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# Comparison of Position Control of Ball and Beam System using Phase lead and PID Controller

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**ABSTRACT:**The ball and beam system is the classical mechanical system having unstable dynamics and strong nonlinear characteristics which makes the control a challenging task. The control problem is difficult as the ball position changes continuously with the beam angle. In this paper a Phase lead controller and Computational Optimization based PID controller is used to enhance a better position control. The controllers are designed based on some design criteria. The simulations of the proposed controllers are analyzed using MATLAB Simulink and the performances of the system using both the controllers are compared and the response is analyzed. The results show that the PID tuned using Computational optimization shows satisfactory response compared to the phase lead controller.

**KEYWORDS:**nonlinear, position control, phase lead, computational optimization.

### I.INTRODUCTION

The Ball and Beam system is one of the widely used and important laboratory models for control system techniques. Nonlinearities and instabilities are considered to be a major challenge for control systems. The ball and beam system is such a system having nonlinearities and instabilities. The ball and beam is an open loop unstable system that is the ball will continuously roll on beam until a controller is used. Different control strategies can be used to control the ball position. It is generally linked to real time control problems such as control of aircraft during landing. There is a high risk of aircraft tumbling without a controller. Learning the dynamics and control of ball and beam system helps to solve such problems. The objective is to control the ball position to a predefined position. The feedback information of the ball position can be used as the control signal. The control voltage signal operates the DC motor and the torque generated drives the beam to a desired angle.

Many research works have been done on ball and beam system. An approximate form of input-output linearization approach is used in [1].This technique does not approximate the system by a linear system or a family of linear systems, but by a single nonlinear system which is input output linearizable. In [2] the control of ball and beam system is described with neural network. An incremental tracking and Sliding Mode Control is used in [3]and have achieved semi global tracking results for the systems and it can be extended to nonlinear systems as well. This method used two genetic algorithm, one to minimize the integral time weighted absolute error and other to maximize the disturbance rejection. In [4], Ant colony optimization is used for the design of PID controllers. In [5],a state observer is developed to estimate the velocity of ball and angular velocity of motor. Sliding Mode Control is proposed and it overcome the problem associated with singular states and experimentally validated the results. The nonlinear factors and the coupling effect is considered in modeling the ball and beam system is described in [6] where Linear Quadratic Regulator is used. The coupling effects dynamics cannot be eliminated as a simplifying assumption while designing the controller. Meta heuristic technique current search method is used to obtain optimal PID control[7].A strict feedback form based on T-S Fuzzy modeling [8] is used to formulate the dynamic ball and beam system. This method is better than classical dynamic surface control. Lyapunov theorem is used to analyze the robust stability of the ball and beam system.

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## II.DYNAMICS OF BALL AND BEAM SYSTEM

The physical setup of ball and beam is shown in Fig 1. A metal ball is placed on the beam where it is allowed to roll along the beam. One end of the beam is attached with a lever arm and the other end is attached with a servo gear. When the servo gear turns by an angle  $\theta$  the beam angle get changed by the lever by  $\alpha$ . This causes movement of the beam which forces the ball to roll as the acceleration is directly proportional to the tilt of beam angle. A linear transducer is fitted along the beam to measure the ball position. The ball and beam system is open loop unstable system because the ball position increases without limit for a fixed beam angle. So balance control is considered to be a difficult control problem for ball and beam system. Hence a suitable controller is necessary to be designed for the system.

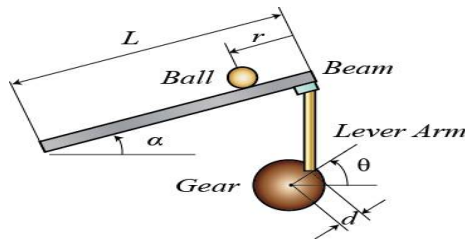


Fig 1: Schematic diagram of Ball and beam system

- $L$  - length of the beam
- $d$  - Distance between centre of the gear and the joint between the gear arms
- $r$  - Ball position
- $\alpha$  - Beam angle
- $\theta$  - Rotation angle of gear

The modeling of Ball and beam system helps to understand the complete dynamics of the system. Lagrange approach is used for the modeling of ball and beam system. It is based on energy balance of the system. Assume that the ball rolls without slipping. The parameters to develop the dynamics of the system is shown in Table 1.

