ISSN (Print) : 2320 – 3765
ISSN (Online) : 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 5.621|| A Monthly Peer Reviewed & Referred Journal |

Vol. 5, Issue 5, May 2016

A Study of Boundary Value Problem and Expansion Formula: A Review

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ABSTRACT: In this paper, the authors make a model of a boundary value problem and then obtain its solution involving products of I -function and a general class of polynomials.

KEYWORDS: General Class of Polynomials, I -function, Expansion Formula, Hermite Polynomial.

I. INTRODUCTION

The general class of polynomials is defined by Srivastava and Panda [12, 13] as:

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r}(x_1,\dots,x_r)=\sum_{k_1=0}^{[n_1/m_1]}\dots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}\dots\frac{(-n_r)_{m_rk_r}}{k_r!}F[n_1,k_1;\dots;n_r,k_r]x_1^{k_1}\dots x_r^{k_r} \quad , \quad (1.1)$$

Where, $m_1,\dots,m_r$ are arbitrary positive integers and the coefficients $F[n_1,k_1;\dots;n_r,k_r]$ are arbitrary constants real or complex . Finally, we derive some new particular cases and find their applications also.

The I-function introduced by Saxena [6] will be represented and defined as follows[1,2,3]:

$$I[Z]=I_{p_i,q_i:r}^{m,n}[Z]=I_{p_i,q_i:r}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]=\frac{1}{2\pi\omega}\int_L\chi(\xi)d\xi \qquad (1.2)$$

where $\omega=\sqrt{-1}$

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Vol. 5, Issue 5, May 2016

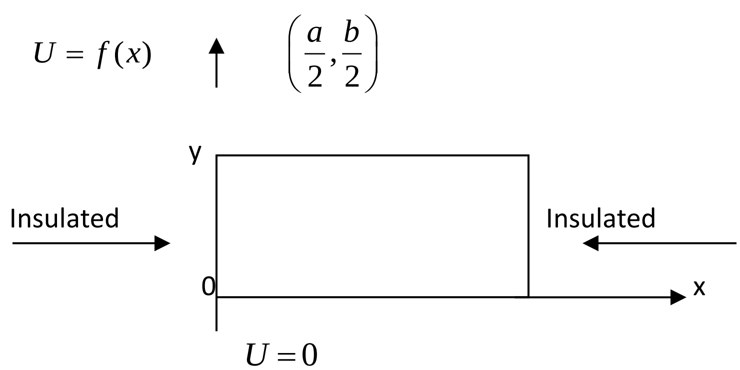
$$\chi(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji})\prod_{j=n+1}^{p_i}\Gamma(a_{ji},\alpha_{ji})\right\}} \qquad (1.3)$$

We shall use the following notations:

$$A^*=(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i};B^*=(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}$$

II. A BOUNDARY VALUE PROBLEM

We consider a rectangular plate such that,



Where the boundary value conditions are[4,5,6]:

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=a,\ 0<x<\frac{a}{2},0<y<\frac{b}{2} \qquad (2.1)$$

$$\left.\frac{\partial U}{\partial x}\right|_{x=0}=\left.\frac{\partial U}{\partial x}\right|_{x=\frac{a}{2}}=0,0<y<\frac{b}{2} \qquad (2.2)$$

$$U(x,0)=0,0<x<\frac{a}{2} \qquad (2.3)$$

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$$U\left(x,\frac{b}{2}\right)=f(x)=\left(\cos\frac{\pi x}{a}\right)S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[y_1\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$
$$A_{p,q}^{m,n}\left[z\left(\cos\frac{\pi x}{a}\right)^{2\sigma}\left|\begin{matrix}{}_1(a_j,\alpha_j)_p\\ {}_1(b_j,\beta_j)_q\end{matrix}\right.\right] \qquad (2.4)$$

Where, $0<x<\frac{a}{2}$ provided that $\mathrm{Re}(\eta)>-1,\sigma>0.$

$U(x,y)$ is the temperature distribution in the rectangular plate at point (x,y).

III. MAIN INTEGRAL

In our investigations, we make an appeal to the modified formula due to Kumar [5,7,8,9] as,

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}dx=\frac{a\Gamma(\eta+1)}{2^{\eta+1}\left(\frac{\eta}{2}+m+1\right)\left(\frac{\eta}{2}-m+1\right)} \qquad (3.1)$$

Where, m is positive integer and $\mathrm{Re}(\eta)>-1$, then we evaluate an applicable integral

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$
$$I_{p_i,q_i:r}^{m,n}\left[z\left(\cos\frac{\pi x}{a}\right)^{2\sigma}\left|\begin{matrix}A*\\B*\end{matrix}\right.\right]dx$$

$$=\frac{a}{2^{\eta+1}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n)_{m_1k_1}}{k_1!}\cdots\frac{(-n)_{m_rk_r}}{k_r!}$$
$$F[n_1,k_1;\ldots,n_r,k]I(k)\left(\frac{y}{4\rho}\right)^{k_1}\cdots\left(\frac{y}{4\rho}\right)^{k_r} \qquad (3.2)$$

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Where,

$$I(k) = I^{m,n+1}_{p_i+1,q_i+1:r}\left[\frac{z}{4^{\sigma}}\left|\begin{matrix}(-\eta-2\rho k_1-\ldots-2\rho k_r;2\sigma),A^*\\ B^*,\left(-\frac{\eta}{2}-m-\rho k_1-\ldots-\rho k_r;\sigma\right)\end{matrix}\right.\right] \quad (3.3)$$

