



Self Balancing Two Wheel Mobile Robot Using Sliding Mode Control

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ABSTRACT: A Two-Wheeled Mobile Robot (2WMR), is one which has two wheels in parallel and an inverse pendulum, and is inherently unstable. In the control design of a 2WMR, the objective here is to use a single actuator to enable position or velocity control of the wheels while balancing the pendulum. Such a system is defined as an under actuated system, since it has fewer actuators than the number of independent variables to be controlled. For control of under actuated systems, many of the conventional control designs for fully actuated systems are not applicable. In addition, various uncertainties such as the joint and the ground frictions, the varying slope angle of the ground, etc., exist in the 2WMR system, which makes the control problem more difficult especially when the uncertain part has unmatched components that are not in the control range space. To sum up, the 2WMR system is a nonlinear, unstable and under actuated system with uncertainties, thus the control system design is challenging. This brief tries to study the control of the system using a Sliding Mode Control. The control of body pitch angle and average angle of the wheels are controlled using the Sliding Mode Controller and results evaluated

KEYWORDS: SMC

I.INTRODUCTION

The research on a two-wheel inverted pendulum, commonly known as the selfbalancing robot, has gained momentum over the last decade. The self-balancing mobile robot on two wheels works on the principle of an inverted pendulum. The robot is inherently unstable and without external control it would roll around the wheels' rotation axis and eventually fall. Driving the motors in the right direction returns the robot to the upward position. Although the robot is inherently unstable, it has several advantages over the statically stable multi-wheeled robots – since it has only two wheels there are only two points of contact with the ground and hence requires less space. Since it is based on dynamic stability, i.e. it constantly needs to correct its tilt angle to remain stable, it exhibits improved dynamic behavior and mobility. This additional maneuverability allows easy navigation on various terrains, turning sharp and traversing small steps or curbs. Due to the difference in system configuration, under actuated 2WMRs can be classified into the class without input coupling where the actuator is mounted on the wheel (class A), and the class with input coupling where the actuator is mounted on the pendulum or chassis (class B). The class A is more complex in mechanical construction but easier in controller design owing to the absence of input coupling between the wheel and pendulum. In contrast, the class B is easier in mechanical construction but more challenging in controller design due to the input coupling between the wheel and the pendulum.

Control of underactuated systems is a popular research topic due to its wide range of applications in robotics, underwater vehicles, aerospace vehicles, etc. [1]. From practical concerns such as cost reduction or weight reduction, many systems are designed to be underactuated. The well known commercial product, two wheeled SEGWAY, is a popular personal transporter. For research and education purposes, prototypes and products of two-wheeled mobile vehicle or robot have been designed in universities and research institutes [2]. The control of inverted pendulum and similar underactuated mechanical systems is a rather challenging problem even no uncertainties are considered existing in the systems, thus, the problem has attracted much attention from researchers whose interest are in the field of control theory. In plenty of theoretical works, stabilizing algorithms based on Lyapunov theory, passivity, feedback linearization, etc., are developed for underactuated mechanical systems [3] Due to the difference in mechanical configuration, underactuated 2 WMRs can be classified into the class without input coupling and the class with input coupling [4]. Stabilizing algorithms based on Lyapunov theory, passivity,

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feedback linearization, etc., are developed for underactuated systems in absence of uncertainties [5]–[7]. The controller design and stability prove are based on the accurate mathematical models without considering any uncertainties. However, uncertainties and model mismatch between the nominal mathematical models and the real-life plants are inevitable. Real-time control of 2 WMRs and similar underactuated systems are presented in [8]. In [9] a novel adaptive output recurrent cerebellar model articulation controller is proposed, which is a model free design. Functions are used to approximate the system model, thus, the designed control algorithm is complex in mathematics and not evident in physics idea, furthermore, there are plenty of controller parameters to be determined. In [10], a fuzzy traveling and position control algorithm is proposed, however it is limited applicable to the 2 WMR without input coupling.

