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System Reduction of Discrete Time Uncertain Model using Stability Preservation Techniques

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ABSTRACT: An extended approach for order reduction of complex discrete uncertain systems is proposed. Using Interval arithmetic Routh Stability arrays are formed to obtained numerator and denominator of reduced order model. The developed approach preserves the stability aspect of reduced system if higher order uncertain system is stable. A numerical example is included to illustrate the proposed algorithm along with the comparison with existing techniques. An extended technique for suppressing the complexity of discrete time uncertain model using stability preservation approach is proposed. The numerator and denominator of uncertain model are suppressed by β & α table respectively. The proposed technique guarantees the preservation of stability in reduced uncertain model and well suited in quality with other existing methods. A numerical illustration is discussed to exemplify the extended technique

KEYWORDS: Discrete Interval System, Integral Square Error (ISE), Model Order Reduction, Routh Stability Array.

I.INTRODUCTION

In present time as the complexity of physical and geometrical model increases many cases so it is desirable to represent reduced order system in place of its original system. System reduction for both continuous and discrete systems has been extensively studied. Conventional methods for system reduction are Aggregation method [1], Moment matching technique [2], Padé approximation [3], and factor division method [4]. Classical technique has many serious disadvantages which often lead to an unstable reduced order model even though the original system is stable. These drawbacks of classical techniques are overcome by stability preservation techniques which allows stable reduced model for stable system and having useful features such as the computational simplicity and the fitting of time-moments. However, a plenty of stability preservation techniques were developed in past. Some famous methods are Routh approximation [6], Stability equation method [7], $\alpha\beta$ table method [8], and many more.

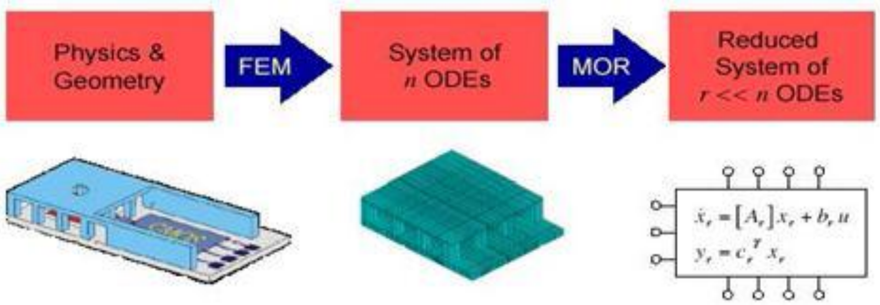

Fig 1. Order reduction of an efficient means to enable a system -level simulation

Developments made in the direction of downsizing the complexity of an uncertain system after the idea of solving the arithmetic of uncertain system by Hansen & Smith [11,12]. However, Practical systems like cold rolling mill, stirred tank reactor, oblique aircraft and electric motors, contains disturbance in model dynamics due to sensor noises, actuator constrains, etc. which are most suitably represented by continuous or discrete type uncertain models, instead of deterministic mathematical models. Many methodologies based on discrete time uncertain system came into picture such as Routh array method by Shamash & Feinmesser [16], minimal Padè type stability method by Bistritz and

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Shaked [17] for discrete multivariable system, Padè type approximations method [18] by Bandyopadhyay&Kande, bilinear Schwarz approximation [19] by Hsieh & Hwang etc. Apart from these, several techniques are proposed to check the novelty of reduced model.

In this brief, an extended technique is discussed for reduction of large system, where denominator is retained by α table of uncertain model while the numerator is obtained by β table of the given discrete interval system, This method shows the stability of the reduced system for a stable model. The outline of the paper is organized as follows: primaries and main result of the paper are deliberated in Section II& III, Numerical problem is illustrated in Section III with the comparison of proposed result with other existing methods. Finally, conclusions and comments are given in Section IV.

II. PRIMALIES

A general higher order single input single output interval system of n^{th} order is defined as

$$G_n(s)=\frac{[q_0^-,q_0^+]+[q_1^-,q_1^+]z+........+[q_{n-1}^-,q_{n-1}^+]z^{n-1}}{[p_0^-,p_0^+]+[p_1^-,p_1^+]z+............+[p_n^-,p_n^+]z^n}=\frac{\sum[q_i^-,q_i^+]s^i}{\sum[p_j^-,p_j^+]s^j}=\frac{N_n(s)}{D_n(s)} \quad (1)$$

where (i=0,1,..., n-1) and (j=0,1,2,..,n) are order of interval parameters.
and general reduced order system of r^{th} order is defined as

$$R_r(s)=\frac{[b_0^-,b_0^+]+[b_1^-,b_1^+]z+........+[b_{r-1}^-,b_{r-1}^+]z^{r-1}}{[a_0^-,a_0^+]+[a_1^-,a_1^+]z+..............+[a_r^-,a_r^+]z^r}=\frac{\sum[q_i^-,q_i^+]s^i}{\sum[p_j^-,p_j^+]s^j}=\frac{N_r(s)}{D_r(s)} \quad (2)$$

where (i=0,1..., r-1) and (j=0,1,..., r) are order of interval parameters.