Provided that $F[n_1,k_1;\ldots;n_r,k_r]$ are arbitrary functions of $n_1,k_1;\ldots;n_r,k_r$, real or complex independent of x,y,ρ, the conditions of (2.4) and (3.1) are satisfied and [10,11,12]

$$\mathrm{Re}\left(\eta+\sigma\frac{b_{ji}}{\beta_{ji}}\right)>-1,\ |\arg z|\le\frac{1}{2}\pi\Omega,$$

IV. SOLUTION OF BOUNDARY VALUE PROBLEM

In this section, we obtain the solution of the boundary value problem stated in the section (2) as using (2.1), (2.2) and (2.3) with the help of the techniques referred to Zill [14] as:

$$U(x,y) = A_0 y+\sum_{p=1}^{\infty}A_p\sinh\frac{2p\pi y}{a}\cos\frac{2p\pi x}{a},0<x<\frac{a}{2},0<y<\frac{a}{2} \quad (4.1)$$

For $y=\frac{b}{2}$, we find that

$$U\left(x,\frac{b}{2}\right)=f(x)=\frac{A_0 b}{2}+\sum_{p=1}^{\infty}A_p\sinh\frac{p\pi b}{a}\cos\frac{2p\pi x}{a},0<x<\frac{a}{2} \quad (4.2)$$

Now making an appeal to (2.4) and (4.2) and then interchanging both sides with respect to

x from 0 to $\frac{a}{2}$, we derive,

$$A_0=\frac{2}{b\sqrt{\pi}}\sum_{k_1=0}^{[n_1/m_1]}\cdots\sum_{k_r=0}^{[n_r/m_r]}(-n_1)_{m_1k_1}\ldots(-n_r)_{m_rk_r}$$

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$$F[n_1,k_1;...;n_r,k_r]I_1(k)\frac{y^{k_1}}{k_1!}...\frac{y^{k_r}}{k_r!} \qquad (4.3)$$

Where

$$I_1(k)=I^{m,n+1}_{p_i+1,q_i+1:r}\left[z\left|\begin{matrix}\left(-\frac{1}{2}-\frac{\eta}{2}-\rho k_1-...-\rho k_r;\sigma\right),A^*\\ B^*,\left(-\frac{\eta}{2}-\rho k_1-...-\rho k_r;\sigma\right)\end{matrix}\right.\right] \qquad (4.4)$$

Where all conditions of (2.4), (3.1) and (3.3) are satisfied.

Again making an appeal to (2.4) and (4.2) and then multiplying by $\cos\frac{2m\pi x}{a}$ both sides and thus integrating that result with respect to x from 0 to $\frac{a}{2}$, we find,

$$A_m=\frac{1}{2^{\eta-1}\sinh\frac{p\pi b}{a}}\sum_{k_1=0}^{[n_1/m_1]}...\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1k_1}}{k_1!}...\frac{(-n_r)_{m_rk_r}}{k_r!}F[n_1,k_1;...,n_r,k_r]$$

$$I(k)\left(\frac{y}{4^\rho}\right)^{k_1}...\left(\frac{y}{4^\rho}\right)^{k_r} \qquad (4.5)$$

Provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

Finally, making an appeal to the result (4.1), (4.3) and (4.5), we derive the required solution of the boundary value problem,

$$U(x,y)=\frac{2y}{b\sqrt{\pi}}\sum_{k_1=0}^{[n_1/m_1]}...\sum_{k_r=0}^{[n_r/m_r]}\left[\prod_{j=1}^{r}\left((-n_j)_{m_jk_j}\frac{y^{k_j}}{k_j!}\right)\right]F[n_1,k_1;...;n_r,k_r] \;+$$

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$$\sum_{m=1}^{\infty} \frac{\sinh \frac{2m\pi y}{a} \cos \frac{2m\pi x}{a}}{2^{\eta-1} \sinh \frac{m\pi b}{a}} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \left[\prod_{j=1}^{r} \left((-n_j)_{m_j k_j} \left(\frac{y}{4^{\rho}} \right)^{k_j} \frac{1}{k_j!} \right) \right]$$

$$F[n_1, k_1; \ldots; n_r, k_r] I(k) \qquad (4.6)$$

Where ,

Provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

V. EXPANSION FORMULA

With the aid of (2.4) and (4.6) and then setting $y = \frac{b}{2}$, we evaluate the expansion formula

$$\left(\cos \frac{\pi x}{a} \right)^{\eta} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r} \left[y \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right]$$

$$I_{p_i,q_i:r}^{m,n} \left[z \left(\cos \frac{\pi x}{a} \right)^{2\sigma} \middle| \begin{matrix} A* \\ B* \end{matrix} \right]$$

$$= \frac{1}{\sqrt{\pi}} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \left[\prod_{j=1}^{r} \left((-n_j)_{m_j k_j} \frac{y^{k_j}}{k_j!} \right) \right] F[n_1, k_1; \ldots; n_r, k_r] I(k)$$

$$\sum_{m=1}^{\infty} \frac{\cos \frac{2m\pi x}{a}}{2^{\eta-1}} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \left[\prod_{j=1}^{r} \left((-n_j)_{m_j k_j} \left(\frac{y}{4^{\rho}} \right)^{k_j} \frac{1}{k_j!} \right) \right]$$

$$F[n_1, k_1; \ldots; n_r, k_r] I(k) \qquad (5.1)$$

where $0 < x < \frac{a}{2}$,

provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.[13,14]

ISSN (Print) : 2320 – 3765
ISSN (Online) : 2278 – 8875

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| e-ISSN: 2278 – 8875, p-ISSN: 2320 – 3765| www.ijareeie.com | Impact Factor: 5.621|| A Monthly Peer Reviewed & Referred Journal |

Vol. 5, Issue 5, May 2016

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