This paper is divided into four sections. Section II describes the system modeling of a Two wheeled Mobile Robot (2WMR). Section III describes the controller formulation. Section IV illustrates the simulation results. Section V discusses a few conclusions.

II SYSTEM DESCRIPTION MODELING OF A TWO WHEELED MOBILE ROBOT

In order to control the dynamic trajectory of the 2WMR, a mathematical model of the same is required to specify how the control variables affect the position and orientation of the robot. The Fig 1 represents a two wheeled inverted pendulum model. The 2WMR used for this study has the following elements: NXT block, which is the heart of the robot, a gyroscopic sensor, which measures the speed of inclination of the robot, and allows to estimate the angle Ψ ; and two electric actuators (left and right wheels) with encoders, which measure the angular position of the wheel θ and estimate its speed $\dot{\theta}$. The model equations are obtained by Lagrange method. The model has four states and two inputs. The states are: the body pitch angle Ψ and the average angular position of the wheels θ and their respective velocities. The system inputs are the motor

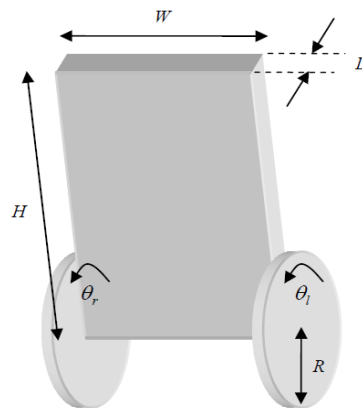


Figure 1 Two-wheeled inverted pendulum

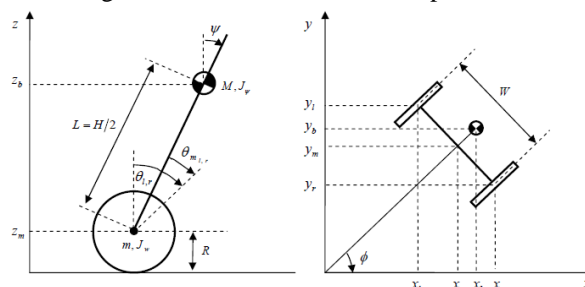


Figure. 2 Side view and plane view of two-wheeled inverted pendulum

ψ : body pitch angle $\theta_{l,r}$: wheel angle (l,r indicates left and right) $\theta_{ml,r}$: DC motor angle
voltages U_1 and U_2 , which are the inputs to the left and right wheels.. Figure 2 shows side view and plane view of it. The coordinate system used in Fig 2. Motion equations of two-wheeled inverted pendulum is described in Figure 2.



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A. MOTION EQUATIONS OF TWO-WHEELED INVERTED PENDULUM

We can derive motion equations of two-wheeled inverted pendulum by the Lagrangian method based on the coordinate system in Figure 3.2. If the direction of two-wheeled inverted pendulum is in x-axis positive direction at t=0, each coordinates are given as the following equations from

$$(x_m, y_m, z_m) = (R\theta\cos\emptyset, R\theta\sin\emptyset, R) \quad (1)$$

$$(\theta, \emptyset) = \left(\frac{1}{2}(\theta_l + \theta_r), \frac{R}{W}(\theta_r - \theta_l) \right) \quad (2)$$

$$(x_l, y_l, z_l) = \left(x_m - \frac{W}{2}\sin\emptyset, y_m + \frac{W}{2}\cos\emptyset, z_m \right) \quad (3)$$

$$(x_r, y_r, z_r) = \left(x_m + \frac{W}{2}\sin\emptyset, y_m - \frac{W}{2}\cos\emptyset, z_m \right) \quad (4)$$

$$(x_b, y_b, z_b) = (x_m + L\sin\psi\cos\emptyset, y_m - L\sin\psi\sin\emptyset, z_m + L\cos\psi) \quad (5)$$

