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# Non-Linear Attitude and Altitude Control of a Quadrotor

Rikesh P Mathew<sup>1</sup>, Aswin R B<sup>2</sup>

<sup>1</sup> P G Student, Dept. of EEE, Mar Baselios College of Engineering and Technology, Thiruvananthapuram, Kerala, India<sup>1</sup>

<sup>2</sup> Assistant Professor, Dept. of EEE, Mar Baselios College of Engineering and Technology, Thiruvananthapuram, Kerala, India<sup>2</sup>

**ABSTRACT:** Quadcopter is now being increasingly used in both military and civilian applications and is also heavily researched. This is because of their high agility and accurate response. They are hence suitable for both indoor and outdoor applications. Autonomous Quadcopter should be capable of taking-off, landing and following a pre-set trajectory without any external control. However due to inherent non linearity and under actuated properties it is not an easy affair to control the quadrotor. A two loop control strategy is the most accurate control methodology. This paper however describes only the attitude and altitude control of the quadrotor. A nonlinear controller named Robust Integral of Sign of Error (RISE) controller is adopted for the inner attitude control loop. RISE controller has superior tracking capability and robustness in presence of bounded external disturbances and model uncertainties. A traditional PID controller is used to control the attitude. Quadrotor helicopter is assumed to be a rigid body and the dynamics were modelled using Newton- Euler equations to describe the six degrees of motion of the Quadcopter. Both the attitude and altitude control schemes are designed as separate loops. MATLAB implementation of the above control structure is also presented.

**KEYWORDS:** Quadrotor, RISE, robustness, PID, feature detection

### I.INTRODUCTION

Quadrotor control have always been an interesting research area for control engineers in the recent past owing to the highly nonlinear nature of the system, its under actuated dynamics and the various challenges faced by it in the real time applications. Many of the system parameters like aerodynamic coefficients are difficult to be calculated beforehand which leads to model uncertainties. The Quadrotor can also be affected by wind gusts and other environmental factors. Since it is no inherent damping properties. Many control techniques have been developed over time to cater these needs. Since the dynamic model of a Quadrotor can be linearized at many operating points, many traditional linear control methods [3], [4] have been used to stabilize the Quadrotor in a small range around the equilibrium point around which they are linearized. But the performance of such controllers may not be satisfactory if we consider the nonlinearities and a wider operating range. Therefore in recent past, researchers tend to look for more and more nonlinear and robust control methodologies and as a result lot of control techniques have been explored. To compensate for the under actuated property, various control techniques have been developed for the Quadrotor and other under actuated mechatronic systems, such as a wheeled mobile robot, an underwater vehicle, and overhead cranes [5], [6]. In [7], a novel robust back stepping-based controller that is based on an integral sliding-mode approach is proposed for an under actuated Quadrotor. Although the design procedure of the back-stepping scheme is very clear and the proof of the stability is standardized, the control gains are not easily tuned and chattering like behavior is observed.

In [8], the dynamic system of the Quadrotor is divided into the inner loop and the outer-loop subsystem. Such scheme is not difficult for implementation, but the stability of the closed-loop system cannot be easily guaranteed. To

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Vol. 6, Issue 4, April 2017

overcome this drawback, an inner- and outer-loop-based flight controller is proposed in [9], and the Asymptotic Stability (AS) of the closed-loop system is proved via a theorem of cascaded systems. To compensate for the parametric uncertainties in the system here, an adaptive nonlinear control method will be a suitable choice and has been properly