Interval Arithmetic [11, 12]

Rules based on interval arithmetic [11, 12] are given below. Let $[a,b]$ and $[c,d]$ be two intervals.

Addition: $[a,b]+[c,d]=[a+c,b+d]$ (3)

Subtraction $[a,b]-[c,d]=[a-d,b-c]$ (4)

Multiplication $[a,b]\times[c,d]=[Min(ac,ad,bc,bd)],[Max(ac,ad,bc,bd)]$ (5)

Division: $\frac{[a,b]}{[c,d]}=[a,b]\times[\frac{1}{d},\frac{1}{c}]$ (6)

III. MAIN RESULT

In this section Routh approximation based α- β table method [9] is extended for discrete uncertain system. The steps to obtain reduced order system are as below:

Step 1: Bilinear transformation of higher order system by applying $z=\frac{1+w}{1-w}$

Step 2: Reciprocal transformation of higher order system is obtained as

$$\hat{G}(w)=\tfrac{1}{w}G(\tfrac{1}{w}) \quad (7)$$

where $\hat{G}(w)$ is reciprocal of higher order system and $G(w)$ is the original higher order system

Step 3: Formation of α-table

The first two rows of tabulation are formed from the coefficient of denominator of $\hat{G}(w)$ and rest of entries of table is by "cross-multiplication rule" using interval arithmetic rules [11, 12] such as

$$[\alpha_1^-,\alpha_1^+]=\frac{[p_0^-,p_0^+]}{[p_1^-,p_1^+]}\begin{cases}[p_0^-,p_0^+][p_2^-,p_2^+]\cdots\cdots\\ [p_1^-,p_1^+][p_3^-,p_3^+]\cdots\cdots\end{cases}$$

$$[\alpha_2^-,\alpha_2^+]=\frac{[p_1^-,p_1^+]}{[k_1^-,k_1^+]}\begin{cases}[p_1^-,p_1^+][p_3^-,p_3^+]\cdots\cdots\\ [k_1^-,k_1^+][k_3^-,k_3^+]\cdots\cdots\end{cases}$$

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$$[\alpha_3^-,\alpha_3^+]=\frac{[k_1^-,k_1]}{[l_1^-,l_1^+]}\begin{cases}[k_1^-,k_1^+][k_3^-,k_3^+]\cdots\cdots\\ [l_1^-,l_1^+][l_3^-,l_3^+]\cdots\cdots\end{cases}$$
$$\vdots\qquad\vdots\qquad\vdots\qquad\vdots$$

where

$[k_i^-,k_i^+]=[p_{i+1}^-,p_{i+1}^+]-[\alpha_1^-,\alpha_1^+][p_{i+2}^-,p_{i+2}^+]$; i=(1,3,…, r-1)

$[l_i^-,l_i^+]=[p_{i+2}^-,p_{i+2}^+]-[\alpha_2^-,\alpha_2^+][k_{i+2}^-,k_{i+2}^+]$; i=(1,3,…, r-3)

$[m_i^-,m_i^+]=[k_{i+2}^-,k_{i+2}^+]-[\alpha_3^-,\alpha_3^+][l_{i+2}^-,l_{i+2}^+]$; i=(1,3,…, r-5)

$$\vdots\qquad\vdots\qquad\vdots\qquad\vdots$$

Step 4: Formation of β-table

The first two rows of table are formed from the coefficient of numerator of $\hat{G}(s)$ and remaining entries are calculated from α table and earlier rows of β table