The translational kinetic energy T_1 , the rotational kinetic energy T_2 , the potential energy U are

$$T_1 = \frac{1}{2}m(\dot{x}_l^2 + \dot{y}_l^2 + \dot{z}_l^2) + \frac{1}{2}m(\dot{x}_r^2 + \dot{y}_r^2 + \dot{z}_r^2) + \frac{1}{2}M(\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2) \quad (6)$$

$$T_2 = \frac{1}{2}J_w\dot{\theta}_l^2 + \frac{1}{2}J_w\dot{\theta}_r^2 + \frac{1}{2}J_\psi\dot{\psi}^2 + \frac{1}{2}J_\emptyset\dot{\emptyset}^2 + \frac{1}{2}n^2J_m(\dot{\theta}_l - \dot{\psi})^2 + \frac{1}{2}n^2J_m(\dot{\theta}_r - \dot{\psi})^2 \quad (7)$$

$$U = mgz_l + mgz_r + Mgz_b \quad (8)$$

The fifth and sixth term in T_2 are rotation kinetic energy of an armature in left and right DC motor. The Lagrangian L has the following expression.

$$L = T_1 + T_2 + U \quad (9)$$

We use the following variables as the generalized coordinates.

θ : Average angle of left and right wheel

ψ : Body pitch angle

\emptyset : Body yaw angle

Lagrange equations are the following

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = F_\theta \quad (10)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} = F_\psi \quad (11)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\emptyset}} \right) - \frac{\partial L}{\partial \emptyset} = F_\emptyset \quad (12)$$



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We derive the following equations by evaluating eqn 10 to 12

$$F_{\theta} = [(2m + M)R^2 + 2J_w + 2n^2J_m]\ddot{\theta} + (MLR\cos\psi - 2n^2J_m)\dot{\psi} - MLR\dot{\psi}^2\sin\psi - [(2m + M)R^2\dot{\theta} + MLR\sin\psi]\dot{\theta}^2 \quad (13)$$

$$F_{\psi} = (MLR\cos\psi - 2n^2J_m)\ddot{\theta} + (ML^2 + J_{\psi} + 2n^2J_m)\ddot{\psi} - MgL\sin\psi - (MLR\dot{\theta} + ML^2\sin\psi)\dot{\theta}\cos\psi \quad (14)$$

$$F_{\phi} = \left[\frac{1}{2}mW^2 + J_{\phi} + \frac{W^2}{2R^2}(J_w + n^2J_m) + (2m + M)R^2\dot{\theta}^2 + 2MLR\dot{\theta}\sin\psi \right] \ddot{\phi} + 2[(2m + M)R^2\dot{\theta}\dot{\theta} + ML^2\dot{\psi}\cos\psi\sin\psi + MLR(\dot{\theta}\sin\psi + \dot{\psi}\cos\psi)]\dot{\phi} \quad (15)$$

Table 1. Physical parameter used in LEGO robot

PARAMETER	UNIT	DESCRIPTION
$g=9.81$	[m/sec ²]	Gravity acceleration
$m=0.03$	[kg]	Wheel weight
$R=0.04$	[m]	Wheel radius
$J_w = mR^2/2$	[kgm ²]	Wheel inertia moment
$M=0.6$	[kg]	Body weight
$W=0.14$	[m]	Body width
$D=0.04$	[m]	Body depth
$H=0.144$	[m]	Body height
$L=H/2$	[m]	Distance of the center of mass from the wheel axle
$J_{\psi}=ML^2/3$	[kgm ²]	Body pitch inertia moment
$J_{\phi}=M(W^2+D^2)/12$	[kgm ²]	Body yaw inertia moment
$J_m= 1*10^{-5}$	[kgm ²]	DC motor inertia moment
$R_m=6.69$	[Ω]	DC motor resistance
$K_b=0.468$	[V sec/rad]	DC motor back EMF constant
$K_v=0.317$	[Nm/A]	DC motor torque constant
$n=1$		Gear ratio
$f_m=0.0022$		Friction coefficient between body and DC motor
$f_w=0$		Friction coefficient between wheel and floor.