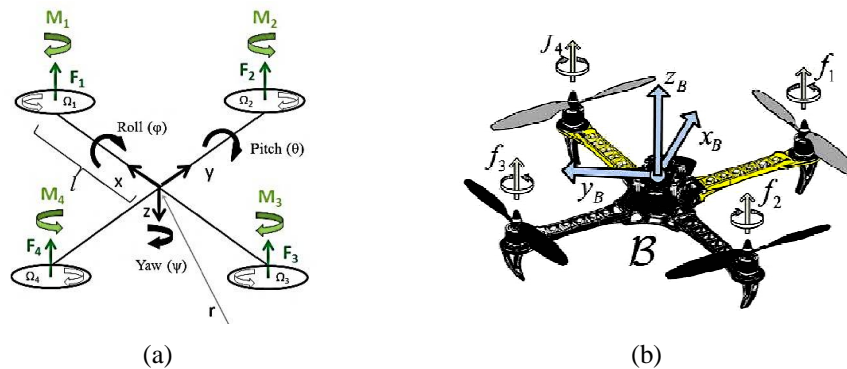


Fig 1 (a) Quadcopter Moments (b) Quadcopter Axes

control method always requires a Linear Parameterization (LP) condition, and sometimes, the controller's singularities will come into action [11]. Recently, a new adaptive control design using the immersion and invariance (I&I) methodology is first proposed in [12] and then was further studied and worked on by many researchers [13]– [16]. This approach, is quite different from the classic adaptive method, and does not require the LP condition, nor does it require certainty equivalence. It also simplifies the stability analysis using the Lyapunov method by showing up by providing cross terms in the Lyapunov function [8]. In [14], an I&I method is used to estimate the unknown mass of a Vertical Take Off and Landing vehicle, and it guarantees that the estimation converges to its true value. In [15], to control a mini Quadrotor UAV and overcome the uncertainties related with the thrust and drag coefficients, Fujimoto *et al.* simplified the dynamic model and developed an adaptive controller via the I&I methodology.

Due to the presence of external disturbances, devoted to the design of a robust controller for a quadrotor. In [13], a general sliding-mode control (SMC) is developed for a class of uncertain under actuated systems and then utilized to stabilize a quadrotor. However, one of the main drawbacks of the SMC is the chattering issue, which might deteriorate the control performance during practical implementation. To achieve a continuous control strategy, a robust integral of the signum of the error (RISE)-based controller is first presented in [14] and further developed by other researchers [15]. The RISE feedback control can compensate for external disturbances and modeling uncertainties while ensuring accurate tracking and semi global asymptotic convergence Unmanned aerial vehicles (UAVs) especially quadrotor drones are becoming the standard in reconnaissance in both military and civilian applications. Object identification and tracking are the typical requirement of most of these missions. So much research is being done in this field. Paula *et al.* [17] described the phases of identification, dynamic modelling and control of an unmanned aerial vehicle of type quadrotor intended to capture pictures and video in high definition with relatively low cost. Kim *et al.* [18] described wearable hybrid interface where eye motions and mental focus specifically impact the control of a quadcopter in three-dimensional space. Kim *et al.* [18] described wearable hybrid interface where eye motions and mental focus specifically impact the control of a quadcopter in three-dimensional space. This brief intends to explore object detection using a monocular camera which may prove useful as the size of these drones are getting smaller by the day and thereby payload capacity is reducing making a need for smaller and lesser sensory devices on board the quadrotor.

The paper is divided into five sections. Section II presents the modeling of the under actuated Quadrotor. Section III details control problem formulation and the controller development. Section IV presents the simulation results of the attitude and altitude control loops. Finally a few conclusions and the possible improvements are listed in Section V



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Vol. 6, Issue 4, April 2017

## II. QUADROTOR MODELLING

A Quad rotor Helicopter or simply a Quadcopter is as mentioned previously an Unmanned Aerial Vehicle (UAV) which has four rotors or propellers producing thrust. It uses these rapidly spinning rotors to push air downwards, thus creating a thrust force in the downward direction thereby keeping the quadcopter in hover. Fig. 1 shows the quadcopter moments during operation and Fig. 2 shows the two reference frames. An inertial frame  $\mathcal{I}$  is taken to be stationary with respect to the earth and a body frame  $\mathcal{B}$  is fixed to the center of mass of the object

To fly a quad rotor helicopter, you have to balance its weight by generating an equivalent force (Lift) and balance moments about its Centre of Gravity (CG) by generating opposite moments. A quad-copter generates these required moments and lift force using its four rotors. Generating lift is restively easy, but the difficult part is generating moments to stabilize the machine and generating control forces to move it to a desired location or on a desired path. To fly stable in an orientation, net moment about the CG should always be zero or resultant of all the forces acting on the quadrotor should pass through its CG. If the resultant of the lift generated by all the rotors doesn't pass through CG, it creates a moment about the CG and tend to tilt the quad-copter until lift again passes through the CG. Also, to balance the angular momentum about the CG, two rotors are made to rotate clockwise and other two anti-clockwise as shown in Fig. 1

### A. QUADROTOR DYNAMICS

The Modelling of the dynamics of the Quadrotor is done following the Newton-Euler equations for a dynamic system. The quadrotor has six degrees of freedom namely the translational position  $(x, y, z)$  and attitude  $(\theta, \phi, \varphi)$ . The following assumptions are also considered while modelling the system equations

1. The structure is rigid and symmetrical.
2. The center of gravity of the quadrotor coincides with the body fixed frame origin
3. The propellers are rigid
4. Thrust and drag are proportional to the square of propeller's speed.

