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# Application of Arithmetic Mean for Reduction of Higher Order Interval Systems

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**ABSTRACT:**A number of techniques for the reduction of interval system have been presented by various researchers. But the validity of the method is based on the resulting error by the model reduction. The system with parameter variations within bounds creates intervals in the coefficients of the system polynomial; hence the system is called interval system. This method represents the reduction of the order of Interval system about a general shifting point 'a' .The selection of this shifting point 'a' done based upon the arithmetic mean of the real parts of the poles of four high order fixed systems obtained by Kharitonov's theorem. The denominator of the reduced model obtained by Least Square method while the numerator is obtained by matching the power series expansion of the original high order system with the reduced model . A numerical example is provided to demonstrate various aspects of theoretical results.

**KEYWORDS:** Least Squares Mean, Large scale Interval system, order reduction.

## I.INTRODUCTION

The dimensionality problem in the analysis of high order systems is well known. In many situations it is desirable to replace the high order system by a lower order model. Several methods are available in the literature for the model reduction of high order dynamic systems. Recently it was shown[1] that care has to be taken, If the system transfer function contains a pole of magnitude less than one, then numerical problems can arise owing to a rapid increase in the magnitude of successive time moments .This gives an ill conditioned set of linear equations to solve for the reduced denominator .To overcome this problem, it is sometimes possible to use a linear shift s→ (s+a) such that the pole of smallest magnitude has the modulus of approximately one, this tends to reduce the sensitivity of the method . However, the focus of the work so far appears to concentrate mainly on the basic idea of extending this technique for order reduction of fixed parameter systems. In this paper this method is extend for order reduction of high order Interval systems. Consequently, the method is more flexible than most other stability preserving methods and is simple to implement.

## II.MAIN PROCEDURE

Given the nth order transfer function of a high order interval systems be represented as

$$H(s) = \frac{[a_0^-, a_0^+]+[a_1^-, a_1^+]s+\cdots+[a_{n-1}^-, a_{n-1}^+]s^{n-1}}{[b_0^-, b_0^+]+[b_1^-, b_1^+]s+\cdots+[b_n^-, b_n^+]s^n} \ldots (1)$$

Where $[a_i^-, a_i^+]$ for i = 0 to n-1 and $[b_i^-, b_i^+]$ for i = 0 to n are the interval parameters. Consider now the set δ(s) of real polynomials of degree 'n' of the form

$$\delta(s) = \delta_0 + \delta_1 s + \cdots + \delta_n s^n \ldots (2)$$

Where the coefficients lie within given ranges $\delta_0 \epsilon [x_o, y_o]$ , $\delta_1 \epsilon [x_1, y_1]$ , …. $\delta_n \epsilon [x_n, y_n]$ .Write $\delta = [\delta_0, \delta_1, \ldots\ldots\ldots\ldots, \delta_n]$ using the Kharitonov's theorem the following four extreme polynomials are derived

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$$K_1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + \cdots$$
$$K_2(s) = x_0 + y_1 s + y_2 s^2 + x_3 s^3 + \cdots$$
$$K_3(s) = y_0 + x_1 s + x_2 s^2 + y_3 s^3 + \cdots$$
$$K_4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + \cdots$$

From the above equations the numerator and the denominator polynomials are obtained.

Thus the four nth order system transfer functions are obtained each defined as

$$G_P(s) = \frac{A_{P0} + A_{P1}s + A_{P2}s^2 + \cdots + A_{Pn-1}s^{n-1}}{B_{P0} + B_{P1}s + B_{P2}s^2 + \cdots + B_{pn}s^n} \ldots (3)$$

Where p = 1, 2, 3, 4. and n = order of the original system.
Replace the $G_p(s)$ by $G_p(s+a)$ where the value of 'a' obtained by Arithmetic mean. Let the nthorder system transfer function of $G_p(s)$ is given by:

$$G_p(s) = k\frac{\prod_{i=1}^{m}(s+Z_i)}{\prod_{i=1}^{n}(s+P_i)}$$

Where, $P_i$ and $Z_i$ are the poles and zeros of the system, respectively. For this system, 'a' is given by the arithmetic mean (A.M) of the magnitude of real parts of $P_i$ $(|P_i|)$.

$$a = \sum_{i=1}^{n}\frac{|P_i|}{n}$$

The above equation gives value for the linear shift point 'a'. If $G_p(s+a)$ is expanded about s=0, then the time moment proportional's, $c_i$ are obtained by:

$$G_p(s+a) = \sum_{i=0}^{\infty} c_{pi} s^i \quad \ldots (4)$$

Similarly, if $G_p(s+a)$ is expanded about $s=\infty$, then the Markov parameters $m_j$ are obtained by:

$$G_p(s+a) = \sum_{j=1}^{\infty} m_{pj} s^{-j} \quad \ldots (5)$$

The four reduced $r^{th}$ order models obtained as

$$R_p(s) = \frac{d_{p0} + d_{p1}s + d_{p2}s^2 + \cdots + d_{pr-1}s^{r-1}}{e_{p0} + e_{p1}s + e_{p2}s^2 + \cdots + e_{pr}s^r} \ldots (6)$$

