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Modelling and Analysis of Active Vehicle Steering Control using nonlinear controller

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ABSTRACT: In this paper, an active trailer steering control strategy is proposed as a means to improve the maneuverability of truck–trailer combinations during cornering. For this, the problem of reducing the swept path width during cornering and eliminating unsafe tail swing of the trailer as a tracking control problem is formulated. A dynamic tractor–trailer model, including off-axle hitching, is designed. Based on this, nonlinear control strategies, namely LQR (Linear Quadratic Regulator) controller is designed to solve this tracking problem for an articulated vehicle. The effectiveness of the control strategy, with the controller design based on LQR controller, is validated using simulations of certain turning maneuver. The results shows that the trailer rear point indeed converges to the path of the tractor front wheel and follows the latter throughout the entire cornering maneuver, while avoiding tail swing at all times.

KEYWORDS: Active trailer steering, dynamics, LQR control, tracking control, Vehicle lateral dynamics**.**

I. INTRODUCTION

In the past few years, with the requirements of high-efficiency goods transportation, articulated vehicles have undergone rapid development. The most popular type of such vehicles consists of a tractor unit coupled to a long, non-steering trailer via a fifth wheel. The conventional tractor-trailer is usually fitted with a trailer that does not steer, which has many negative consequences, such as bad maneuverability at low speed, trailer swing, and rollover at high speed, which are key factors leading to fatal traffic accidents In particular, the space required by a (conventional) tractor–trailer combination during a turning maneuver, the so-called swept path, is an important maneuverability/safety aspect in urban areas or on narrow roads, the available space is limited and turning a maneuver, such as taking a 90° turn or taking a roundabout, can be a difficult and even an unsafe task, especially for long combination (truck–trailer) vehicles.

A solution to reduce the swept path width of a tractor trailer combination is found in the application of trailer axle steering, as evidenced by existing trailer steering systems on the market [1], [2]. Even though these systems reduce the swept path width, a further reduction in swept path width can be obtained using more advanced control strategies for trailer axle steering. Furthermore, these systems generate tail swing during transient cornering, i.e., during entering and exiting a turn, which represents a serious safety hazard. Passive steering systems can reduce tire wear and help improve low-speed steady state maneuverability of the articulated vehicles. However, they cannot achieve good performance in transient steering at high speeds [3].

To overcome the negative consequences above, many researchers are aware that handling of articulated vehicles can benefit a lot from active steering. Their research can be categorized into two groups. In the first group, the focus is on the active steering of a single trailer to improve the maneuverability and stability of the trailer. A control method was presented to reduce tail swing on a rigid truck to make the rear of the truck body follow the path of the front axle [4]. Their results showed that this could effectively reduce swept path without tail swing. Their theory laid the foundation for the path following controller. [5] studied the turning behaviour of the tractor-trailer with steerable wheels. The trailer followed the trajectory of the front unit by active trailer steering. The controller performed well at low speeds and along gentle curves; however, the performance along sharp curves and at high speeds was not presented. An optimal linear quadratic regulator (LQR) controller for active trailer steering to minimize the rear amplification ratio, as a surrogate for minimum off-tracking was proposed[6].Later a virtual trailer steering control model virtual driver model of the trailer) for the trailer based on the LQR theory was introduced [7]. The idea of lateral position deviation preview LQR controller has also been researched [8]. The active steering controller based on path following performs well along gentle curves; however, it cannot work along sharp curves, especially along the rectangular curves. In the second

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group, since some researchers hold that the driver's steering wheel control depends mainly on his/her perception and feeling of the tractor unit's behaviour, the improvement of the tractor is crucial. The second group focuses mainly on the

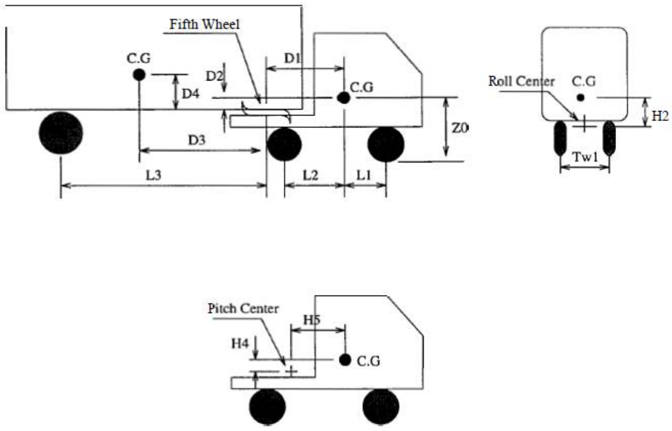

Fig. 1. Schematic Diagram of Complex Vehicle Model

control to improve the tractor handling behaviour. An LQR to minimize the tractor yaw rate, tractor sideslip angle, etc. by active steering also gained attention [9].