The dynamics of the quadrotor were adopted from [2], [19]. A few assumptions from [20] has also been adapted to obtain the model as required for our analysis

$$\begin{aligned}
 m\ddot{x} &= -K_1\dot{x} + u_t[\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\varphi] \\
 m\ddot{y} &= -K_2\dot{y} + u_t[\cos\phi\sin\phi\sin\theta - \sin\phi\cos\varphi] \\
 m\ddot{z} &= -K_3\dot{z} - mg + u_t[\cos\phi\cos\theta] \\
 J_1\ddot{\phi} &= -K_4l\dot{\phi} + d_1(t) + l\tau_1 \\
 J_2\ddot{\theta} &= -K_5l\dot{\theta} + d_2(t) + l\tau_2 \\
 J_3\ddot{\varphi} &= -K_5l\dot{\varphi} + d_3(t) + c\tau_2
 \end{aligned} \tag{1}$$

where  $m \in \mathbb{R}^+$  denotes the mass,  $J_i \in \mathbb{R}^+$  for  $i=1,2,3$  are the moments of inertia,  $K_i \in \mathbb{R}^+$  for  $i=1,2,3,4,5,6$  denotes the aerodynamic damping coefficients,  $g$  is the acceleration due to gravity,  $u_t(t)$ ,  $\tau_1(t)$ ,  $\tau_2(t)$ , and  $\tau_3(t)$  denote the total thrust and the three rotational forces produced by the four rotors and  $c \in \mathbb{R}^+$  represents a constant force-moment factor. The rotation from the body frame to inertial frame describes the orientation of the quadrotor.

$$R = \begin{bmatrix} c\theta c\varphi & s\theta s\phi c\varphi & s\theta c\phi c\varphi + s\phi s\varphi \\ c\theta s\varphi & s\theta s\phi s\varphi + c\theta c\varphi & s\theta c\phi s\varphi - s\theta c\varphi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{2}$$



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Vol. 6, Issue 4, April 2017

The rotation matrix is given by R (2), where s and c denote sin and cosine. The control inputs  $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  can be obtained from motor thrusts as

$$\begin{aligned} U_1 &= T_1 + T_2 + T_3 + T_4 \\ U_2 &= -T_1 - T_2 + T_3 + T_4 \\ U_3 &= -T_1 + T_2 + T_3 - T_4 \\ U_4 &= -T_1 - T_2 + T_3 - T_4 \end{aligned} \quad (3)$$

The inertial matrix for the quadrotor is a diagonal matrix, the diagonal elements, which are the product of inertia, are zero due to the symmetry of the quadrotor.

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (4)$$

where  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  are the moments of inertia about the principle axes in the body frame. The gyroscopic moment of a rotor is a physical effect in which gyroscopic torques or moments attempt to align the spin axis of the rotor along the inertial z-axis. The quadrotor is also subject to aerodynamic forces and moments.

$$F_i = \frac{1}{2} \rho A C_T r^2 w_i^2 \quad (5)$$

$$M_i = \frac{1}{2} \rho A C_D r^2 w_i^2 \quad (6)$$

where  $\rho$  is the density of surrounding air,  $C_T$ ,  $C_D$  are the aerodynamic coefficients,  $w_i^2$  is the angular velocity,  $A$  blade area. Clearly, the aerodynamic forces and moments depend on the geometry of the propeller and the air density. Since for the case of quadrotor, the maximum altitude is usually limited, thus the air density can be considered constant. The following section describes the non-linear control structure capable of providing accurate tracking even in the presence of external disturbances and modelling uncertainties.