Therefore

$$[\beta_1^-,\beta_1^+]=\frac{[q_0^-,q_0^+]}{[p_1^-,p_1^+]}\begin{cases}[q_0^-,q_0^+][q_2^-,q_2^+]\cdots\cdots\\ [p_1^-,p_1^+][p_3^-,p_3^+]\cdots\cdots\end{cases}$$

$$[\beta_2^-,\beta_2^+]=\frac{[q_1^-,q_1^+]}{[k_1^-,k_1^+]}\begin{cases}[q_0^-,q_0^+][q_2^-,q_2^+]\cdots\cdots\\ [k_1^-,k_1^+][k_3^-,k_3^+]\cdots\cdots\end{cases}$$

$$[\beta_3^-,\beta_3^+]=\frac{[e_1^-,e_1^+]}{[l_1^-,l_1^+]}\begin{cases}[e_1^-,e_1^+][e_3^-,e_3^+]\cdots\cdots\\ [l_1^-,l_1^+][l_3^-,l_3^+]\cdots\cdots\end{cases}$$
$$\vdots\qquad\vdots\qquad\vdots\qquad\vdots$$

where

$[e_i^-,e_i^+]=[q_{i+1}^-,q_{i+1}^+]-[\beta_1^-,\beta_1^+][p_{i+2}^-,p_{i+2}^+]$; i=(1,3,…, r-3)

$[f_i^-,f_i^+]=[p_{i+2}^-,p_{i+2}^+]-[\beta_2^-,\beta_2^+][k_{i+2}^-,k_{i+2}^+]$; i=(1,3,…, r-5)

$[g_i^-,g_i^+]=[r_{i+2}^-,r_{i+2}^+]-[\beta_3^-,\beta_3^+][l_{i+2}^-,l_{i+2}^+]$; i=(1,3,…, r-7)

$$\vdots\qquad\vdots\qquad\vdots\qquad\vdots$$

Step 5: Let $A_r(s)$ and $B_r(s)$ represent the denominator and numerator respectively, of the r^{th} order reduced system in general [19], i.e.

$$A_r(s)=[\alpha_r^-,\alpha_r^+]A_{r-1}(s)+A_{r-2}(s) \tag{8}$$

$$B_r(s)=[\alpha_r^-,\alpha_r^+]sB_{r-1}(s)+B_{r-2}(s)+[\beta_r^-,\beta_r^+] \tag{9}$$

Where r =(1, 2, ..., n)are obtained with $A_{-1}(s)=0$, $B_{-1}(s)=0$, $A_0(s)=1$, $B_0(s)=0$;

Step 6: The r^{th} order reciprocal transformation system is evaluated as

$$\hat{R}_r(s)=\frac{B_r(s)}{A_r(s)} \tag{10}$$

Step 7: Finally reduced order system is obtained as

$$R_r(s)=\frac{1}{s}\hat{R}_r\left(\frac{1}{s}\right) \tag{11}$$

Step 8: Inverse Bilinear transformation of reduced order system by applying $z=\dfrac{1-w}{1+w}$

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IV. NUMERICAL EXAMPLE

Consider the fourth order discrete interval system described by transfer function [27, 28]

$$G(z)=\frac{[1,2]z^2+[3,4]z+[8,10]}{[6,6.6]z^3+[9,9.5]z^2+[4.9,5]z+[0.8,0.85]}$$

Step 1: Applying bilinear transformation in higher order transfer function.

$$G(w)=\frac{[5,9]w^2+[-18,-12]w+[12,16]}{[0.55,1.2]w^3+[5.9,6.65]w^2+[19.45,20.2]w+[20.7,21.35]}$$

Step 2: After reciprocal transformation of $G(s)$

$$\hat{G}(w)=\frac{[12,16]w^2+[-18,-12]w+[5,9]}{[20.7,21.35]w^3+[19.45,20.2]w^2+[5.9,6.65]w+[0.55,1.2]}$$

Step 3: The α table is obtained as:

$\alpha-table$	$[20.7,21.35]$ $[19.45,20.2]$	$[5.9,6.65]$ $[0.55,1.2]$
$[1.024,1.097]$ $[3.1956,4.407]$	$[4.583,6.0864]$	

Step 4: The β table is calculated as:

$\beta-table$	$[12,14]$ $[-18,-12]$	$[5,9]$
$[0.59,0.82]$ $[-2.95,-2.62]$	$[6,7.5]$	

Step 5: Now transformation reduced system, using Eq.10 is evaluated as

$$\hat{R}_2(w)=\frac{[1.88,3.60]w+[-2.95,-2.62]}{[3.25,4.79]w^2+[3.19,4.40]w+1}$$

Step 6: Then, reciprocal transformation of above reduced order system using Eq. 11 is obtained as

$$R_2(w)=\frac{[-2.95,-2.62]w+[1.88,3.60]}{w^2+[3.19,4.40]w+[3.25,4.29]}$$

Step 7: Applying inverse bilinear transformation in reduced order transfer function as

$$R_2(z)=\frac{[-1.07,0.98]z+[4.5,6.55]}{[7.44,10.19]z^2+[4.5,7.58]z+[-0.15,2.6]}$$

