Comparison of Tuning PI Controller for Non Linear Conical Tank

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ABSTRACT: The control of the liquid level is significant for the process industries. But the conical tank is basically non-linear due to the change in area of cross section. Many industries use conical tank which contributes better drainage for solid mixtures, slurries & viscous liquids. In conical tank, maintaining the liquid level is a challenging task. This non-linear process is done using mathematical model to find a first order pulse dead time model. The objective is to sustain the conical tank in desired level and we implemented the suitable controller design to the non-linear system. This process is to intend the mathematical modelling to the conical tank using Internal Model Control (IMC) and Ziegler Nichols (ZN). by comparing both the tuning methods, we obtained the best of these. This controller is simulated using MATLAB SIMULINK. By using the control model, we expect to have enhanced closed loop performance & robustness when we compare to. The servo and regulatory responses are obtained with PI by both ZN and IMC method.

KEYWORDS: Conical Tank, ZN Tuning method, Non linear process, IMC, PI controller

I.INTRODUCTION

The proposed work’s objective is to perform model based controller design which uses model of the process to compute the controller setting, but the structure of the model has not been openly involved in the controller design. There are numerous choices of controller design methods that make more clear use of a process model. The tuning of level controllers can be tricky because of the excessive variation in the process dynamics and tuning settings. Many researchers have done research in the level control of the non linear tank process.

R.Ramya, S.Sakinath Nisa and P.Aravind [1] explained the level control of spherical tank, a non linear process using Z-N and IMC method. D.Mercy, S.M.Girirajkumar [2] controlled the conical tank level using intelligent techniques. P.Aravind, M.Valluvan, S.Ranganathan [3] designed PID controller using direct synthesis method and skogestad method. Control system studies have shown that the most frequent method of optimum vales of controller parameters. Conventional PID controllers are simple, robust provided the system is linear. But the process considered here is the level control in conical tank which has nonlinear characteristics and so it is represented as piecewise linear models; multiple linear models of tank with many conventional controllers were implemented. The PID tuner allows achieving a good balance between performance and robustness.

The crucial task of a controller is to retain the process at the desired set point and to attain optimum performance when facing various types of disturbances. This paper compares two methods Ziegler-Nichols method (ZN method) and Internal Model Control method (IMC) and proposes the more suitable controller tuning method for this conical tank level control process.

II. MATHEMATICAL MODELLING OF CONICAL TANK

Consider a conical tank whose radius varies at different regions due to the shape of the tank. Here the conical tank is divided into different regions. Each region is approximated as First Order Process with dead time. This approach will help us to control the non-linear system as we consider it as a linear system for a particular region.

The mathematical model of the conical tank is determined by the following assumptions.

1. Level as the control variable
2. Inflow as the manipulated variable. This can be achieved by controlling the input flow of the conical tank.

Inflow rate of the tank (Fin) is regulated using the valve and the input flow through the conical tank.

The difference between the inflow and the outflow rate will be based on the cross-sectional area of the tank and level with respect to time. The flow and the level of the tank can be regulated by proper modelling the tank.

Operating Parameters are,
Fin - Inflow rate of the tank
Fout - Outflow rate of the tank
H - Total height of the conical tank.
R - Top radius of the conical tank
h - Nominal level of the tank
r - Radius at nominal level

Mass balance Equation is given by

\[ \text{Fin} - \text{Fout} = A \frac{dh}{dt} \]  
\[ \text{Outflow rate of the tank, } Fout = b \sqrt{h} \]  

Where, \( b = a \sqrt{2g} \)

By substituting the values,

Area of the tank, \( A = \pi r^2 \)  
\[ A = \pi R^2 \frac{h^2}{H^2} \]  

Where radius,
\( r = \frac{\text{(Top radius of the tank)}^2 \text{(Nominal level of the tank)}^2}{\text{(Total height of the tank)}^2} \)

Therefore, \( r = \frac{R^2 h^2}{H^2} \)

The differential equations describe the process in time domain. Transfer function of the system can be obtained by converting the process from time domain to frequency domain. Transfer function of the conical tank is given by taking the partial differentiation of the linearized equation and its corresponding Laplace transform,

\[ \frac{H(s)}{Fout(s)} = \frac{K e^{-\theta s}}{\tau s + 1} \]  

Where, \( \tau = \frac{2A \sqrt{h}}{b} \)
\( K = \frac{2 \sqrt{h}}{b} \)

The transfer function is FOPTD process and the proper tuning technique is used to optimize the system.