### B. MOTOR DYNAMICS

Most motors used on Quadcopters are now out-runner which means, the rotating part is on the outside, and not on the inside as in the case of in-runner motors). With such a layout these motors can generate much more torque as compared to their in-runner counterparts. High torque is required for a quadcopter, as we maintain balance by changing the revolutions of the motor. The higher the torque the faster, can change the speed of the propellers be brought about. High torque implies that we don't essentially need gearbox, and hence can save a lot of mass. But out-runners are not very practical for brushed commutation, as it require lots of wires and additional contacts. Hence most of the out-runners are brushless. The brushless motors are way more powerful for their weight than available brushed motors and are also comparatively efficient. The voltage and current equation for BLDC motor is given below.

$$\begin{aligned} v &= R_{mot} i_a + L_{mot} \frac{di_a}{dt} + K_{mot} \omega \\ i_a &= \frac{v - K_{mot} \omega}{R_{mot}} \end{aligned} \quad (7)$$

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Vol. 6, Issue 4, April 2017

Motor Power is given by product of voltage ( $v$ ) and current ( $i_a$ ) and the torque equation is given by (5)

$$\tau = k_t(I - I_0) \quad (8)$$

From Momentum Theory we can obtain

$$v_h = \sqrt{\frac{T}{2\rho A}} \quad (9)$$

Reducing and combining equations from (4) – (6), we can obtain

$$T = k\omega^2 \quad (10)$$

i.e. the motor torque is proportional to the propeller speed

### III. PROBLEM FORMULATION AND CONTROL DEVELOPMENT

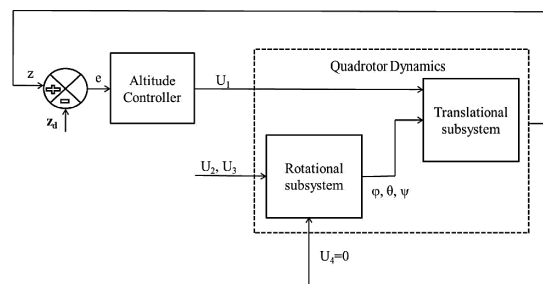


Fig. 2. Structure of Altitude-Attitude Control System

The control development of a Quadrotor is a tad difficult process owing to the highly under actuated nature of the quadcopter meaning that the number of independent control inputs (which is in this case the thrust produced by the four rotors) is lesser than the number of variables that need to be controlled (which in case of a quadrotor are the three translational positions and the three attitude angles). Therefore it is always advisable to formulate a two loop control structure to control the two set of dynamics. Also from analyzing the quadrotor dynamics derived by Newton-Euler Method we can see that the translational dynamics are dependent on the attitude but the attitude dynamics are completely independent of the translational dynamics. Therefore a good strategy would be to control the position of the quadrotor in the outer loop and generate the desired two attitude values theta ( $\theta$ ) and phi ( $\varphi$ ) from the control outputs of the position controllers. The heading psi ( $\psi$ ) is given as the reference. The calculation of desired values of theta and psi are not discussed in this brief as it aims to develop only a non-linear controller for the attitude and an altitude controller

The general structure of the control system is shown in Fig. 2. This section presents the attitude and altitude control for a quadrotor. The altitude control is separated from the attitude control loop and implements different controller as well. The altitude controller used here is a traditional PID controller which has the general structure as given in (4). The proportional ( $K_p$ ) accounts for the current values of error, the integral ( $K_i$ ) term accounts for the past values of error and derivative term ( $K_d$ ) compensates for future trends in error based on the rate of its change in the present

$$\mu_z = K_{pz} e_z + K_{dz} \dot{e}_z + K_{iz} \int_0^t e_z \quad , e_z = z_d - z \quad (11)$$



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Vol. 6, Issue 4, April 2017

However for the attitude control loop, a novel nonlinear controller is used which was first proposed in [19]. This nonlinear control methodology, named as RISE (robust integral of the signum of the error) feedback control, has fewer requirements for the dynamic model knowledge and maintains robustness with respect to structural or non-structural uncertainties and non-vanishing external disturbance. The objective is to design a robust controller that guarantee tracking of desired attitude for proper flight control. To implement this control strategy the attitude errors are first calculated as follows

$$e_{\phi_1} = \phi_d - \phi, e_{\theta_1} = \theta_d - \theta, e_{\varphi_1} = \varphi_d - \varphi \quad (12)$$

Next step is to define the auxiliary errors of the above system as follows

$$\begin{aligned} e_{\phi_2} &= e_{\phi_1} + \alpha_{\phi} \dot{e}_{\phi_1} \\ e_{\theta_2} &= e_{\theta_1} + \alpha_{\theta} \dot{e}_{\theta_1} \\ e_{\varphi_2} &= e_{\varphi_1} + \alpha_{\varphi} \dot{e}_{\varphi_1} \end{aligned} \quad (13)$$

where  $\alpha_{\phi}, \alpha_{\theta}, \alpha_{\varphi}$  are positive constants.

