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Fractional Calculus of Multivariable Mittag Leffler Function

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ABSTRACT: Multivariate Mittag-Leffler functions are a strong generalisation of the univariate and bivariate Mittag-Leffler functions which are known to be important in fractional calculus. We consider the general functional operator defined by an integral transform with a multivariate Mittag-Leffler function in the kernel. We prove an expression for this operator as an infinite series of Riemann–Liouville integrals, thereby demonstrating that it fits into the established framework of fractional calculus, and we show the power of this series formula by straightforwardly deducing many facts, some new and some already known but now more quickly proved, about the original integral operator. We illustrate our work here by calculating some examples both analytically and numerically, and comparing the results on graphs. We also define fractional derivative operators corresponding to the established integral operator. As an application, we consider some Cauchy-type problems for fractional integro-differential equations involving this operator, where existence and uniqueness of solutions can be proved using fixed point theory. Finally, we generalise the theory by applying the same operators with respect to arbitrary monotonic functions.

KEYWORDS: multivariable, mittag, leffler, functions, fractional, calculus, operator, monotonic

I. INTRODUCTION

Because of its widespread applications in almost all applied sciences, particularly numerical analysis, mathematical physics, and engineering, the theory of FC has attracted a lot of attention in recent decades. For example, FC has enabled the creation of a theoretical model based on experimental data ,. Fractional integral inequalities, which include the integrals of functions and their derivatives, have a century of history in mathematics, with far-reaching applications in differential equations theory, approximation theory, and probability theory, among other topics.

The most important applications of fractional differential inequalities to fractional differential equations are the validation of uniqueness of initial problem solutions and the provision of upper bounds for their solutions .Many researchers in the domain of integral inequalities have been prompted by these applications to explore additional extensions by employing various fractional differential and integral operators.[1,2] In several authors in the domain of integral inequalities have been inspired by these applications to investigate other extensions and refinements of inequalities by employing multiple fractional differential and integral operators.

The work of demonstrates the wide applications of fractional differential equations (FDEs) in physics, economics, engineering, and many other disciplines of research. Because there is no universal way for solving every FDE analytically, one of the most essential and difficult topics is to build acceptable methods for discovering analytical solutions to certain classes of FDEs . Fractional formulations of conventional integral transforms,[3,4] namely the Laplace and Fourier transforms, have attracted the focus of research in recent years.Integral transforms namely the Laplace, Fourier, Generalized Laplace, and Weighted-Laplace transforms were shown to be effective in obtaining analytical solutions to specific types of FDEs. The application of fractional calculus and FDEs can be found in the work of

The M-L functions[5,6] and their generalized version appear as a solution to fractional order differential or integral equations. Using the Mittag–Leffler function and its generalizations, researchers have been able to generate fractional integral inequalities of various types. As a result, additional results for more generalized fractional integral operators using the Mittag–Leffler function in their kernels have been achieved. [7,8]Saxena et al. introduced the multvariate Mittag–Leffler function. For more information, the reader may follow the work

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Inspired by the work done in we aim to define a generalized weighted fractional integral and differential operators and as well as their applications.[9]

$$\begin{aligned}
\left({}_c\mathfrak{C}^{\delta;\omega_1,\omega_2}_{\alpha,\beta,\gamma} f\right)(x) &= {}_c\mathfrak{I}^{-\delta;\omega_1,\omega_2}_{\alpha,\beta,n-\gamma}\left(\frac{\mathrm{d}^n}{\mathrm{d}x^n} f(x)\right) \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-\delta)_{k+l}\omega_1^k\omega_2^l}{k!l!}\, {}^{RL}_{c}I^{\alpha k+\beta l+(n-\gamma)}_{x}\left(\frac{\mathrm{d}^n}{\mathrm{d}x^n} f(x)\right) \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-\delta)_{k+l}\omega_1^k\omega_2^l}{k!l!}\Bigg[{}^{RL}_{c}I^{(\alpha k+\beta l+n-\gamma)-n}_{x} f(x) \\
&\quad - \sum_{j=0}^{n-1} \frac{(x-c)^{(\alpha k+\beta l+n-\gamma)-n+j}}{\Gamma((\alpha k+\beta l+n-\gamma)-n+j+1)} f^{(j)}(c)\Bigg] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-\delta)_{k+l}\omega_1^k\omega_2^l}{k!l!}\, {}^{RL}_{c}I^{\alpha k+\beta l-\gamma}_{x} f(x) \\
&\quad - \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{j=0}^{n-1} \frac{(-\delta)_{k+l}\omega_1^k\omega_2^l}{k!l!}\cdot\frac{(x-c)^{\alpha k+\beta l-\gamma+j}}{\Gamma(\alpha k+\beta l-\gamma+j+1)} f^{(j)}(c) \\
&= \left({}_c\mathfrak{D}^{\delta;\omega_1,\omega_2}_{\alpha,\beta,\gamma} f\right)(x) - \sum_{j=0}^{n-1}(x-c)^{-\gamma+j} E^{-\delta}_{\alpha,\beta,-\gamma+j+1} \\
&\quad \left(\omega_1(x-c)^{\alpha}, \omega_2(x-c)^{\beta}\right) f^{(j)}(c)
\end{aligned}$$