In consideration of DC motor torque and viscous friction, the generalized force are given as the following



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$$(F_{\theta}, F_{\psi}, F_{\phi}) = \left(\frac{1}{2}(F_1 + F_r), F_{\psi}, \frac{R}{W}(F_r - F_1) \right) \quad (16)$$

$$F_1 = nK_t i_1 + f_m(\dot{\psi} - \dot{\theta}_1) - f_w \dot{\theta}_1 \quad (17)$$

$$F_r = nK_t i_r + f_m(\dot{\psi} - \dot{\theta}_r) - f_w \dot{\theta}_r \quad (18)$$

$$F_{\psi} = -nK_t i_1 - nK_t i_r + f_m(\dot{\psi} - \dot{\theta}_1) - f_w(\dot{\psi} - \dot{\theta}_r) \quad (19)$$

where $i_{1,r}$ is DC motor current

We cannot use DC motor current directly in order to control it because it is based on PWM (voltage) control. Therefore, we evaluate the relation between current $i_{1,r}$ and voltage $v_{1,r}$ using DC motor equation. The DC motor equation is generally as follows

$$L_m \dot{i}_{1,r} = v_{1,r} + K_b(\dot{\psi} - \dot{\theta}_{1,r}) - R_m i_{1,r} \quad (20)$$

Here we consider that the motor inductance is negligible and is approximated as zero. Therefore the current is

$$i_{1,r} = \frac{v_{1,r} + K_b(\dot{\psi} - \dot{\theta}_{1,r})}{R_m} \quad (21)$$

From Eq.(3.21), the generalized force can be expressed using the motor voltage

$$F_{\theta} = \frac{\alpha}{2}(v_1 + v_r) - (\beta + f_w)\dot{\theta} + \beta\dot{\psi} \quad (22)$$

$$F_{\psi} = -\alpha(v_1 + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\psi} \quad (23)$$

$$F_{\phi} = \frac{R}{W}\alpha(v_r - v_1) - \left(\beta + \frac{W}{R}f_w\right)\dot{\phi} \quad (24)$$

$$\alpha = \frac{nK_t}{R_m}, \beta = \frac{nK_t K_b}{R_m} + f_m \quad (25)$$

B. STATE EQUATIONS OF TWO-WHEELED INVERTED PENDULUM

We can get state equations based on modern control theory by linearizing motion equations at a balance point of NXTway-GS. It means that we consider the limit $\psi \rightarrow 0$ ($\sin \psi \rightarrow \psi$, $\cos \psi \rightarrow 1$) and neglect the second order term like ψ^2 . The motion equations (3.12) – (3.14) are approximated as the following

$$F_{\theta} = [(2m + M)R^2 + 2J_w + 2n^2J_m]\ddot{\theta} + (MLR - 2n^2J_m)\ddot{\psi} \quad (26)$$

$$F_{\psi} = (MLR - 2n^2J_m)\ddot{\theta} + (ML^2 + J_{\psi} + 2n^2J_m)\ddot{\psi} - MgL\psi \quad (27)$$

$$F_{\phi} = \left[\frac{1}{2}mW^2 + J_{\phi} + \frac{W^2}{2R^2}(J_w + n^2J_m) \right] \ddot{\phi} \quad (28)$$

Here we consider the following variables X_1 X_2 as state, U and as input.

$$x_1 = [\theta \quad \psi \quad \dot{\theta} \quad \dot{\psi}]^T, x_2 = [\phi \quad \dot{\phi}]^T$$

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$$u = [v_1 \quad v_r]^T \quad (29)$$

$$\dot{x}_1 = A_1 x_1 + B_1 u \quad (30)$$

$$\dot{x}_2 = A_2 x_2 + B_2 u \quad (31)$$

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_1(3,2) & A_1(3,3) & A_1(3,4) \\ 0 & A_1(4,2) & A_1(4,3) & A_1(4,4) \end{bmatrix} \quad (32)$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -I/K \end{bmatrix} \quad (33)$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_1(3) & B_1(3) \\ B_1(4) & B_1(4) \end{bmatrix} \quad (34)$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ -J/K & J/K \end{bmatrix} \quad (35)$$

III CONTROLLER DESIGN

Sliding mode control (SMC) is a nonlinear control technique featuring remarkable properties of accuracy, robustness, and easy tuning and implementation. SMC systems are designed to drive the system states onto a particular surface in the state space, named sliding surface.

