ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Relation between Type-II Discrete Sine Transform and Discrete Fourier Transform

M.N.Murty[1], H.Panda[2], B.padhy[3]

Visiting Faculty, Dept. of Physics, Sri Sathya Sai Institute of Higher Learning, Prasanthi Nilayam , Andhra Pradesh, India[1]

Dept. of Physics, S.K.C.G. Autonomous College, Paralakhemundi, Odisha, India[2]

Dept. of Physics, Khallikote Autonomous College, Berhampur, Odisha, India[3]

ABSTRACT: In this paper, a relation between type-II discrete sine transform (DST-II) of even length *N* and *N*- point discrete Fourier transform (DFT) is derived. Also, another two relations between inverse discrete sine transform (IDST) of even length *N* and *N*- point inverse discrete Fourier transform (IDFT) are established for even and odd DST-II input samples respectively.

KEYWORDS: Discrete Sine Transform, Discrete Cosine Transform, Discrete Fourier Transform.

I.INTRODUCTION

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT), discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are eight types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity. The DCT and DST transform of types I, II, III and IV, form a group of so-called “even” sinusoidal transforms. Much less known is group of so-called “odd” sinusoidal transforms: DCT and DST of types V, VI, VII and VIII.

The DST was first introduced to the signal processing by Jain [1], and several versions of this original DST were later developed by Kekre*et al.*[2], Jain [3] and Wang *et al.* [4]. Ever since the introduction of the first version of the DST, the different DSTs have found wide applications in several areas in Digital signal processing (DSP), such as image processing[1,5,6], adaptive digital filtering[7] and interpolation[8]. The performance of DST can be compared to that of the DCT and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates [9]. Yip and Rao [10] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT.

Among all the discrete transforms, the discrete Fourier transform (DFT) is the most popular transform, and it is mainly due to its usefulness in very large number of applications in different areas of science and technology. The DFT plays a key role in various digital signal processing and image processing applications [11, 12]. Not only it is frequently encountered in many different applications, but also it is computation-intensive. DFT and inverse discrete Fourier transform (IDFT) have been regarded as the key technologies for signal processing in orthogonal frequency division multiplexing (OFDM) communication systems.

In this paper, the relation between DST-II of even length *N* and *N*- point DFT has been established. Another two relations between IDST-II of even length *N* and *N*- point IDFT are derived for even and odd input samples of DST-II respectively.

The rest of the paper is organized as follows. The proposed relation between DST-II and DFT is presented in Section-II. Section-III establishes the relation between IDST-II and IDFT. Conclusion is given in Section-IV.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

II.RELATION BETWEEN DST-II AND DFT

The type-II DST for input sequence $\{x_n; n = 0,1,2,...,N-1\}$ is defined as

$$Y_{II}(k) = \sqrt{\frac{2}{N}} c_k \sum_{n=0}^{N-1} x_n \sin\left[\frac{(2n+1)k\pi}{2N}\right] \qquad for\, k = 1,2,...,N \tag{1}$$

Where

$$c_k = \begin{cases} \frac{1}{\sqrt{2}} & if \;\; k = N \\ 1 & otherwise \end{cases} \tag{2}$$

The $Y_{II}(k)$ values represent the transformed data. Without loss of generality, the scale factor $\sqrt{\frac{2}{N}} c_k$ in (1) may be ignored in the rest of the paper.

The 1-D DFT of input sequence $\{u_n; n = 0,1,2,...,N-1\}$ is defined by

$$X(k) = \sum_{n=0}^{N-1} u_n e^{-\frac{j2nk\pi}{N}} \qquad for\, k = 0,1,2,...,N-1 \tag{3}$$

Where $j = \sqrt{-1}$

Separating even and odd input samples for even length N , (1) can be expressed as

$$Y_{II}(k) = \sum_{n=0}^{\frac{N}{2}-1} x_{2n} \sin\left[\frac{(4n+1)k\pi}{2N}\right] + \sum_{n=0}^{\frac{N}{2}-1} x_{2n+1} \sin\left[\frac{(4n+3)k\pi}{2N}\right] \tag{4}$$

Define even and odd input samples of DST-II in terms of input sequence u_n of DFT in (3) as given below.

$$\begin{cases} x_{2n} = u_n \;\; for \;\; n = 0,1,2,...\frac{N}{2}-1 \\ x_{2n+1} = -u_{N-1-n} \;\; for\, n = 0,1,2,..,\frac{N}{2}-1 \end{cases} \tag{5}$$