V. RESULT AND DISCUSSION

To check the superiority of the proposed method over other existing methods [27, 28] integral-square-error for reduced order models are tabulated in Table I. The integral square error is determined between transient step response of original system and its reduced order model which can be represented as

$$ISE=\int_0^\infty [y(t)-y_r(t)]^2\,dt \tag{12}$$

Where $y(t)$ is the step response of the original system and $y_r(t)$ is the step response of the original system

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TABLE I. COMPARISON OF ISE FOR REDUCED ORDER MODELS

Model Order Reduction	Reduced Models	ISE (lower)	ISE (upper)
Proposed Method	$G_2(z)=\frac{[-1.07,0.98]z+[4.5,6.55]}{[7.44,10.19]z^2+[4.5,7.58]z+[-0.15,2.6]}$	0.0852	0.0377
Padé& Dominant Pole Retention Method [27]	$G_2(z)=\frac{[0.5921,0.6055]z+[0.8845,0.9]}{z^2+[0.8041,1.2465]z+[0.1437,0.3805]}$	0.1810	0.0.741
$\gamma-\delta$ method [28]	$G_2(z)=\frac{[-1.328,1]z+[3.522,5.85]}{[6.89,8.14]z^2+[3.94,5.44]z+[0.55,1.8]}$	1.1292	0.0443

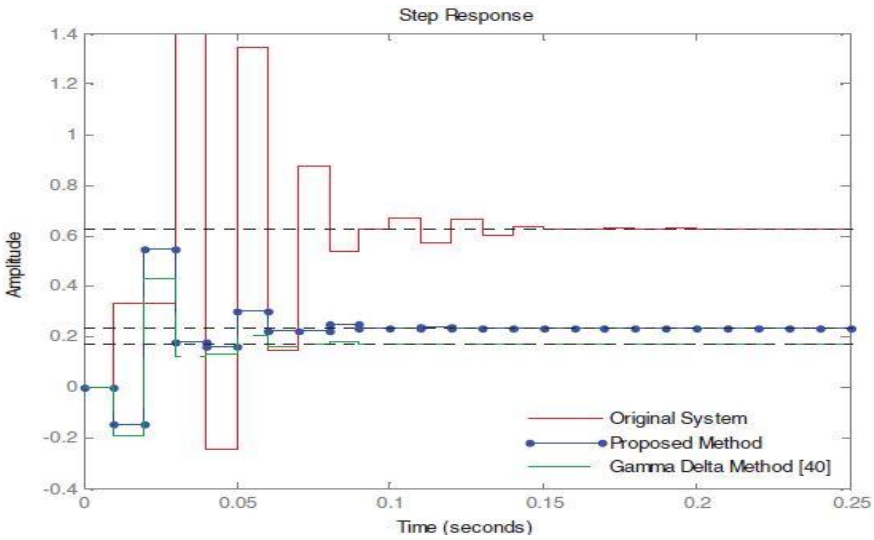

Fig 1(a). Step response of Original and reduced systems using Kharitonov's theorem

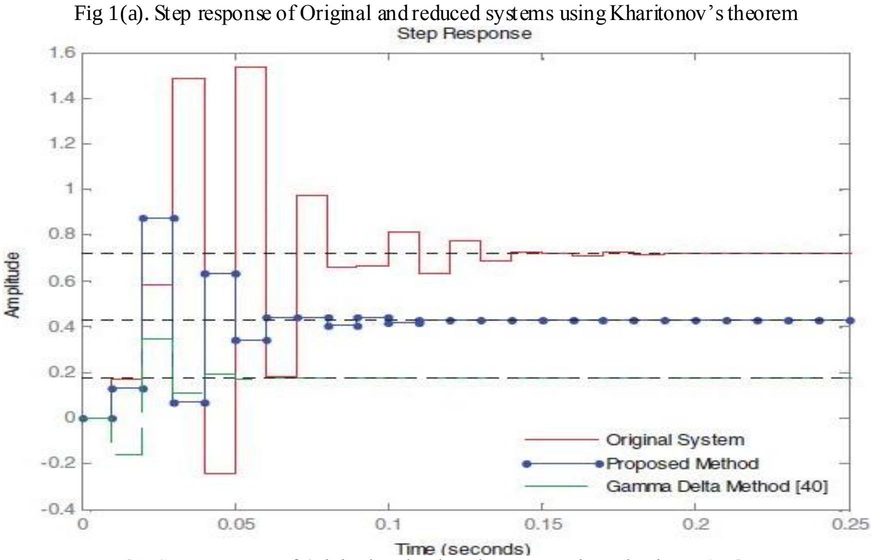

Fig 1(b). Step response of Original and reduced systems using Kharitonov's.theorem

In the fig 1(a) & fig 1(b), it shows the step response of original system and reduced order systems using Kharitonov's theorem in which graph is plotted for amplitude Vs time (seconds). Throughput is the average rate of successful message delivery over a communication channel. It is observed from fig 1(a) & fig 1(b) that step responses of original system and reduced order interval system using proposed method are close to each other which show the accuracy of proposed method.