Generalized fractional operators recover the classical ones as special cases. A new definition and properties of a fractional operator without a singular kernel were studied . Yang et al. defined a new fractional derivative without a singular kernel and described a potential application for modeling the steady heat-conduction problem . Agarwal and Nieto discussed the Marichev–Saigo–Maeda fractional integral operators by involving the Mittag–Leffler type function with four parameters . Some fractional integrals and derivatives with general analytic kernels were briefly studied . The (k,s)-fractional calculus of the generalized Mittag–Leffler function was discussed . For further details and applications of fractional operators, we refer the readers .[10,11]

The classical Mittag–Leffler function was proposed in 1903. It has been generalized in various ways by introducing the two parameters or three parameters. Further, Dorrego and Cerutti represented the k Mittag–Leffler function in the following form:

Eγk,ρ,η(θ)=∑n=0∞(γ)n,kθnΓk(ρn+η)n!,

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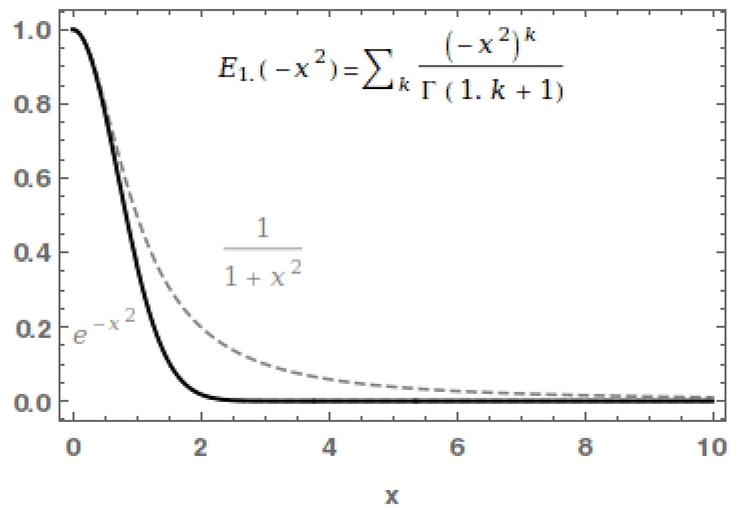

II. DISCUSSION

The importance of fractional calculus has a significant role in all sciences due to its widespread applications. Fractional calculus represents nature more accurately than integer-order calculus does. Numerous mathematicians have studied the Mittag–Leffler function and its applications. In the present work, we introduced the multivariate Mittag–Leffler function and used it to explore a fractional integral operator and its inverse derivative operator. The cases presented just after the definitions proved the generalization of the introduced operators. Both operators are bounded in the Xp[a,b] space. The fundamental properties of the fractional operators were established. Moreover, we evaluated the modified Laplace transform of both the derivative and integral operators. These results cannot be obtained with the classical Laplace transform. By using the new operators, a fractional kinetic differintegral equation was developed, and its solution was obtained via the modified Laplace transform. A growth model was explored as a real-life application, and its graph was sketched. The authors of this study strongly encourage readers to explore more multivariate forms of special functions and new forms of fractional operators.[12,13]

Example 1.

If we substitute s=0,Θ(x)=x,a=0,N0=0,α=β=1,k=1,p=1,n=2,h1=1,μ=ν=1, and δi=σi=0 for all i, then the solution of Equation with the condition becomes

N(x)=∑m=0∞x2mΓ(2m+1)

