1999, eq.(3.14)].

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If in the Theorem-2.2 we take $A \to 0,\ \lambda = \rho$ and $\beta = \gamma - \alpha$ let the function $\psi\left[x_1,\dots,x_s;t_1u^{\mu_1}(1-u)^{\sigma_1},\dots,t_su^{\mu_s}(1-u)^{\sigma_s}\right]$ defined in (1.12) remain uniformly convergent for $u \in (0,1)$ then it reduces to an integral contained in

**Corollary 3.6**

**Let** $\delta_i > 0 (i = 1,\dots,n)$ , $\max\{|x_1|,\dots,|x_n|\} < 1$ and $\mu_i, \sigma_i > 0$ Then we have

$$\int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}\prod_{i=1}^{r}\left[(1-y_iu)^{-\delta_i}\right]\psi\left[x_1,\dots,x_s;t_1u^{\mu_1}(1-u)^{\sigma_1},\dots,t_su^{\mu_s}(1-u)^{\sigma_s}\right]du$$

$$= \sum_{n_1,\dots,n_s=0}^{\infty} C(n_1,\dots,n_s)f_{n_1,\dots,n_s}(x_1,\dots,x_s)t_1^{n_1},\dots,t_s^{n_s}$$

$$B\left(\alpha+\sum_{i=1}^{s}\mu_in_i,\gamma-\alpha+\sum_{i=1}^{s}\sigma_in_i\right)(\delta_1)_{m_1},\dots(\delta_r)_{m_r}\frac{y_1^{m_1}}{m_1!},\dots,\frac{y_r^{m_r}}{m_r!}$$

$$\sum_{m_1,\dots,m_r=0}^{\infty}\frac{\left(\alpha+\sum_{i=1}^{s}\mu_in_i\right)_{m_1+,\dots,+m_r}}{\left(\gamma+\sum_{i=1}^{s}(\mu_i+\sigma_i)_{n_i}\right)_{m_1+,\dots,+m_r}}$$

$$= \sum_{n_1,\dots,n_s=0}^{\infty} C(n_1,\dots,n_s)f_{n_1,\dots,n_s}(x_1,\dots,x_s)B\left(\alpha+\sum_{i=1}^{s}\mu_in_i,\gamma-\alpha+\sum_{i=1}^{s}\sigma_in_i\right)$$

$$F_D^{(r)}\left[\alpha+\sum_{i=1}^{s}\mu_in_i,\ \delta_1,\dots,\delta_r;\gamma+\sum_{i=1}^{s}(\mu_i+\sigma_i)n_i\ ;y_1\dots y_r\right]\ t_1^{n_1},\dots,t_s^{n_s} \tag{3.6}$$

Where $F_D^{(r)}[\cdot]$ is the generalized Lauricella function of $r$ variable and the series on the right-hand side of (3.6) is absolutely convergent and the result in (3.6) exists.

The result (3.6) in turn at $s = 1$ reduce to the known result due to Jaimini and Sharma [ 3, p.357, eq.(17)]

$$\int_0^1 u^{\alpha-1}(1-u)^{\gamma-\alpha-1}\prod_{i=1}^{r}[1-x_iu]^{-\delta_i}F\left[x,tu^{\mu}(1-u)^{\sigma}\right]du$$

$$F(x,t) = \sum_{n=0}^{\infty}c_nf_n(x)t^n \tag{3.7}$$

Where $F(x,t) = \sum_{n=0}^{\infty}c_nf_n(x)t^n$ is the two variable generating function. The result (3.7) at $\mu = \sigma = 1$ provides the known result [4, p.2000, eq.(5.5)] at $r = 2$ the result (3.7) reduces immediately to the known result [4, p.2000, eq.(5.4)] which is turn on setting $F(x,t) = (1-xt)^{-a}$ with $x = 1$ provide an another known result in [4, p.2000, eq.(5.6)].