Table 1: PARAMETERS OF BALL AND BEAM SYSTEM

SL NO	PARAMETERS	VALUE
1	Mass of ball	0.064kg
2	Radius of ball	0.0127m
3	Beam length	0.4255m
4	Distance between servo gear shaft and coupled joint	0.0254 m
5	Acceleration due to gravity	-9.8m/s <sup>2</sup>

The Euler-Lagrange is used to define the kinetic and potential energy for the system is shown in equations (1) and (2).

$$T = \frac{1}{2} \left[ (J_b + m_b x^2) \dot{\alpha}_{(1)}^2 + \frac{7}{5} m_b \dot{x}^2 \right]$$

$$V = m_b g x \sin \alpha \tag{2}$$



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$J_b$  - Moment of inertia of beam

$m_b$  - Mass of the ball

$x$  - Position of the ball

$\alpha$  - Beam angle

$g$  -acceleration due to gravity

The Lagrange function is the difference between kinetic energy and potential energy of the system.

$$L = T - V \quad (3)$$

Lagrange equations are obtained as follows.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = \tau \quad (4)$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0 \quad (5)$$

Where  $x$  -ball position on beam

$\alpha$  - beam angle

$$\left( J_b + m_b \dot{x}^2 \right) \ddot{\alpha} + 2m_b \dot{x} \ddot{x} \dot{\alpha} + m_b g x \cos \alpha = \tau \quad (6)$$

$$\ddot{x} + \frac{5}{7} \left( g \sin \alpha - x \dot{\alpha}^2 \right) = 0 \quad (7)$$

Linearizing equation (7) around the operating points, the equation becomes as follows.

$$\ddot{x} = \frac{-5}{7} g \sin \alpha \quad (8)$$

The beam angle and the angle of the gear can be related by equating the arc distance is specified as follows

$$\alpha = \frac{r_{arm}}{L_{beam}} \theta \quad (9)$$

On substituting equation (9) in (8) and taking Laplace transforms, the transfer function of the ball and beam transfer function with system parameters can be represented as

$$\frac{X(s)}{\theta(s)} = \frac{0.4182}{s^2} \quad (10)$$

The ball and beam system is an inherent open loop unstable system. The open loop step response is shown in Fig 2. The response shows that the system is unstable. So some controllers are required to bring the system to a stable one.

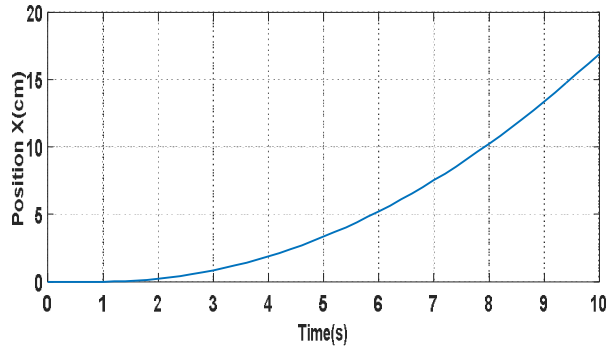


Fig 2 : Open loop response of ball and beam system

### III. DESIGN OF PHASE LEAD CONTROLLER

The lead controllers are extensively used which can increase the stability and speed of response of the system. This controller is designed by determining  $\alpha$  from the amount of phase needed to satisfy the required phase margin and determine T to place the added phase at new gain crossover frequency.

The design criteria for ball and beam system

- a. Settling time should be less than 3.5 sec
- b. Overshoot should be less than 10%

The open loop transfer function of ball and beam system is shown in equation (10) and bode plot of open loop response is shown in Fig 3