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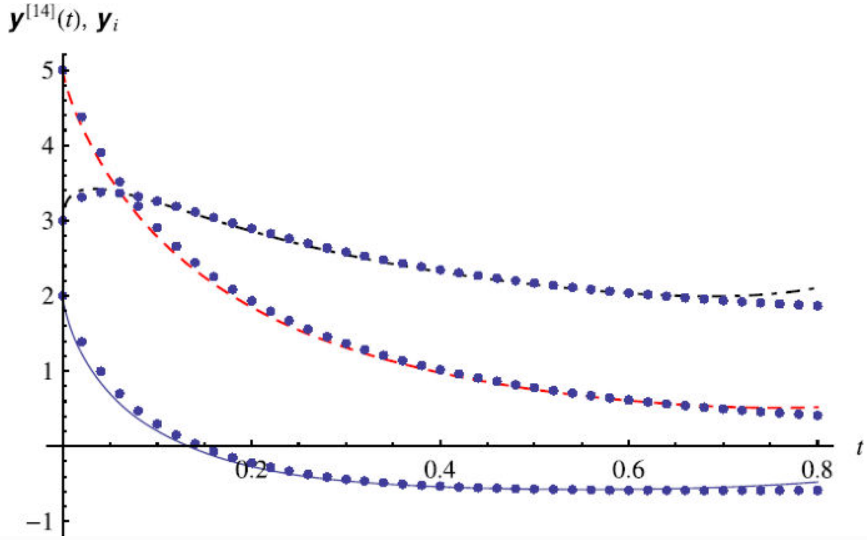
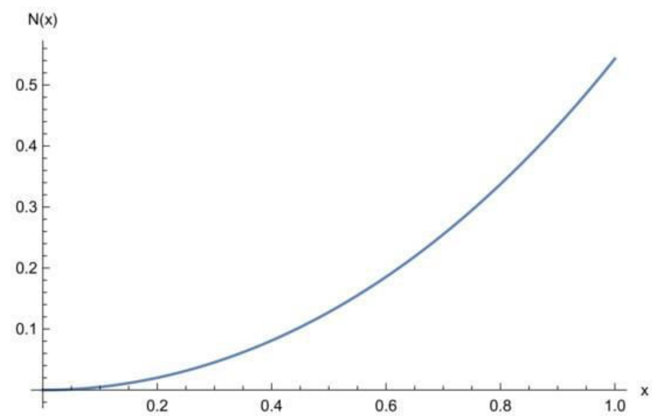
A graph of the function :



The solutions of system of linear fractional differential equations of incommensurate orders are investigated in this paper. First, we introduce an n-variable Mittag–Leffler function with n+1n+1 parameters. Then we derive the analytic expressions for the solutions of the fractional system by using the Laplace transform and multi-variable Mittag–Leffler functions of matrix arguments. Finally, we verify the analytic result with numeric solutions by an example, where the numeric solutions are given by generalizing the L1 algorithm to the fractional system.
We generate the plots of analytic approximate solutions and numeric solutions with the help of MATHEMATICA 8. The obtained series solutions are convergent on the entire interval 0<t<+∞0<t<+∞ and are easy to program and are approximated by any symbolic computation software.[14,15]

III. RESULTS

In applied mathematics and mathematical analysis, a fractional derivative is a derivative of any arbitrary order, real or complex. Its first appearance is in a letter written to Guillaume de l'Hôpital by Gottfried Wilhelm Leibniz in 1695.[2]



For the function N(x), we get Figure 1, where 0<x<1.
Around the same time, Leibniz wrote to one of the Bernoulli brothers describing the similarity between the binomial theorem and the Leibniz rule for the fractional derivative of a product of two functions. Fractional calculus was introduced in one of Niels Henrik Abel's early papers[3] where all the elements can be found: the idea of fractional-order integration and differentiation, the mutually inverse relationship between them, the understanding that fractional-order differentiation and integration can be considered as the same generalized operation, and even the

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unified notation for differentiation and integration of arbitrary real order.[4] Independently, the foundations of the subject were laid by Liouville in a paper from 1832.[5][6][7] The autodidact Oliver Heaviside introduced the practical use of fractional differential operators in electrical transmission line analysis circa 1890.[8] The theory and applications of fractional calculus expanded greatly over the 19th and 20th centuries, and numerous contributors have given different definitions for fractional derivatives and integrals.[9]
This is a MATLAB routine for evaluating the Mittag-Leffler function with two parameters (sometimes also called generalized exponential function).[16]

The Mittag-Leffler function with two parameters plays an important role and appears frequently in solutions of fractional differential equations (i.e. differential equations containing fractional derivatives).