Based on the specification, the transfer function of the conical tank
Region 5: (23 cm to 36 cm)
\[ y(s) = \frac{0.196 e^{-1s}}{(44.5s+1)} \]  
---------- (5)
Region 6: (36 cm to 52 cm)
\[ y(s) = \frac{0.24 e^{-1s}}{(98.25s+1)} \]  
---------- (6)

III. CONTROLLER DESIGN

PI controller is a type of feedback controller whose output, a control variable (CV), is generally based on the error (e) between some user-defined set-point (SP) and some measured process variable (PV). Each element of the PI controller refers to a particular action to be taken on the error.

\[ e(t) = r(t) - y(t) \]

The transfer function of PI controller is given by the formula

\[ G_{PI}(s) = K_c \left[ 1 + \frac{1}{T_i s} \right] \]

Where \( K_c \) is the proportional gain, \( T_i \) is the integral time constant. The objective of designing a PI controller is to determine the PI parameters \((K_c, T_i)\) to get the optimized performance.

Ziegler Nichols method:

Controller standardization is required to achieve the response. In Ziegler-Nichols method, PI controller is tuned based upon continuous oscillation or ultimate cycle method. This method is also known as the closed-loop method (or) on-line tuning method.

Like all the other tuning methods, Z-N Method consists of two steps:

- Determination of the dynamic characteristics of the control loop.
- Estimation of the controller tuning parameters that produce a desired response for the dynamic characteristics.

The tuning formula for Z-N method is shown in Table 1.

<table>
<thead>
<tr>
<th>CONTROLLER TYPE</th>
<th>( K_p )</th>
<th>( K_i )</th>
<th>( K_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.5 Ku</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PI</td>
<td>0.45 Ku</td>
<td>1.2 ( K_p/Pu )</td>
<td>-</td>
</tr>
<tr>
<td>PID</td>
<td>0.6 Ku</td>
<td>2 ( K_p/Pu )</td>
<td>( K_p Pu/8 )</td>
</tr>
</tbody>
</table>

TABLE 1: (Z-N formula)

The Z-N method is more robust because it does not require a specific process model. To tune a controller using the Z-N method, the integral element and derivative element of the PID controller are ignored. The proportional element is used to find a \( K_c \) that will sustain oscillation. This gain value is considered as \( Ku \), or the ultimate gain. The time interval between the two peaks is \( Pu \), or the ultimate period.

Internal Model Control Method:

Morari and his co-workers have developed an important control system strategy known as internal model control or IMC. IMC provides better time delay composition than any other tuning methods. The internal model control relies on the internal model principle, which states that control can be achieved only if the control system encapsulates either implicitly or explicitly, some representation of the process to be controlled.

The IMC approach has two important advantages:
a) It explicitly takes into account model uncertainty
b) It allows the designer to trade-off control system performance against control system robustness to process changes and modelling errors.

The tuning formula for IMC method is shown in TABLE 2

| $K_p$ | $T_i$ | $T_d$ |
| $\tau / K_p(\tau_c + 0.5 \tau_d)$ | $T$ | $\tau_d / 2$ |

<table>
<thead>
<tr>
<th>Region 5</th>
<th>Region 6</th>
<th>Region 5</th>
<th>Region 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$:198</td>
<td>$K_p$:373.95</td>
<td>$K_p$:115.97</td>
<td>$K_p$:204.68</td>
</tr>
<tr>
<td>$K_i$:74.71</td>
<td>$K_i$:144.94</td>
<td>$K_i$:2.577</td>
<td>$K_i$:2.07</td>
</tr>
</tbody>
</table>