Substituting (5) in (4), we obtain

$$Y_{II}(k) = \sum_{n=0}^{\frac{N}{2}-1} u_n \sin\left[\frac{(4n+1)k\pi}{2N}\right] - \sum_{n=0}^{\frac{N}{2}-1} u_{N-1-n} \sin\left[\frac{(4n+3)k\pi}{2N}\right] \tag{6}$$

It can proved that

$$-\sum_{n=0}^{\frac{N}{2}-1} u_{N-1-n} \sin\left[\frac{(4n+3)k\pi}{2N}\right] = \sum_{n=\frac{N}{2}}^{N-1} u_n \sin\left[\frac{(4n+1)k\pi}{2N}\right] \tag{7}$$

Using (7) in (6), we have

$$Y_{II}(k) = \sum_{n=0}^{\frac{N}{2}-1} u_n \sin\left[\frac{(4n+1)k\pi}{2N}\right] + \sum_{n=\frac{N}{2}}^{N-1} u_n \sin\left[\frac{(4n+1)k\pi}{2N}\right]$$

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$\Rightarrow Y_{II}(k) = \sum_{n=0}^{N-1} u_n \sin\left[\frac{(4n+1)k\pi}{2N}\right] \quad (8)$$

The above expression can be written as

$$\begin{aligned} Y_{II}(k) &= \mathrm{Im}\left[\sum_{n=0}^{N-1} u_n e^{-j\frac{(4n+1)k\pi}{2N}}\right] \\ &= \mathrm{Im}\left[e^{-j\frac{k\pi}{2N}} \sum_{n=0}^{N-1} u_n e^{-j\frac{2nk\pi}{N}}\right] \end{aligned} \quad (9)$$

The symbol Im denotes imaginary part.

Substituting (3) in (9), we have

$$Y_{II}(k) = \mathrm{Im}\left[e^{-j\frac{k\pi}{2N}}\ DFT\ X(k)\right] \quad (10)$$

The above expression gives the relation between DST-II and DFT.

III.RELATION BETWEEN IDST-II AND IDFT

The inverse discrete sine transform (IDST) is defined as

$$x_n = \sum_{k=1}^{N} Y_{II}(k) \sin\left[\frac{(2n+1)k\pi}{2N}\right] \qquad for\ n = 0,1,2,...,N-1 \quad (11)$$

The inverse discrete Fourier transform (IDFT) is given by

$$u_n = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2nk\pi}{N}} \qquad for\ n = 0,1,2,...,N-1 \quad (12)$$

By neglecting the normalization factor $1/N$ in (12), IDFT can be written as

$$u_n = \sum_{k=0}^{N-1} X(k) e^{j\frac{2nk\pi}{N}} \qquad for\ n = 0,1,2,...,N-1 \quad (13)$$

Let us now find out IDST in terms of IDFT for n even and n odd.

3.1 IDST FOR EVEN n

For even n , we take $n = 2r$, where $r = 0,1,2,..,\frac{N}{2}-1.$

Substituting $n = 2r$ in (11), the IDST for even n is given by

$$\begin{aligned} x_{2r} &= \sum_{k=1}^{N} Y_{II}(k) \sin\left[\frac{(4r+1)k\pi}{2N}\right] \\ &= \sum_{k=0}^{N} Y_{II}(k) \sin\left[\frac{(4r+1)k\pi}{2N}\right] \\ &= \sum_{k=0}^{N-1} Y_{II}(k) \sin\left[\frac{(4r+1)k\pi}{2N}\right] + Y_{II}(N) \sin\left[\frac{(4r+1)\pi}{2}\right] \\ &= \sum_{k=0}^{N-1} Y_{II}(k) \sin\left[\frac{(4r+1)k\pi}{2N}\right] + Y_{II}(N) \end{aligned}$$

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

$$= \mathrm{Im}\left[\sum_{k=0}^{N-1} Y_{II}(k) e^{j\frac{k\pi}{2N}} e^{j\frac{2rk\pi}{N}}\right] + Y_{II}(N)$$

$$\Rightarrow x_{2r} = \mathrm{Im}\left[\sum_{k=0}^{N-1} P(k) e^{j\frac{2rk\pi}{N}}\right] + Y_{II}(N) \tag{14}$$

Where

$$P(k) = Y_{II}(k)\, e^{j\frac{k\pi}{2N}} \tag{15}$$

According to (13), the value within the brackets of (14) represents the $r\,th$ component of N - point IDFT of $P(k)$, denoted by $IF_r^N\{P(k)\}$ .Hence (14) can be written as

$$x_{2r} = \mathrm{Im}\left[IF_r^N\{P(k)\}\right] + Y_{II}(N) \tag{16}$$

The above expression gives the relation between IDST and IDFT for even n .