IV. CONCLUSIONS

Fractional calculus (FC) is a discipline of mathematics that derives from the conventional definitions of integral and derivative operators by considering fractional values. The reason for attracting the scientist towards FC is that fractional derivatives have been recognized as powerful modeling and simulation tools for engineering problems. Many physical laws are expressed more accurately in terms of differential equations of arbitrary order. The fractal calculus can efficiently deal with kinetics, which is termed the fractal kinetics [17,18].The Mittag-Leffler (M-L) function and its generalizations are widely used in the field of fractals. The generalized M-L law with fractal calculus appears . The use of M-L function in the medical field with fractals , authors defined the M-L function on fractal sets. For more details about the use of the Mittag-Leffler function in the field of fractal calculus and applications, interested readers can refer .FC has potential applications in the variational iteration method (VIM). The, authors used the local fractional operators to investigate the application of local fractional VIM for solving the local fractional Laplace equations. A new VIM for a class of fractional convection-diffusion equations . Numerous papers on VIM and its various applications are found in many research articles[19,20]

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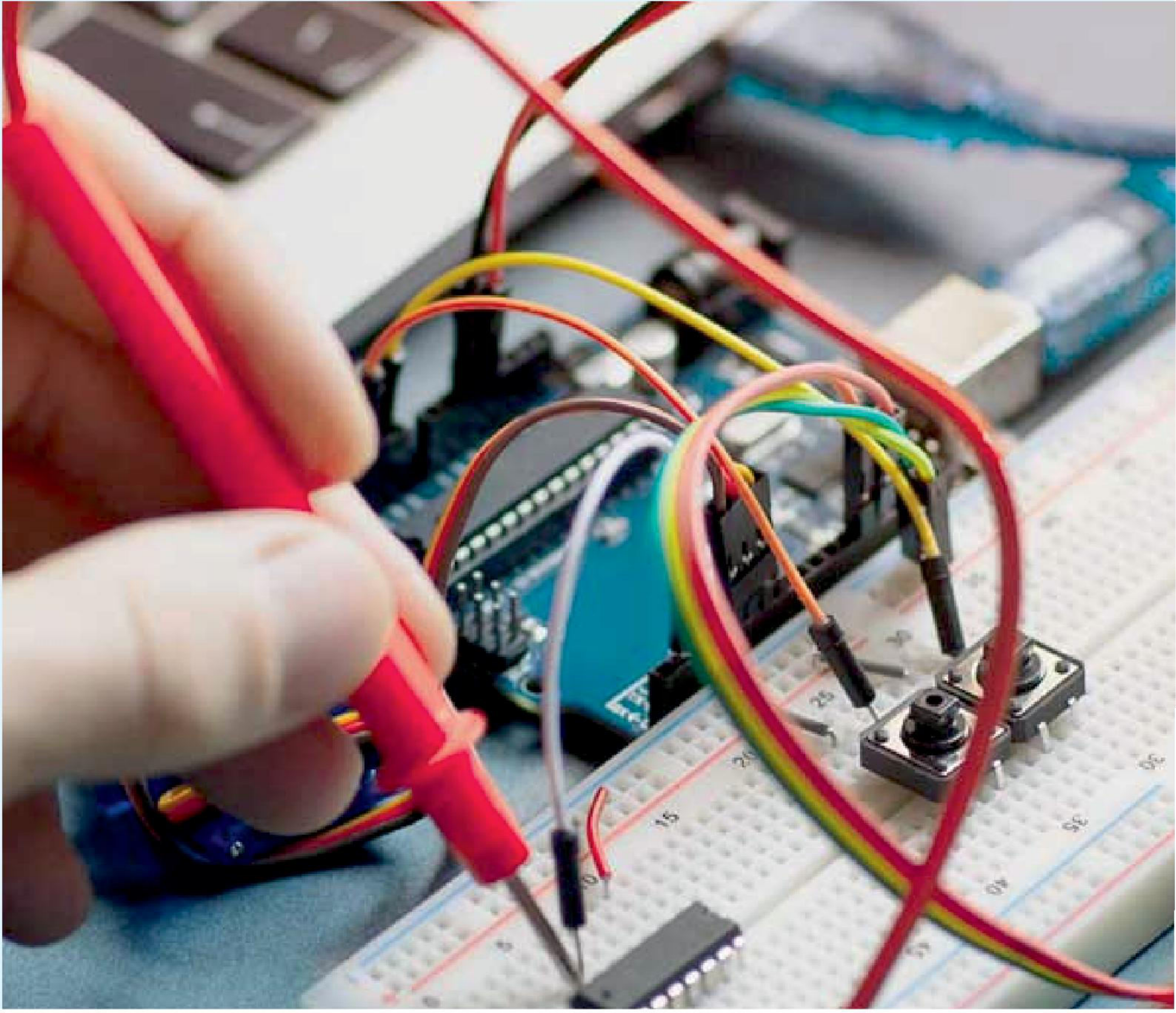

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