TABLE 2: (IMC formula)

FIGURE 2: basic block diagram of IMC controller

(Kp, Ki VALUES OF CONTROLLER USING Z-N METHOD AND IMC METHOD)

<table>
<thead>
<tr>
<th>Z-N METHOD</th>
<th>IMC METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 5</td>
<td>Region 6</td>
</tr>
<tr>
<td>$K_p$:198</td>
<td>$K_p$:373.95</td>
</tr>
<tr>
<td>$K_i$:74.71</td>
<td>$K_i$:144.94</td>
</tr>
</tbody>
</table>

TABLE 3

IV. RESULT AND DISCUSSION

RESPONSE CURVE: [(Region 5: 23 cm to 36 cm), (Region 6: 36 cm to 52 cm)]

In MATLAB simulation, we tend to settle the Region 5 of the conical tank to 28 cm (Level in cm) using Z-N method and IMC method and we take the response for the same to compare the results.
Similarly, for Region 6 of the conical tank, set point is given as 38 cm. The response is taken for Region 6 using both Z-N and IMC method.

In the following table, we tabulated the time domain parameters to compare the result.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Operating region</th>
<th>Set point</th>
<th>Rise time</th>
<th>Overshoot</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Region 5</td>
<td>28</td>
<td>4.8 sec</td>
<td>22</td>
<td>21 sec</td>
</tr>
<tr>
<td>2</td>
<td>Region 6</td>
<td>38</td>
<td>5.2 sec</td>
<td>33</td>
<td>27 sec</td>
</tr>
</tbody>
</table>

The following table shows the time domain parameters of the curve that we get from IMC method.
SERVO AND REGULATORY RESPONSE:

A servo control loop is one which responds to a change in set point. The set point may be changed as a function of time, and therefore the controlled variable must follow the set point.

A regulatory control loop is one which responds to a change in some input value, bringing the system back to steady state. Regulatory control is by far more common than servo control in the process industries.

The following graph shows the servo responses of Region 5 and Region 6 of the conical tank.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Operating region</th>
<th>Set point</th>
<th>Rise time</th>
<th>Overshoot</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Region 5</td>
<td>28</td>
<td>6.2sec</td>
<td>2</td>
<td>14sec</td>
</tr>
<tr>
<td>2</td>
<td>Region 6</td>
<td>38</td>
<td>6.3sec</td>
<td>2</td>
<td>15sec</td>
</tr>
</tbody>
</table>

TABLE 5

For Region 5, after the process settles at the given set point 28 cm, we change the set point to 35 cm which is being tracked by the process and analyse the response of the process. We did the servo response using both ZN and IMC method.
Similarly, for Region 6, the servo response has taken using ZN and IMC method by changing the set point to 45 cm which initially settles at 38 cm.

The following graph shows the regulatory response:

**FIGURE 7**

For Region 5, after the process has settled at 28 cm, a disturbance is given to the process by changing the set point to 35 cm. Initially the process disturbs and then it comes back and settles to the steady state.

**FIGURE 8**

For Region 6, a disturbance is given by changing the set point to 45 cm. The process get disturbs for some time and again settles at 38 cm.

The regulatory response is taken by giving set point disturbance at some instant of time. The process responds to the disturbance, and then it comes back and settles to the given set point.
V. CONCLUSION

IMC tuning method gives better response than the Z-N tuning method. IMC method has less overshoot than ZN method. In IMC tuning method, the process settles faster compared with Ziegler Nichols method. While comparing the two methods based on servo and regulatory response of the process, IMC method settles faster and there is no overshoot either. From the system response, it is observed that the IMC tracks the set point with smooth transition and with less oscillation compared with Z-N method. From the simulation results, it is observed that the performance of IMC is better than Z-N method for the nonlinear conical tank process. Hence we can conclude that IMC method is better to control the non-linear conical tank process

REFERENCES