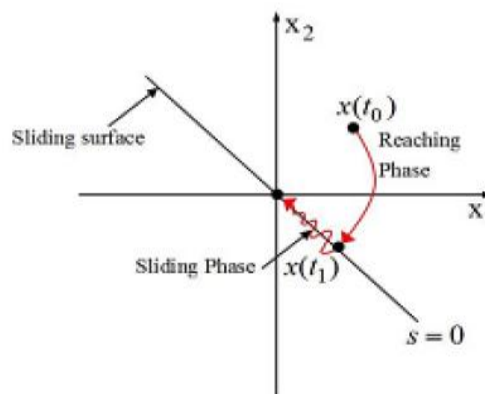


Fig. 3 Phase plane plots of SMC

Once the sliding surface is reached, sliding mode control keeps the states on the close neighborhood of the sliding surface. Hence the sliding mode control is a two part controller design. The first part involves the design of a sliding



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surface so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law that will make the switching surface attractive to the system state

Consider a Single Input Single Output (SISO) system with its state space model as:

$$\dot{x}_1 = x_2 \quad (36)$$

$$\dot{x}_2 = -a_1x_1 - a_2x_2 + bu \quad (37)$$

Where $0 < a < \bar{a}, 0 < b < \bar{b}$

For design of SMC of above mentioned system, sliding surface is chosen as:

$$\sigma = \lambda x_1 + x_2 \quad (38)$$

where λ is positive.

For above sliding surface, a control law should be designed so that the sliding surface is reached to zero, i.e. made attractive towards the origin. This can be achieved by Lyapunov stability technique.

Consider a Lyapunov function as:

$$V = \frac{1}{2} \sigma^2 \quad (39)$$

which implies that on differentiating

$$\begin{aligned} \dot{V} &= \sigma \dot{\sigma} \\ &= \sigma(-a_1x_1 - a_2x_2 + bu) \\ &= \rho|\sigma| \end{aligned} \quad (40)$$

$$\begin{aligned} u &= b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2 - \rho \frac{|\sigma|}{\sigma}] \\ &= b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2 - \rho \text{sign}(\sigma)] \\ &= u_{eq} + u_{sw} \end{aligned} \quad (41)$$

where

$$\text{sign}(\sigma) = \frac{|\sigma|}{\sigma}$$

$$u_{eq} = b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2]$$

$$u_{sw} = b^{-1} \rho \text{sign}(\sigma)$$

IV SIMULATION

In this paper simulation is done using MATLAB SIMULANK program. By running the program I obtain the following response. MATLAB program is shown in the Table 2

Table 2 MATLAB program

<pre>%% two wheel mobile robot clear all close all A=[0 0 1 0 0 0;0 0 0 1 0 0;0 -409.7184 -162.1273 162.1273 0 0;0 269.6273 78.1496 -78.1496 0 0;0 0 0 0 1;0 0 0 0 0 -95.5684]; B=[0 0;0 0;157.5798 157.5798;-75.9576 -75.9576;0 0;- 53.0787 53.0787]; dt=0.01; k=1;</pre>	<pre>c=[0.0003 0.1002;-0.10005 0.1002;-0.22 -0.3;-1 0.005;-0.0011 -0.003;0.000023 -0.00022]; for tim=0:dt:30 Xdt=A*X+B*U; X=X+Xdt*dt; Xt=X-Xd e=Xt s=c*Xt U=-inv(c*B)*c*A*X-k*sign(s) thet(ii)=X(1); thetdt(ii)=X(2); thetdtdt(ii)=X(3); thetdtdtdt(ii)=X(4);</pre>
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<pre> x1d=0/57.3; x2d=0/57.3; x3d=0/57.3; x4d=0; x1=45/57.3; x2=5/57.3;x3=20/57.3;x4=2/57.3; X=[x1;x2;x3;x4;x5;x6]; Xd=[x1d;x2d;x3d;x4d;x5d;x6d]; ii=1; U=[0;0]; </pre>	<pre> ut(ii)=U(1); utt(ii)=U(2); et(ii)=e(1); ft(ii)=e(2); gt(ii)=e(3); jt(ii)=e(6); x(ii)=x1d; y(ii)=x2d; z(ii)=x3d; v(ii)=x4d; time(ii)=tim; ii=ii+1; end </pre>
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By running the program coded , the response are shown below