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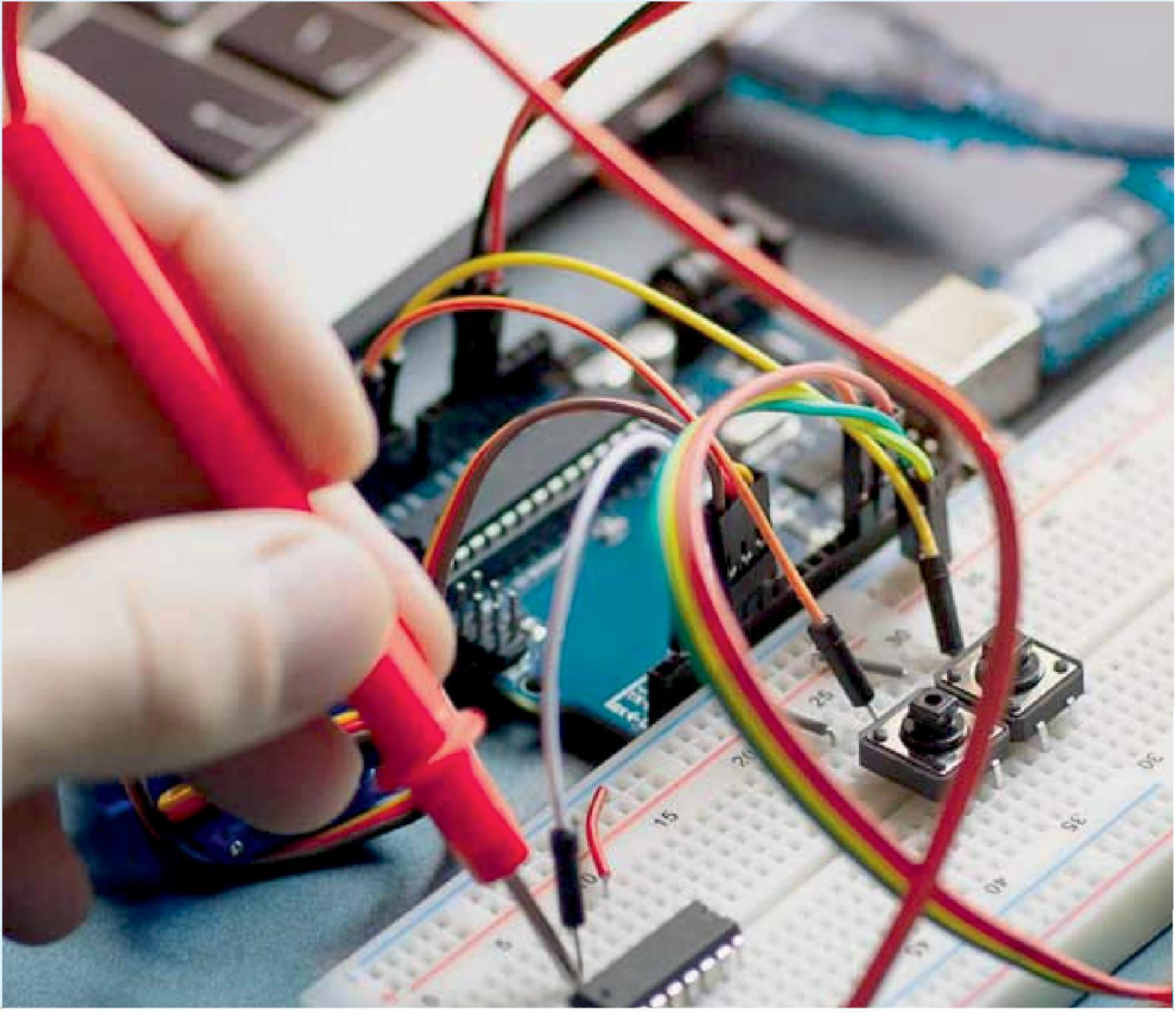

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