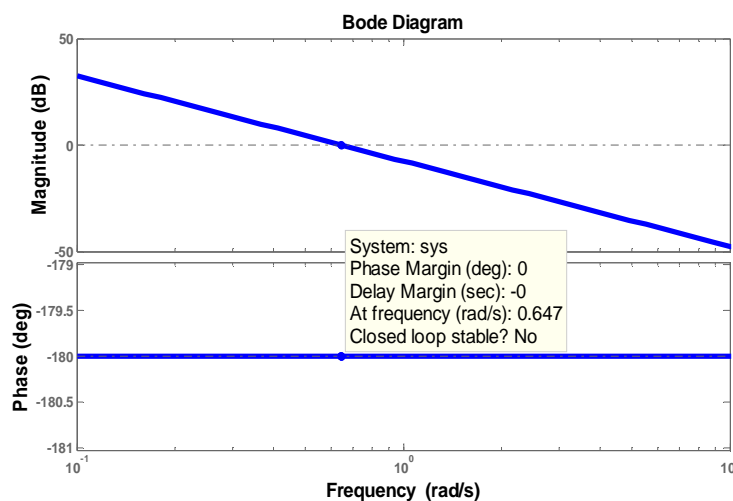


Fig 3: Bode plot of open loop system

From the plot, we obtain the phase margin as zero. This clearly indicates that the open loop system is unstable. Increase the phase margin in order to obtain a stable system response. Phase lead controller is such a compensator controller which helps to attain a stable response using increased phase margin.



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A first order phase lead controller can be represented by

$$C(s) = K \left( \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \right) \quad (11)$$

Where  $T > 0$  and  $\alpha < 1$

$$C(s) = K \left( \frac{1 + Ts}{1 + \alpha Ts} \right) \quad (12)$$

$1/\alpha T$  and  $1/T$  are the corner frequencies. The maximum added phase for phase lead compensator is 90 degree. The lead compensator helps to improve the speed of response and reduces the overshoot. This improves the transient response of the system.

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \quad (13)$$

$\xi$  corresponds to 10% peak overshoot is 0.591. Generally  $\xi * 100$  will give the minimum phase margin needed to obtain the desired overshoot. Therefore we require a phase margin of  $59.1^\circ \approx 60^\circ$

### A. Design steps for obtaining T and $\alpha$

- Find the positive phase needed.  
Here we need at least 60 degree for the controller.
- Find the center frequency  
The relation between bandwidth frequency and settling time gives 1.66 rad/s. Therefore we want a centre frequency just before this. Choose 1
- Obtain the value of  $\alpha$

$$\alpha = \frac{1 - \sin \phi}{1 + \sin \phi}$$

For  $\phi = 60^\circ$   $\alpha = 0.0718$

- Find T and  $\alpha T$  from the given equations

$$T = \frac{1}{w\sqrt{a}}$$

$$\alpha T = \frac{\sqrt{a}}{w}$$

Substituting these values  $\alpha T = 0.1614$  and

$$T = 3.732$$

- Choose the value of K.. It should be chosen such that the gain will make the response faster and should not make the overshoot worse.

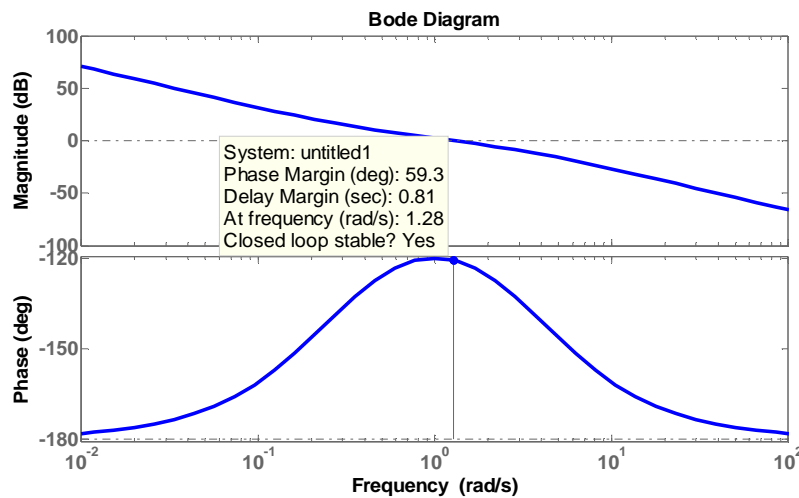


Fig 4: Bode plot of phase lead controller to the system

From the plot it is clear that the phase margin now obtained is near to 60 degree and the closed loop is stable. The transfer function of phase lead controller obtained is

$$C(s) = 5 \left( \frac{1 + 3.73s}{1 + 0.1614s} \right) \quad (14)$$

#### IV. DESIGN OF PID CONTROLLER

PID controllers are widely used in the industry due to its simple structure and robust control. PID controller helps to acquire the most optimum results. The major task of PID controller design is to obtain the controller parameters for achieving desired response. There are many tuning rules available for unstable and integrating systems but only a few tuning methods for double integrating system. So it is essential to develop new control strategies for double integrating system for attaining good response. Here Computational Optimization approach is used to obtain an optimal set of PID values. The PID controller is given by

$$C(s) = K \frac{(s+a)^2}{s} \quad (15)$$

PID controller with computational optimization approach for ball and beam system is shown in fig 5.