The idea of minimization using an LQR controller was followed by [9] and [10]. However, the minimization of yaw rate may not be an optimal choice, and following the desired yaw rate is better. As indicated in the studies mentioned above, much effort has been made in active steering control for tractor or trailer and great progress has been achieved. The current research on active steering of articulated vehicles focuses mainly on improving the tractor's or trailer's handling behaviour. In this study, an active steering controller based on the LQR theory is designed to improve the maneuverability at low speed and lateral stability at high speed of the whole tractor-trailer combination. The tractor and trailer active steering controller is designed to follow the desired yaw rate and minimize the side-slip angle of the tractor's center of gravity (CG) and trailer's CG at the same time, which is very important in the vehicle's stability evaluation. The results show that the active steering controller has the potential to improve the maneuverability and lateral stability of the articulated vehicles.

The main contributions of this paper are as follows. First, we reformulate the problem of reducing the swept path width and avoiding tail swing as a tracking problem for a dynamic (reference) model of the tractor–trailer system including off-axle hitching. We note that most of the work on the control of robotic tractor–trailer systems has focused on path following problems where the tractor needs to follow a certain path [11]–[14], while here we focus on ensuring path following by the trailer of path driver by the tractor in order to support the minimization of the swept path width while avoiding tail swing. The latter problem is highly relevant in the control of trailers of heavy-duty vehicles [15], [16], [17] and the reformulation of the problem into a tracking problem for the variables describing the kinematics of the trailer supports the design of nonlinear controllers formally guaranteeing the stabilization of the desired trajectory. Second, nonlinear controllers solving this tracking problem are proposed, including stability certificates.

The remainder of this paper is organized as follows. In Section II, we derive the kinematic and dynamic tractor–trailer model and the control problem is formulated. In Section III, we propose our controller design scheme, and present related stability results.In Section IV, a simulation study is presented to evidence the effectiveness of the proposed approach and to support controller gain tuning. Finally, in Section V, we present conclusions.

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II. MODEL OF THE TRACTOR TRAILER

In this section, a dynamic model is derived for the off-axle tractor-trailer. We aim to construct a state space model formulation such that front wheel steering angle and its forward velocity are given time-varying inputs and the trailer axle steering velocity is the only control input. Therefore in this formulation the tractor of the robot is steered and driven by an (emulated) driver and the trailer axle is steered automatically, thus improving maneuverability. One Vehicle model and one Tire model are combined to form the System.

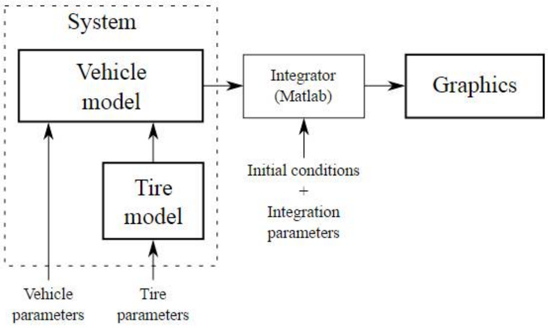

Fig. 2. Block diagram representation of Tractor-trailer system

The kinetic energy of the tractor-semitrailer vehicle can be obtained by adding the kinetic energy component of the tractor and that of the trailer.

$$T_1 = \frac{1}{2}\mathrm{m}_1.v_{CG1}.v_{CG1} + \frac{1}{2}\omega_{s1}.\mathrm{I}_1.\omega_{s1} \tag{1}$$

Similarly, the kinetic energy of the trailer, denoted **as** *T2*, can be obtained from the translational velocity at the trailer ***C.G.*** and the angular velocity of the trailer's sprung mass coordinate, or

$$T_2 = \frac{1}{2}\mathrm{m}_2.v_{CG2}.v_{CG2} + \frac{1}{2}\omega_{s2}.\mathrm{I}_2.\omega_{s2} \tag{2}$$