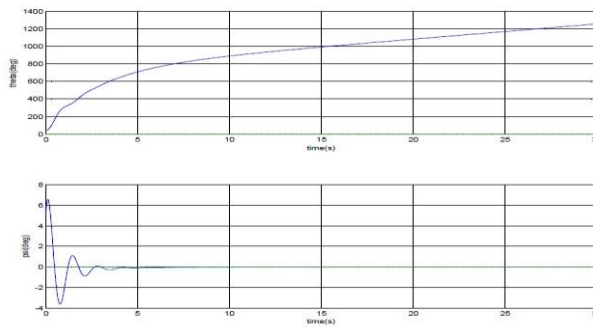


Fig 4 MATLAB Simulink Response of Lego 2WMR θ and Ψ vs time

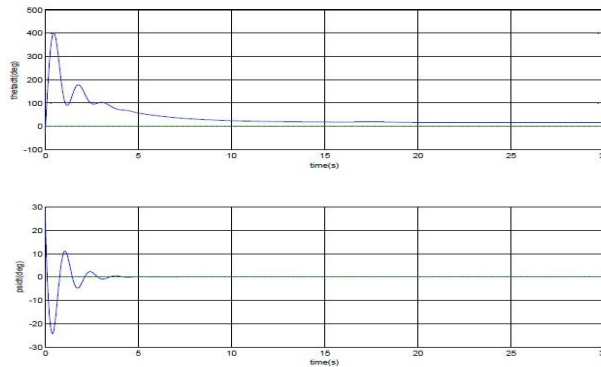


Fig 4 MATLAB Simulink Response of Lego 2WMR $\dot{\theta}$ and $\dot{\Psi}$ vs time

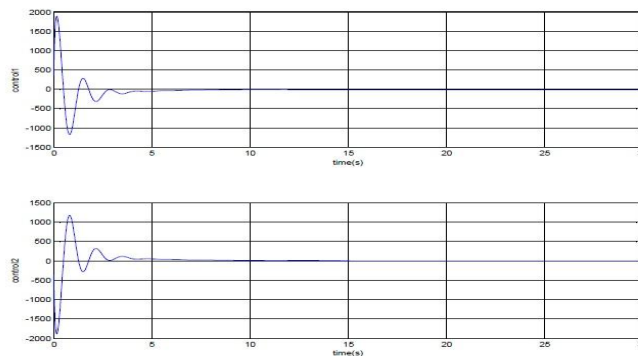


Fig 5 MATLAB Simulink Response of Lego 2WMR Control Input



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Table 3. Result obtained in SMC

Parameters	SMC
Theta	Doesn't settled
phi	In 4sec
thetadot	In 10sec
phidot	In 4sec

Table 3 shows the result of SMC. SMC gives more robust control. SMC have high oscillations at initial state but it settles to zero in a faster rate. We can see that Theta in SMC is not settled due to the improper selection of C value. We can settle Theta to zero by proper selection of C value. By this we can attain the desire value of Theta.

V CONCLUSION

The study of nonlinear, unstable under actuated system of Two Wheel Mobile Robot is carried out. The dynamic modeling of the system using Lagrange Approach is studied and being used here. Using this study, the various forces acting on the system is observed. This modeling will be used for the controller design. The thesis aims to implement both the velocity and position control of Two Wheel Mobile Robot. Sliding Mode control will be used to study the theta and psi.

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