Following the similar steps in [24], a continuous RISE feedback control law to attain the mentioned control objective is as follows

$$\begin{aligned} \mu_{\phi} &= K_{s\phi} e_{\phi_2} + \int_0^t (K_{s\phi} \alpha_{\phi} e_{\phi_2} + \beta_{\phi} \text{sgn}(e_{\phi_2}(\tau))) d\tau \\ \mu_{\theta} &= K_{s\theta} e_{\theta_2} + \int_0^t (K_{s\theta} \alpha_{\theta} e_{\theta_2} + \beta_{\theta} \text{sgn}(e_{\theta_2}(\tau))) d\tau \\ \mu_{\varphi} &= K_{s\varphi} e_{\varphi_2} + \int_0^t (K_{s\varphi} \alpha_{\varphi} e_{\varphi_2} + \beta_{\varphi} \text{sgn}(e_{\varphi_2}(\tau))) d\tau \end{aligned} \quad (14)$$

where  $K_{s\phi}, K_{s\theta}, K_{s\varphi}$  are positive constants and  $\text{sgn}(\cdot)$  represents the signum function. The MATLAB implementation of the RISE controller is given in Fig.4. This control technique may also be extended to any other similar system. The stability analysis for the above controller is mentioned in [14]

#### IV. SIMULINK VALIDATION

The attitude and altitude controllers were implemented in MATLAB Simulink platform to verify the effectiveness of the same. The Quadrotor dynamics were first simulated in open loop to verify the performance. The quadrotor parameters were adopted from [16]. The rotors speed required for the quadrotor to hover was calculated using the following equation

$$mg = 4F_i = 4K_f \omega^2 \quad (15)$$

Where  $K_{fi}$  is the motor constant and  $\omega$  is the propeller velocity. The input to the quadrotor dynamics is steadily increased in parts to check whether the increase in applied thrust only causes variation to the altitude. If so the system modelling is satisfactory. The other dynamics can be checked by providing respective inputs to the various channels to verify the model accuracy. Then an altitude controller is introduced followed by an attitude controller as the resulting structure is as in Fig 3.

In order to analyze the tracking capability of the system a reference signal is designed which zero value at the start of simulation and followed by both positive and negative peaks and the system is made to follow the trajectory using the controllers. The reference are given individually for each of the controllers. It should be noted that yaw angle



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Vol. 6, Issue 4, April 2017

should be preferably kept as low as possible if not zero. Therefore simulations are also done with yaw reference as zero which is shown in Fig 14. The control output of altitude is given to the attitude loop for the desired value of thrust in the system dynamic equations as given is (1). The results from Simulink Implementation shows that the suggested controller is capable of accurately tracking the attitude angles of the quadrotor to the best possible extend. The pitch, roll and yaw dynamics were subjected to varying trajectories to study the tracking capability of the controller. The pitch and roll response are quite similar owing to the almost equal dynamics due to the symmetry of the system.