The change of the potential energy for the tractor of a tractor semitrailer vehicle is primarily due to the roll motion. However, the change of potential energy for the semitrailer is affected by both the roll and pitch motion at the linking joint (fifth wheel). Furthermore, the compliance at the fifth wheel will be significant in describing the roll motion of the trailer. For simplicity, these complicated coupling will not be modeled. Instead, the roll motion is approximated **as** if the articulation angle is zero, that is, the truck is in straight configuration. This approximation for roll motion will be examined by comparing simulation results and experimental data in section 7. Thus the potential energy can be obtained as

$$V = \mathrm{m}_1 \mathrm{g}\, \mathrm{h}_2 (\cos\phi - 1) + \mathrm{m}_2 \mathrm{g}\, (h_2 - d_2 + d_4)(\cos\phi - 1) \tag{3}$$

The Lagrangian, *L*, is defined **as**

$$L = T_1 + T_2 - V \tag{4}$$

and will be used to derive vehicle body dynamics

PACEJKA TIRE MODEL

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In this Pacejka tire model, nonlinear relationship between tire lateral force and slip angle expressed by a semi empirical model with experimental coefficients. The lateral force can be written as

$$F_y = -\frac{\mu_y}{\mu_{yn}}(F_{y,n}(\alpha_{eq}) + S_v \tag{5}$$

where αeq is the equivalent slip angle

$$F_y = -\frac{\mu_{y0}}{\mu_y}\frac{F_{z0}}{F_z}(\alpha) + S_h \tag{6}$$

and Fy,n is the reference function of the lateral force

$$F_{y,n} = D\sin(C\arctan(B_\alpha - E(B_\alpha - \arctan(B_\alpha)))) \tag{7}$$

The coefficients BB, CC, DD and EE can be written as

$$\begin{aligned} C &= a_o \\ D &= \mu_{y,n}F_z = (\mathrm{a}_1\mathrm{F}_z + \mathrm{a}_2)\,\mathrm{F}_z \\ \mathrm{B} &= \frac{\mathrm{BCD}}{\mathrm{CD}} = \mathrm{a}_3\sin\left\{2\arctan(\frac{\mathrm{F}_z}{\mathrm{a}_4})\right\}(1-a_5|\gamma|) \\ E &= a_6F_z + a_7 \end{aligned} \tag{8}$$

The horizontal and vertical shifts of the curve are calculated as

$$\begin{aligned} S_h &= a_8\gamma + a_9F_z + a_{10} \\ S_v &= a_{11}F_z\gamma + a_{12}F_z + a_{13} \end{aligned} \tag{9}$$

The model implemented here converts the slip angle using the following equation:

$$ALPHA = a\sin(\sin(alpha)); \tag{10}$$

This equation alters the slip angle in such way that the characteristic equation becomes symmetric in relation to the vertical line at 90 degrees and the lateral force becomes zero at 180 degrees. The same analogy can be made with negative values of the slip angle.

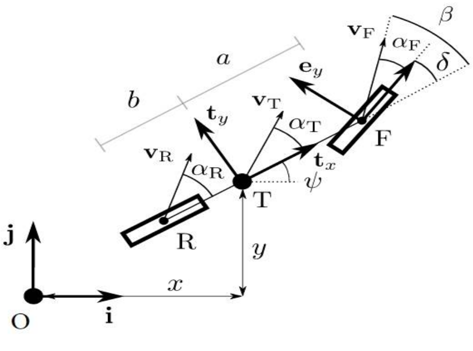

Fig. 3. Bicycle-like schematic model representation of the tractor-trailer

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NON-LINEAR MODEL:

Step-1: State vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \varphi \\ \vartheta_T \\ \alpha_T \\ \dot{\varphi} \end{bmatrix} \tag{11}$$

Step-2: State equation

$$\begin{aligned} \dot{x}_1 &= x_4 \cos(x_3 + x_5) \\ \dot{x}_2 &= x_4 \sin(x_3 + x_5) \\ \dot{x}_3 &= x_6 \\ \dot{x}_4 &= \frac{F_{y,F} \sin(x_5 - \delta) + F_{y,R} \sin x_5}{m_T} \\ \dot{x}_5 &= \frac{F_{y,F} \cos(x_5 - \delta) + F_{y,R} \cos\alpha_T - m_T x_4 x_6}{m_T x_4} \\ \dot{x}_6 &= \frac{F_{y,F}\alpha \cos\delta - F_{y,R} b}{I_T} \end{aligned} \tag{12}$$