Which retains't' Time moments and 'm' Markov parameters, the coefficients $e_{pk}, d_{pk}$ in (6) are derived from following set of equations

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$$\left.\begin{array}{r}
d_{p0} = e_{p0}c_{p0} \\
d_{p1} = e_{p1}c_{p0+}e_{p0}c_{p1} \\
d_{p2} = e_{p2}c_{p0} + e_{p1}c_{p1+}e_{p0}c_{p2} \\
. \quad . \quad . \quad . \quad . \\
. \quad . \quad . \quad . \quad . \\
. \quad . \quad . \quad . \quad . \\
d_{pr-1} = e_{pr-1}c_{p0} + \cdots + e_{p0}c_{pr-1} \\
0 = e_{pr-1}c_{p1} + \cdots + e_{p0}c_{pr} \\
0 = e_{pr-1}c_{p2} + \cdots + e_{p0}c_{pr+1} \\
. \quad . \quad . \quad . \quad . \\
. \quad . \quad . \quad . \quad . \\
0 = e_{pr-1}c_{pt} + \cdots + e_{p0}c_{pr+t-1}
\end{array}\right\} \ldots (7)$$

And

$$\left.\begin{array}{r}
d_{pr-1} = m_{p1} \\
d_{pr-2} = m_{p1}e_{pr-1} + m_{p2} \\
. \quad . \quad . \quad . \\
. \quad . \quad . \quad . \\
d_{pt} = m_{p1}e_{pt+1} + m_{p2}e_{pt+2} + \cdots + m_{pr-t}
\end{array}\right\} \ldots (8)$$

Where the $c_j$ and $m_k$ are the Time moment proportional and Markov parameters of the system, such that j=(0,1,…..t-1) and k=(1,2,…..m) respectively. The denominator coefficients of the reduced model are obtained by substituting (8) in (7) and are given by the solution set.

$$\begin{bmatrix}
c_{pr+t-1} & c_{pr+t-2} \cdots\cdots\cdots\cdots & & c_{pt} \\
c_{pr+t-2} & c_{pr+t-3} \cdots\cdots\cdots & c_{pt} & c_{pt-1} \\
. & . \quad \cdots\cdots\cdots & . & . \\
c_{pr-1} & c_{pr-2} \cdots\cdots\cdots & c_{p1} & c_{p0} \\
c_{pr-2} & c_{pr-3} \cdots\cdots\cdots c_{p0} - m_{p1} & & \\
c_{pr-3} & c_{pr-4} \cdots\cdots c_{p0} - m_{p1} - m_{p2} & & \\
. & . & . & . \\
. & . & . & . \\
. & . \quad \cdots\cdots & . & . \\
c_{pt} & c_{pt-1} & \cdots c_{p0} - m_{p1} \cdots - m_{pr-t-1} &
\end{bmatrix} \mathrm{X} \begin{bmatrix} e_{p0} \\ e_{p1} \\ . \\ . \\ . \\ . \\ . \\ . \\ . \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ m_{p1} \\ m_{p2} \\ m_{p3} \\ m_{p4} \\ . \\ . \\ m_{pr-t} \end{bmatrix} \ldots (9)$$

or, the above equation can be represented as H e = m in matrix vector form and 'e' can be calculated from,

$$e = \left(H^T H\right)^{-1} H^T m \qquad \ldots (10)$$

are the coefficients of the reduced model denominator. If this estimate still does not yield a stable reduced denominator then H and m in (10) are extended by another row, which corresponds to using the next Markov parameter from the full system in Least Squares match. Once the reduced denominator obtained, formed by 'e', apply the inverse shift s→(s-a) to this reduced denominator. Later calculate the reduced numerator as before by matching proper number of Time moments of $G_p(s+a)$ to that of reduced model.

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## III. ILLUSTRATIVE EXAMPLE

**Example:** Consider the 6th order interval system given by its transfer function:

$$G(s) = \frac{[2\ 3]s^5 + [70\ 71]s^4 + [762\ 763]s^3 + [3610\ 3611]s^2 + [7700\ 7701]s^1 + [6000\ 6001]}{[1\ 2]s^6 + [41\ 42]s^5 + [571\ 572]s^4 + [3491\ 3492]s^3 + [10060\ 10061]s^2 + [13100\ 13101]s + [6000\ 6001]}$$