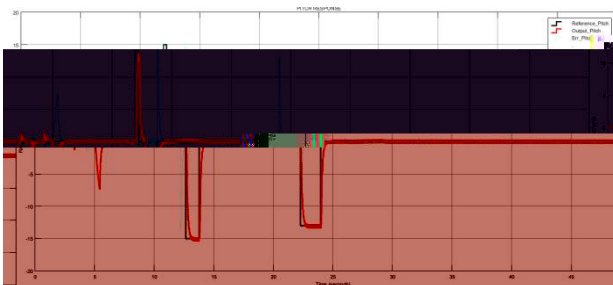


Fig 3. Pitch Response to Varying trajectory

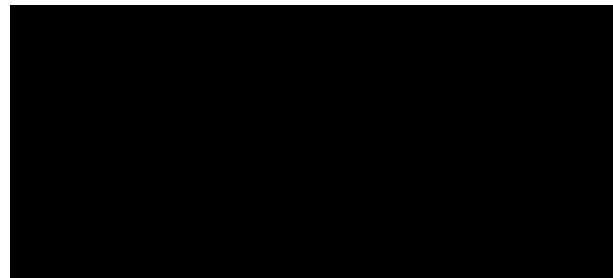


Fig 4. Roll Response to Varying trajectory

Pitch response (Fig 3) is characterized by a rise time of 1.1 seconds with a constant steady state error of 15 percent. The roll response (Fig 4) is much faster with a rise time of about 0.3 seconds and steady state error less than 1 percent. The yaw moment (Fig 5) maintains a constant error which is owing to the inherent non linearity of the system. The response of the yaw dynamics (Fig 6) to zero tracking gives you a clearer picture of the above mentioned case in Fig 8. In order to check the ability of the PID controller to achieve a required height it's a subject to step input which forces the quadrotor from an initial height of zero meters from the ground to above 5 meters above the ground. Fig 9 gives the response characteristics of the altitude controller and as seen has very fast tracking abilities. It has a rise time of about 0.5 seconds and overshoots to about 13 percent more than the required value (Fig 7) before settling to the desired altitude in less than 0.5 seconds after overshoot and with almost zero steady state error. Thus the proposed control structure with nonlinear RISE controller for the attitude control and PID controller for the altitude is capable of providing better tracking performance. The response characteristic of RISE has been compared with PID response characteristic from the reference and is found to have better tracking capabilities and disturbance tolerance.

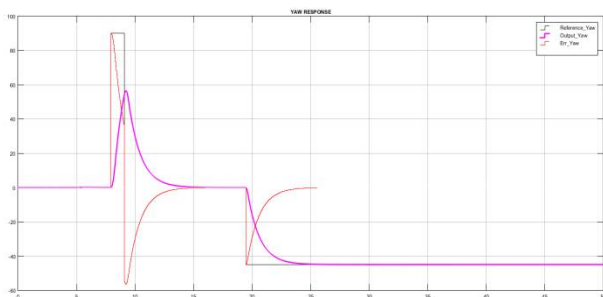


Fig 5. Yaw Response to Varying trajectory

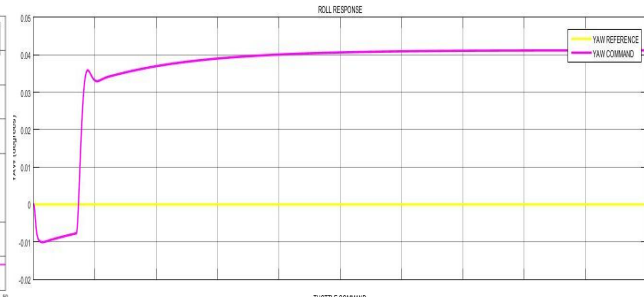


Fig 6. Yaw being tracked to zero



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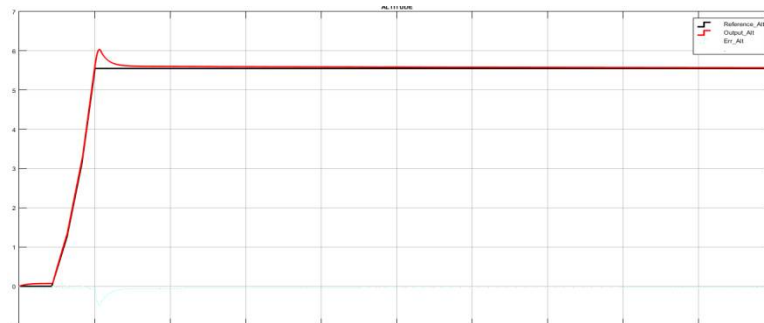


Fig 7. Altitude Tracking

## V. CONCLUSION

This brief aimed to point out the importance of non-linear controllers in designing tracking algorithms for under actuated nonlinear systems. The system considered was a highly under actuated Quadrotor system and it was modelled using Newton- Euler equations. A fairly novel controller named RISE was studied and formulated for the attitude control of the quadrotor and a traditional PID controller was implemented for the altitude control. The modelling and controllers were implemented in MATLAB/Simulink and results analyzed. It has been found that the nonlinear controller has very good tracking capabilities and is less susceptible to disturbances. The PID controller was also sufficient to track the system to the desired altitude. Further advancements to the work may include a complete tracking control of the quadrotor using a nonlinear two loop which provides both accurate translational position tracking.

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Vol. 6, Issue 4, April 2017

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