The above equations can be linearized as

$$\begin{aligned} \dot{x} &= \vartheta_T \\ \dot{y} &= \vartheta_T, o(\varphi + \alpha_T) \\ \dot{\varphi} &= \dot{\varphi} \\ \dot{\vartheta}_T &= 0 \\ \dot{\alpha}_T &= \frac{F_{y,F} + F_{y,R}}{m_T \vartheta_{T,0}} - \dot{\varphi} \\ \ddot{\varphi} &= \frac{\alpha F_{y,F} - b F_{y,R}}{I_T} \end{aligned} \tag{13}$$

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By neglecting

$$\begin{bmatrix} \dot{y} \\ \dot{\varphi} \\ \dot{\alpha}_T \\ \ddot{\varphi} \end{bmatrix} = \begin{pmatrix} 0 & v_{T,0} & v_{T,0} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{K_F + K_R}{m_T v_{T,0}} & -\frac{m_T v_{T,0} + \frac{aK_F - bKR}{v_{T,0}}}{m_T v_{T,0}} \\ 0 & 0 & -\frac{aKy - bK_R}{I_T} & -\frac{a^2 Ky + b^2 K_R}{I_T v_{T,0}} \end{pmatrix} \begin{bmatrix} y \\ \varphi \\ \alpha T \\ \dot{\varphi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_F}{m_T v_{T,0}} \\ \frac{\alpha K_F}{I_T} \end{bmatrix} \delta \tag{14}$$

Step-3: Slip angle

$$\alpha_F = \arctan\left(\frac{v_T \sin_{\alpha T} + a\dot{\varphi}}{v_T \cos_{\alpha T}}\right) - \delta$$

$$\alpha_R = \arctan\left(\frac{v_T \sin_{\alpha T} - b\dot{\varphi}}{v_T \cos_{\alpha T}}\right) \tag{15}$$

By approximating

$$\alpha_{F,lin} = \alpha_T + \frac{a}{v_{T,o}} \dot{\varphi} - \delta$$

$$\alpha_{F,lin} = \alpha_T - \frac{b}{v_{T,o}} \dot{\varphi} \tag{16}$$

<u>TIRE MODEL</u>

Pacejka

$$F_y = D\sin[C\arctan B_\alpha - E(B_\alpha - \arctan(B_\alpha))] \tag{17}$$

Linear

$$F_y = K_\alpha \tag{18}$$

VEHICLE SIMPLE NONLINEAR

mT	Mass of the car (tractor) [kg]
IT	Moment of inertia the car (tractor) [kg * m2]
a	Distance from front axle of the car (tractor) to the center of mass of the car (tractor) [m]
b	Distance from center of mass of the car (tractor) to the front axle of the car (tractor) [m]
mF0	Mass over the front axle [kg]
mR0	Mass over the rear axle [kg]

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lT	Wheelbase [m]
nF	Number of front tires
nR	Number of rear tires
wT	Track of the car (tractor) [m]
muy	Operational friction coefficient
tire	Tire model
deltaf	Steering angle [rad]
Fxf	Longitudinal force at F [rad]
Fxr	Longitudinal force at R [rad]

Table.1 Nomenclature of control models.

III. CONTROLLER DESIGN

In the stability control of passenger cars, the yaw rate and the side-slip angle are universally chosen as the control variables. The control strategies focus mainly on making the actual yaw rate follow the ideal yaw rate and ensuring the side-slip angle as much as possible. Usually, the reference yaw rate is obtained from the bicycle model according to the steering wheel angle. Here we introduce the stability control strategy into the articulated vehicles. The LQR controller aims to make both the tractor and trailer follow the desired yaw angle and side-slip angle by actively steering the tractor's rear axle and the trailer's axles.