3.2 IDST FOR ODD n

For odd n , we take $n = 2r+1$, where $r = 0,1,2,..,\frac{N}{2}-1$.

Substituting $n = 2r+1$ in (11), the IDST for odd n is given by

$$x_{2r+1} = \sum_{k=1}^{N} Y_{II}(k) \sin\left[\frac{(4r+3)k\pi}{2N}\right]$$

$$= \sum_{k=0}^{N} Y_{II}(k) \sin\left[\frac{(4r+3)k\pi}{2N}\right]$$

$$= \sum_{k=0}^{N-1} Y_{II}(k) \sin\left[\frac{(4r+3)k\pi}{2N}\right] + Y_{II}(N) \sin\left[\frac{(4r+3)\pi}{2}\right]$$

$$= \sum_{k=0}^{N-1} Y_{II}(k) \sin\left[\frac{(4r+3)k\pi}{2N}\right] - Y_{II}(N)$$

$$= \sum_{k=0}^{N-1} Y_{II}(k)\ \mathrm{Im}\left[e^{j\left\{2\pi - \frac{(4r+3)k\pi}{2N}\right\}}\right] - Y_{II}(N)$$

$$= \mathrm{Im}\left[\left\{\sum_{k=0}^{N-1} Y_{II}(k) e^{j\frac{k\pi}{2N}}\right\} e^{j\frac{2(N-r-1)k\pi}{N}}\right] - Y_{II}(N)$$

Using (15) in the above expression, we obtain

$$x_{2r+1} = \mathrm{Im}\left[\sum_{k=0}^{N-1} P(k) e^{j\frac{2(N-r-1)k\pi}{N}}\right] - Y_{II}(N) \tag{17}$$

According to (13), the value within the brackets of the above expression (17) represents the $(N-r-1)th$ component of N - point IDFT of $P(k)$, denoted by $IF_{N-r-1}^N\{P(k)\}$ .Hence (17) can be written as

$$x_{2r+1} = \mathrm{Im}\left[IF_{N-r-1}^N\{P(k)\}\right] - Y_{II}(N) \tag{18}$$

The above expression gives the relation between IDST and IDFT for odd n .

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

IV. CONCLUSION

In this paper, the relation between DST-II of even length N and N- point DFT has been established. Another two relations between IDST-II of even length N and N- point DFT are derived for even and odd input samples of DST-II respectively.

REFERENCES

[1] A.K. Jain, "A fast Karhunen-Loeve transform for a class of random processes", IEEE Trans. Commun., vol. COM-24, pp.1023-1029, September 1976.
[2] H.B. Kekre and J.K. Solanka, " Comparative performance of various trigonometric unitary transforms for transform image coding", Int. J. Electron., vol. 44,pp.305-315,1978.
[3] A.K. Jain, "A sinusoidal family of unitary transforms", IEEE Trans. Patt. Anal. Machine Intell., vol. PAMI-I, pp.356-365, September 1979.
[4] Z. Wang and B. Hunt, "The discrete W transform", Applied Math Computat., vol. 16, pp.19- 48,January 1985.
[5] S. Cheng, "Application of the sine transform method in time of flight positron emission image reconstruction algorithms", IEEE Trans. BIOMED. Eng., vol. BME-32, pp.185-192, March 1985.
[6] K. Rose, A. Heiman, and I. Dinstein, " DCT/DST alternate transform image coding", Proc. GLOBECOM 87, vol. I,pp.426-430, November 1987.
[7] J.L. Wang and Z.Q. Ding, "Discrete sine transform domain LMS adaptive filtering", Proc. Int. Conf. Acoust., Speech, Signal Processing,pp.260-263, 1985.
[8] Z. Wang and L. Wang, "Interpolation using the fast discrete sine transform", Signal Processing, vol. 26, pp.131-137, 1992.
[9] A. K. Jain, Fundamentals of Digital Image Processing, Englewood Cliffs, NJ: Prentice - Hall, 1989.
[10] P. Yip and K.R. Rao, "On the computation and the effectiveness of discrete sine transform", Comput. Electron., vol. 7,pp.45-55,1980.
[11] R.N.Bracewell, The Fourier transform and its applications, 3rd edition, New York: McGraw-Hill, 2000.
[12] J.G.Proakis and D.G.Manolakis, Digital signal processing: principles, algorithms and applications, Prentice-Hall, 1996.