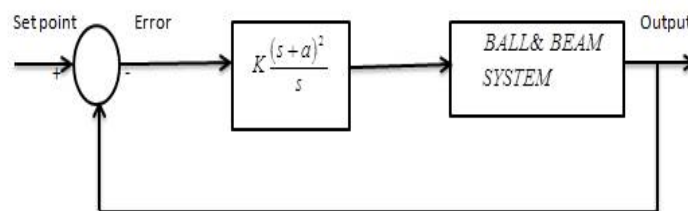


Fig 5: Block diagram for PID controller using computational optimization

It is desired to find the value of 'K' and 'a' such that the maximum overshoot in the unit step response is less than 10% and the settling time should be less than the specified value that is 3.5 s. In this method the optimal set of PID



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parameter values are obtained using MATLAB code. Assume the region of search for 'K' and 'a'. We have to expand it, if the solution does not exist in this region. Select the step size for 'K' and 'a'. The step size should be small in order to avoid large number of computations. Here the step size chosen is 0.2 for 'K' and 0.1 for 'a'. The MATLAB code should be written such that the nested loops in the code first compute for the lowest values and then compute for the highest. We select those values of 'K' and 'a' that gives the smallest overshoot and better settling time.

The values obtained are

$$K=15, \\ a=0.4.$$

The transfer function of the controller is given by

$$C(s) = \frac{15(s + 0.4)^2}{s} \quad (16)$$

$$C(s) = \frac{15s^2 + 12s + 2.4}{s} \quad (17)$$

The transfer function of the PID controller is given by

$$C(s) = \frac{K_d s^2 + K_p s + K_i}{s} \quad (18)$$

The PID parameters obtained are

$$K_p = 12 \quad K_i = 2.4 \quad K_d = 15$$

## V. SIMULATION RESULTS

Fig 6 shows the step response of phase lead controller and PID controller implemented in the ball and beam system. Table 2 shows the performance of step response of Phase Lead controller and computational Optimization PID controller implemented in the Ball and Beam system

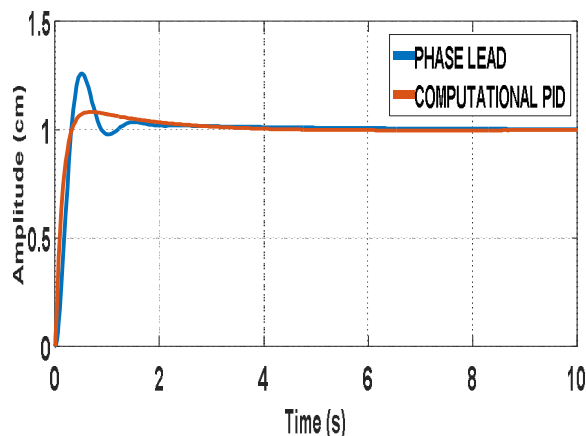


Fig 6: Comparison of step response of Phase lead and computational optimization PID controller.



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Table 2: COMPARISON OF PHASE LEAD AND PID CONTROLLER

CONTROL TECHNIQUE	MAXIMUM OVERSHOOT (%)	SETTLING TIME (s)
PHASE LEAD	26.1	1.92
COMPUTATIONAL PID	8.03	2.1

It is clear that PID tuned by Computational Optimization satisfies the design criteria that is maximum overshoot is less than 10% and settling time is less than 3.5 s. It shows better response than Phase lead controller.

## VI.CONCLUSION

The ball and beam system have been widely used for studying new control techniques. Phase lead controller and Computational Optimization based PID controller is used for the analysis of Ball and Beam System. The proposed controllers are analyzed using MATLAB SIMULINK platform. Simulation results show that the Computational Optimization based PID controller satisfies the design criteria and can perform well for controlling the ball position. The Computational Optimization approach can be applied to any double integrating unstable system in order to obtain better closed loop system performance.

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