The 6-DOF reference model is used to provide the desired states according to the tractor's front axle steering angle input, which is governed by the driver.Here LQR controller is designed by linearization and pole placement design is incorporated .

$$A = \begin{pmatrix} 0 & v_{T,0} & v_{T,0} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{K_F + K_R}{m_T v_{T,0}} & -\dfrac{m_T v_{T,0} + \dfrac{aK_F - bKR}{v_{T,0}}}{m_T v_{T,0}} \\ 0 & 0 & -\dfrac{aKy - bK_R}{I_T} & -\dfrac{a^2 Ky + b^2 K_R}{I_T v_{T,0}} \end{pmatrix} \quad ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_F}{m_T v_{T,0}} \\ \dfrac{aK_F}{I_T} \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

```
Klqr = lqr(A,B,Q,R);
```
(19)

```
Kplace = place(A,B,poles);
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(20)

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IV. SIMULATION RESULTS

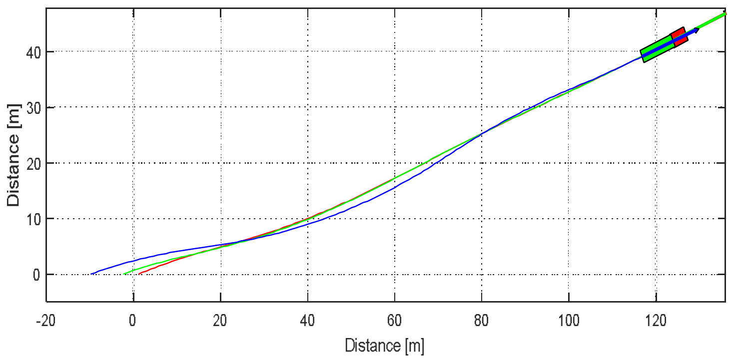

Fig. 4. Tractor-trailer response without trailer steering.

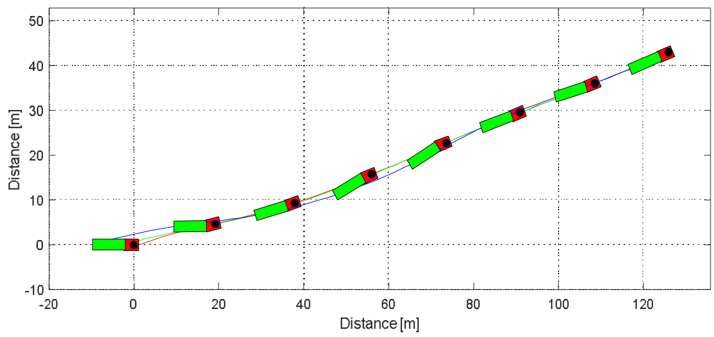

Fig. 5. Swept path for the case without active trailer steering of an articulated vehicle.

Fig.4 shows the response of an articulated vehicle without trailer steering. The detailed path indicating the swept path and tail swing are represented by Fig.5

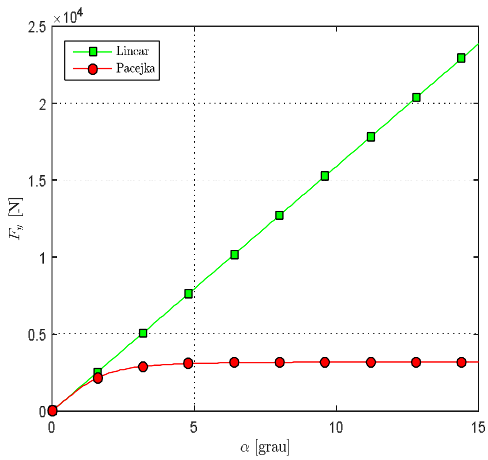

Fig. 6. Comparison of Linear and Pacejka tire models.

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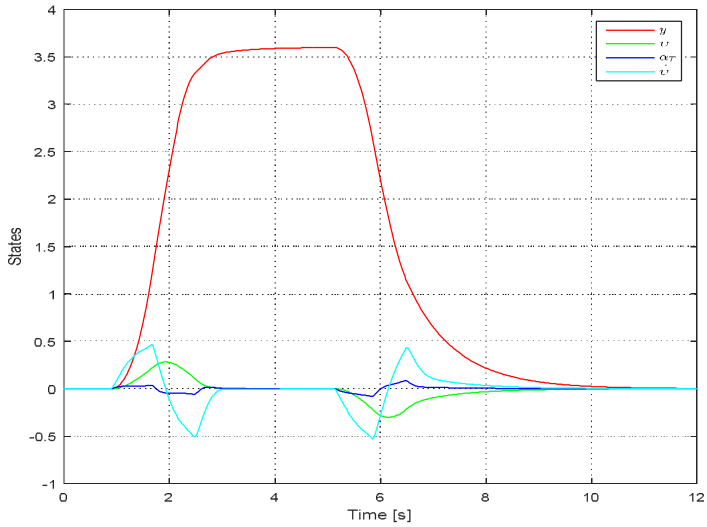

Fig. 7. Graphical representation of various states of Tractor-trailer.

Fig.6 shows comparison between linear and pacejka tire models The Pacejka tire model calculates lateral force and aligning torque based on slip angle.

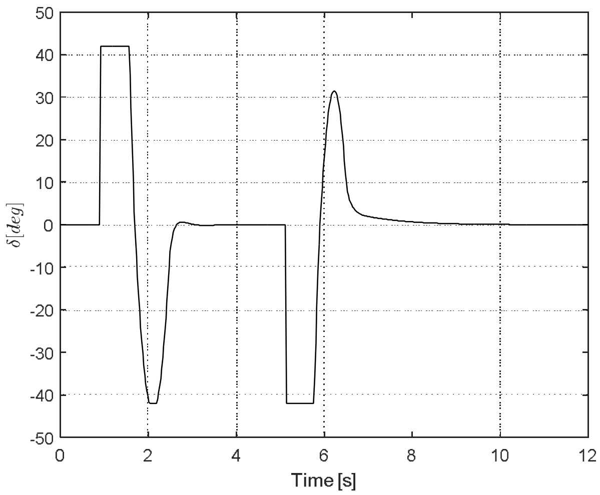

Fig. 8. Control input u (δ in deg) during certain turning maneuver.

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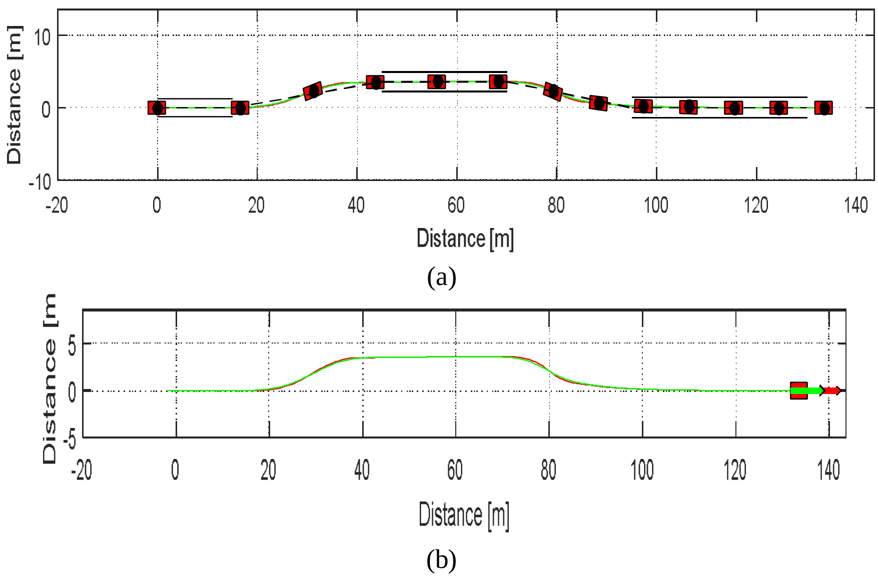

Fig. 9. Tractor-trailer response (a) double lane change (b) swept path width reduction using LQR controller.

V. CONCLUSIONS

An active trailer steering controller for articulated vehicles based on the LQR theory was proposed to reduce the swept path width of a tractor trailer. An LQR controller was designed, and simulated in such a way that the rear point of the trailer tracks the path driven by the tractor front wheel and thus tail swing can be avoided. The simulation results suggest an extension of the controller to dynamic stabilization, especially in