The four transfer functions obtained using the Kharitonov's theorem

$$G_1(s) = \frac{2s^5 + 70s^4 + 763s^3 + 3611s^2 + 7700s^1 + 6000}{2s^6 + 41s^5 + 571s^4 + 3492s^3 + 10061s^2 + 13100s + 6000}$$

$$G_2(s) = \frac{3s^5 + 70s^4 + 762s^3 + 3611s^2 + 7701s^1 + 6000}{2s^6 + 42s^5 + 571s^4 + 3491s^3 + 10061s^2 + 13101s + 6000}$$

$$G_3(s) = \frac{2s^5 + 71s^4 + 763s^3 + 3610s^2 + 7700s^1 + 6001}{s^6 + 41s^5 + 572s^4 + 3492s^3 + 10060s^2 + 13100s + 6001}$$

$$G_4(s) = \frac{3s^5 + 71s^4 + 762s^3 + 3610s^2 + 7701s^1 + 6001}{s^6 + 42s^5 + 572s^4 + 3491s^3 + 10060s^2 + 13101s + 6001}$$

The reduced models obtained using four time moments

$$R_1(s) = \frac{2.189706s+0.517962}{s^2+3.28587s+0.517962} \text{ With AM=3.416683}$$

$$R_2(s) = \frac{2.443738s+5.173420}{s^2+7.09981s+5.173420} \text{ With AM=3.49998}$$

$$R_3(s) = \frac{2.376220s+9.280991}{s^2+10.727719s+9.280991} \text{ With AM=6.8333}$$

$$R_4(s) = \frac{-1.088832s+38.670380}{s^2+33.708710s+38.670380} \text{ With AM=6.99998}$$

Thus the transfer function of the reduced interval model obtained as

$$R(s) = \frac{[-1.088832\ 2.819706]s + [0.517962\ 38.670380]}{[1\ 1]s^2 + [3.285871\ 33.708710]s + [0.517962\ 38.670380]}$$

Simulation results in figure1 and figures 2 , shows the accuracy of the step response when the reduced model compared with the original interval system.

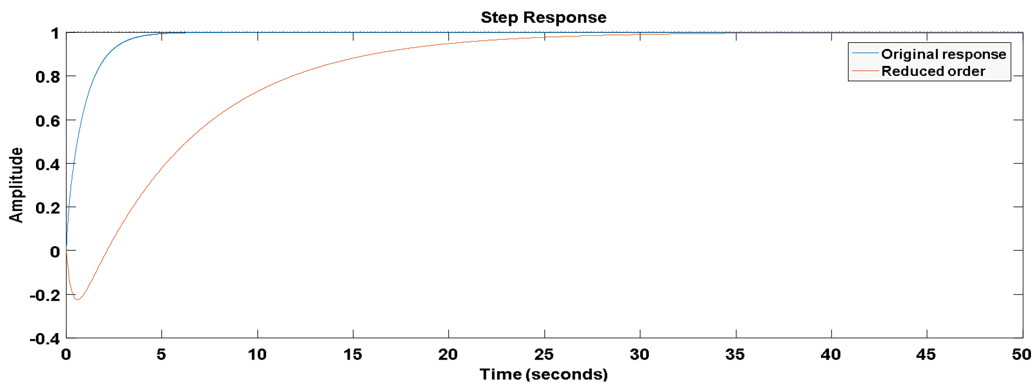


Fig 1: Lower Boundary Response

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In the fig 1, it shows the accuracy of the lower boundary step response when the reduced model compared with the original interval system.

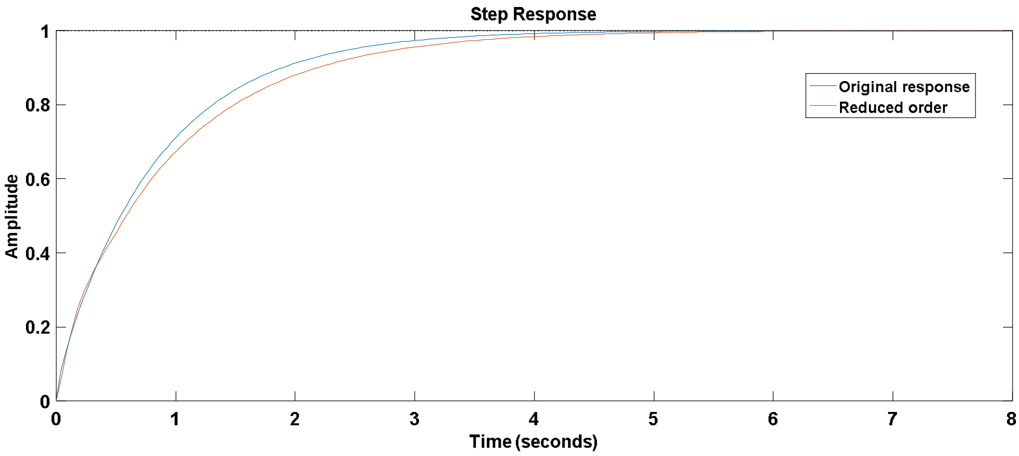


Fig 2: Upper boundary Response

In the fig 2, it shows the accuracy of the upper boundary step response when the reduced model compared with the original interval system.

## IV.CONCLUSION

A Novel method is suggested for the order reduction of high order Interval system based on Least Square moment matching method. Stability is guaranteed in the reduced models for linear time invariant interval systems.

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