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On Complex Inductance in Dissipative Electrical Circuits

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ABSTRACT: It is known that variation principle of least action has been traditionally used in electrical engineering for the conservative i.e. lossless electrical circuits. On the base of very general variation consideration a complex inductance concept is introduced and considered in the article. It was proven in the article that the Euler equation contained the complex inductance (via the Lagrangianappropriated) corresponds to the movement law of oscillatory dissipative electrical circuit. It is shown that this concept lets extend use of variation methods to dissipative electrical circuits. This lets formally use the traditional variation theory of electrical circuits to the wider class of dissipative electrical circuits.

KEYWORDS:complex inductance, Lagrangian, principle of least action, Euler equation, dissipative electrical circuit, conservative electrical circuit.

I.INTRODUCTION

The Lagrangian Ł of a conservative electrical circuit consisting of the inductive coil with inductance L and capacitor with capacitance C is determined as

$$Ł = \frac{1}{2}Li^2 - \frac{1}{2C}q^2 + qe(t),$$

where the electrical charge q is a generalized co-ordinate, the current i = dq/dt is a generalized velocity [1].

The movement equation corresponded to this Lagrangian is the following:

$$L\frac{di}{dt} + \frac{1}{C}\int idt = e(t), \text{ or}$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = e(t)$$

if express it via electric charge and its second temporal derivative.
The Euler equation for the LC circuit is as follows,

$$\frac{d}{dt}\frac{\partial Ł}{\partial q'_t} - \frac{\partial Ł}{\partial q} = 0.$$

Since

$$\frac{\partial Ł}{\partial q'_t} = Lq'_t,$$

$$\frac{d}{dt}\frac{\partial Ł}{\partial q'_t} = Lq''_t,$$

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$$\frac{\partial Ł}{\partial q} = -\frac{1}{C}q + e(t),$$

then we can state that, the above given Lagrangian and movement law are mutually corresponding expressions.

II. CONCEPT OF COMPLEX INDUCTANCE

Both Lagrangian and the principle of least action were determined for conservative (lossless) systems. In the above considered case it was oscillatory circuit with no losses (i.e. with no resistance). Evidently this restrikes possibility of variation analyze of dissipative electrical circuits.
Consider series LR circuit with instantaneous currentflowing through it given below,

$$i = I_m sin\omega t$$

Here I_m is current's peak value, ω is angular frequency.
A rate of the current change will be equal to:

$$v = \frac{di}{dt} = \omega I_m cos\omega t = \omega I_m \sin\left(\frac{\pi}{2} - \omega t\right).$$

To express v via complex amplitude in correspondence with [2] use the following expression:

$$\omega I_m \sin\left(\frac{\pi}{2} - \omega t\right) = I_m\left(j\omega \dot{I}_{km} \exp(-j\omega t)\right) = \dot{V}_m \exp(-j\omega t),$$

where

$$\dot{V}_m = j\omega \dot{I}_{km}$$

is the complex amplitude of the current's rate of change, $\dot{I}_{km}$ is the current's complex amplitude, j is imaginary unit .
The voltage across the terminals of RL circuit is equaled to

$$U = L\frac{di}{dt} + Ri = \omega L I_m \cos\omega t + RI_m \sin\omega t.$$

The complex amplitude of this voltage is

$$\dot{U} = (j\omega L + R)\dot{I}_{km}.$$

Determine a complex inductance as a ratio of complex amplitudes of voltage and current's change velocity i.e.

$$\dot{L} = \dot{U} / \dot{V}.$$

Get

$$\dot{L} = \frac{(j\omega L + R)\dot{I}_{km}}{j\omega \dot{I}_{km}} = L - j\frac{R}{\omega.}$$

Note that the similar approach was used for introducing the concept of complex magnetic permeability in [3] where the hysteresis losses in transformers' steel were studied. It may be also noticed that in general necessity of taking into consideration dissipative factor in mathematical expressions makes use of complex representation of some parameters

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expedient. Remind in this view use of complex dielectric permittivity in some high-frequency applications of electric circuits and electromagnetic fields [1], [4].

III.USE OF COMPLEX INDUCTANCE CONCEPT IN LAGRANGIAN

Now consider the Lagrange function for dissipative electrical circuit (i.e. RLC circuit) contained the complex inductance as follows:

$$Ł = \frac{1}{2}\dot{L}i^2 - \frac{1}{2C}q^2 + qe(t).$$

The Euler equation for dissipative electrical circuits in accordance with [1] is as follows:

$$\frac{d}{dt}\frac{\partial Ł}{\partial q'_t} - \frac{\partial Ł}{\partial q} = -\frac{\partial P}{\partial q'_t}.$$

Implement the following transformations:

$$\frac{\partial Ł}{\partial q'_t} = Lq'_t,$$

$$\frac{d}{dt}\frac{\partial Ł}{\partial q'_t} = \dot{L}q''_t = L\frac{di}{dt} + Ri,$$

$$\frac{\partial Ł}{\partial q} = -\frac{1}{C}q + e(t).$$

After substituting the last two equations into the Euler equation for dissipative electrical circuits we will get the expression

$$L\frac{d^2q}{dt^2} + Ri + \frac{1}{C}q = e(t),$$

which is just a movement law of the oscillatory dissipative circuit.

Thus, equivalency of the Euler equation and movement low for dissipative electrical circuits at use the complex inductance concept is proven.

IV. CONCLUSION

At use the complex inductance concept variation principles and methods can be applied to analyze electrical circuits containing resistive elements.

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