obstacle avoidance maneuvers. The tractor and trailer active steering controller was designed to follow the desired yaw rates of the tractor and trailer and minimize the tractor and trailer's CG side-slip angles at the same time. The test results at a low speed showed that the active steering controller can improve the maneuverability of the articulated vehicle along sharp curves (rectangular curves).The results showed that the stability indexes of the tractor and the trailer at a high speed including the effects of lateral tire forces can be controlled simultaneously, which indicates the controller's significant effects in improving the lateral stability at a high speed for articulated vehicles. The effectiveness of the control strategy, with the controller design based on LQR ,is validated using simulations of certain turning maneuver of an articulated vehicle. The results shown in Fig.9 shows that the trailer rear point indeed converges to the path of the tractor front wheel and follows the path throughout the entire cornering, while avoiding tail swing at all times. The maximum swept path width is significantly reduced using controlled trailer steering compared with the case without trailer wheel steering. Here swept path analysis is performed with bi-cycle like representation, and therefore width of the vehicle is not taken into account in this analysis. This analysis shows that proposed control strategy is indeed effective in simulations.

REFERENCES

[1] B. Jujnovich and D. Cebon, "Comparative performance of semi-trailer steering systems," in *Proc. 7th Int. Symp. Heavy Vehicle Weights Dimensions*, 2002, pp. 195–214.

[2] Y. Nakamura, H. Ezaki, Y. Tan, and W. Chung, "Design of steering mechanism and control of nonholonomic trailer systems," *IEEE Trans. Robot. Autom.*, vol. 17, no. 3, pp. 367–374, Jun. 2001..

[3] B. A. Jujnovich and D. Cebon, "Path-following steering control for articulated vehicles," *J. Dyn. Syst., Meas. Control*, vol. 135, no. 3, p. 031006, 2013.

[4] Hata, N., Hasegawa, S., Takahashi, S., *et al.*, 1989. A Control Method for 4WS Truck to Suppress Excursion of a Body Rear Overhang. SAE Technical Paper 892521, p.754-760.

ISSN (Print) : 2320 – 3765
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International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Website: www.ijareeie.com

Vol. 6, Issue 4, April 2017