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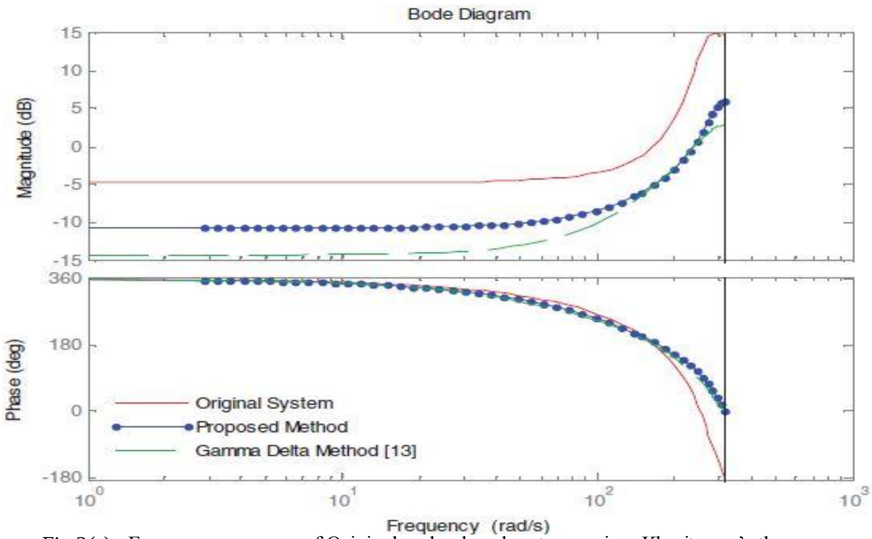

Fig 2(a). Frequency response of Original and reduced systems using Kharitonov's theorem

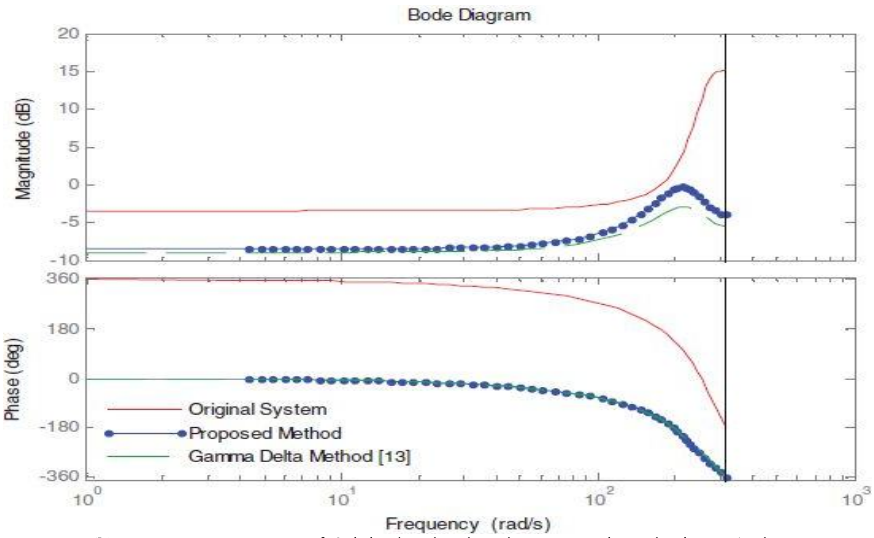

Fig 2(b). Frequency response of Original and reduced systems using Kharitonov's theorem

In the fig 2(a) & fig 2(b), it shows the frequency response of higher order system and lower order systems using Kharitonov's theorem in which graph is shown for magnitude (dB) & phase (deg) Vs time (seconds).

VI.CONCLUSION

This paper presents an extension of the α- β table technique [13] for reduction of higher order discrete time interval systems. The numerator polynomial and denominator polynomial is obtained by using β table and α table respectively. The developed technique is conceptually easy and preserves the accuracy and stability of reduced order model if the higher order system is stable. The proposed method produces lesser values of error indices when compared with other existing methods [27,28]. A numerical example is discussed and compared with some existing techniques.

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