[5] Notsuet, I., Takahashi, S., Watanabe, Y., 1991. Investigation into Turning Behaviour of Semi-trailer with Additional Trailer-Wheel Steering – A Control Method for Trailer- Wheel Steering to Minimize Trailer Rear – Overhang Swing in Short Turns. SAE Technical Paper 912570,

[6] Rangavajhula, K., Tsao, H.S.J., 2007. Active trailer steering control of an articulated system with a tractor and three full trailers for tractor-track following. *Int. J. Heavy Veh. Syst.*, **14**(3):271-293.

[7] Cheng, C.Z., Cebon, D., 2008. Improving roll stability of articulated heavy vehicles using active semi-trailer steering. *Int. J. Veh. Mech. Mob.*, **46**(S1):373-388.

[8] Islam, M., He, Y., 2011. An Optimal Preview Controller for Active Trailer Steering Systems of Articulated Heavy Vehicles. SAE Technical Paper 2011-01-0983.

[9] Palkovics, L., El-Gindy, M., 1996. Examination of different control strategies of heavy-vehicle performance. *J. Dyn. Syst. Meas. Contr.*, **118**(3):489-498.

[10] El-Gindy, M., Mrad, N., Tong, X., 2001. Sensitivity of rearward amplification control of a truck/full trailer to tyre cornering stiffness variations. *Proc. Inst. Mech. Eng. Part D*, **215**(5):579-588..

[11] Hac, A., Fulk, D., Chen, H., 2008. Stability and control considerations of vehicle-trailer combination, *SAE Int. J. Passeng. Cars Mech. Syst.*, **1**(1):925-937.

[12] C. Samson, “Control of chained systems application to path following and time-varying point-stabilization of mobile robots,” *IEEE Trans. Autom. Control*, vol. 40, no. 1, pp. 64–77, Jan. 1995.

[13] D. Tilbury, J. P. Laumond, R. Murray, and S. Sastry, “Steering car-like systems with trailers using sinusoids,” in *Proc. IEEE Int. Conf. Robot. Autom.*, May 1992, pp. 1993–1998.

[14] P. Rouchon, M. Fliess, J. Lévine, and P. Martin, “Flatness and motion planning: The car with *n* trailers,” in *Proc. Int. Conf. ECC*, Groningen, Holland, 1993, pp. 1518–1522.

[15] A. Astolfi, P. Bolzern, and A. Locatelli, “Path-tracking of a tractortrailer
vehicle along rectilinear and circular paths: A Lyapunov-based approach,” *IEEE Trans. Robot. Autom.*, vol. 20, no. 1, pp. 154–160, Feb. 2004.

[16] D. de Bruin and P. P. J. van den Bosch, “Modelling and control of a double articulated vehicle with four steerable axles,” in *Proc. IEEE Amer. Control Conf.*, vol. 5. Jun. 1999, pp. 3250–3254.

[17] Y. He and M. M. Islam, “An automated design method for active trailer steering systems of articulated heavy vehicles,” *J. Mech. Design*, vol. 134, no. 4, p. 041002, 2012.