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Radix-2 Algorithm for Realization of Type-II Discrete Sine Transform using Recursive Filter

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ABSTRACT: In this paper, radix-2 algorithm for realization of type –II discrete sine transform (DST-II) of length $N = 2^m$ (m = 2, 3, 4,......) is presented. The $(N – 1)$ output components of DST-II are arranged in m groups. The number of output components in these groups is 2^r ($r = m – 1, m – 2,....., 0$). Recursive expressions for these $(N – 1)$ output components have been derived and they are realized using simple recursive filter structure. The last N^{th} output component of DST-II is excluded from grouping. The recursive algorithms are suitable for VLSI implementation.

KEYWORDS: Discrete Sine Transform, Discrete Cosine Transform, Radix-2 Algorithm, Recursive Filter.

I.INTRODUCTION

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT), discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are eight types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity. The DCT and DST transform of types I, II, III and IV, form a group of so-called "even" sinusoidal transforms. Much less known is group of so-called "odd" sinusoidal transforms: DCT and DST of types V, VI, VII and VIII.

The DST, originally introduced to the signal processing by Jain [1] (type-1) and Kekre *et al.* [2] (type-II), has found applications to image processing [1], adaptive filtering [3], transmultiplexing [4], and interpolation [5]. The performance of DST can be compared to that of the DCT and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates [6]. Yip and Rao [7] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT. Recursive algorithms for DST can be developed in a similar manner to the recursive algorithms for DCT.

In this paper, radix-2 algorithm for realization of type –II discrete sine transform (DST-II) of length $N = 2^m$ (m = 2, 3, 4,......) is derived by arranging the $(N – 1)$ output components of DST-II in m groups. These output components can be realized using simple recursive filter structure. The last N^{th} output component of DST-II is excluded from grouping.

The rest of the paper is organized as follows. The proposed radix-2 recursive algorithm for DST-II is presented in Section-II. An example for realization of DST-II of length N = 8 is presented in Section-III. Conclusion is given in Section-IV.

II.RADIX-2 RECURSIVE ALGORITHM FOR DST-II

The type-II discrete sine transform (DST-II) for input data sequence $\{x_o(n);\, n = 1, 2, \ldots, N\}$ is defined as

$$Y(k) = \sqrt{\frac{2}{N}}\, c_k \sum_{n=1}^{N} x_o(n)\, \sin\left[\frac{(2n-1)k\pi}{2N}\right] \qquad for\ k = 1, 2, \ldots, N \qquad (1)$$

Where,

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$$c_k = \begin{cases} \frac{1}{\sqrt{2}} & if\ k = N \\ 1 & if\ k = 1, 2, \ldots, N-1 \end{cases}$$

The $Y(k)$ values represent the transformed data. Without loss of generality, the scale factor $\sqrt{\frac{2}{N}} c_k$ in (1) can be ignored in the rest of the paper. After ignoring the scale factor, (1) is written as

$$Y(k) = \sum_{n=1}^{N} x_o(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \qquad for\ k = 1, 2, \ldots, N \tag{2}$$

Taking $N = 2^m (m = 2, 3, 4, \ldots)$, the $(N-1)$ output components $Y(1), Y(2), \ldots, Y(N-1)$ are arranged in m groups, namely

$$Y(2p+1), Y[2(2p+1)], Y[2^2(2p+1)], \ldots\ldots, Y[2^{m-1}(2p+1)] \tag{3}$$

The last output component $Y(N)$ is excluded from grouping.

Where,

$$p = 0, 1, 2, \ldots, 2^{m-(q+1)} - 1 \tag{4}$$

The symbol q denotes the group number starting from zero. That is, $q = 0$ for first group, $q = 1$ for second group, $q = 2$ for third group,...., $q = m-1$ for the last m^{th} group. The number of output components in the groups is 2^r, where $r = m$ -1, m -2,..., 0. That is, $r = m$ -1 for the first group, $r = m$ -2 for the second group and so on.

Substituting $k = N$ in (2), the last output component $Y(N)$ is given by

$$Y(N) = \sum_{n=1}^{N} x_o(n) \sin\left[\frac{(2n-1)\pi}{2}\right] \tag{5}$$

When $N = 2^m (m = 2, 3, 4, \ldots)$, (2) can be expressed as

$$Y(k) = \sum_{n=1}^{\frac{N}{2}} \left[x_0(n) + (-1)^{k+1} x_0(N+1-n)\right] \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{6}$$

$$\Rightarrow Y(k) = \sum_{n=1}^{\frac{N}{2}} x_1(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{7}$$

Where

$$x_1(n) = \left[x_o(n) + (-1)^{k+1} x_o(N+1-n)\right] \tag{8}$$

Decomposing (7) further, we have

$$Y(k) = \sum_{n=1}^{\frac{N}{4}} \left[x_1(n) + (-1)^{k+1} x_1\left(\frac{N}{2} + 1 - n\right)\right] \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{9}$$

$$\Rightarrow Y(k) = \sum_{n=1}^{\frac{N}{4}} x_2(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{10}$$

Where,

$$x_2(n) = \left[x_1(n) + (-1)^{k+1} x_1\left(\frac{N}{2} + 1 - n\right)\right] \tag{11}$$

Similarly, (10) can be decomposed further. Hence, in general we have

$$Y(k) = \sum_{n=1}^{\frac{N}{2^{q+1}}} \left[x_q(n) + (-1)^{k+1} x_q\left(\frac{N}{2^q} + 1 - n\right)\right] \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{12}$$

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$for\ q = 0,1,2,\ldots,m-1$

Substituting $k = 2^q(2p+1)$ in (12), we obtain

$$Y\left[2^q(2p+1)\right] = \sum_{n=1}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]\sin\left[\frac{(2n-1)2^q(2p+1)\pi}{2N}\right] \tag{13}$$

$$= \sum_{n=1}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]\sin\left[(2n-1)2^q\theta_p\right] \tag{14}$$

$$\Rightarrow Y\left[2^q(2p+1)\right] = \sum_{n=1}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]\sin\left[(2n-1)\theta_{qp}\right] \tag{15}$$

Where,

$$\theta_p = \frac{(2p+1)\pi}{2N} \tag{16}$$

and

$$\theta_{qp} = 2^q\theta_p = \frac{2^q(2p+1)\pi}{2N} \tag{17}$$

As explained in Section-III, $\sin\left[(2n-1)\theta_{qp}\right]$ can be expanded as

$$\sin\left[(2n-1)\theta_{qp}\right] = \sum_{j=1}^{n} A_{nj}\sin^{2j-1}\theta_{qp} \tag{18}$$

Where $A_{nj}s$ are some integers

Substituting (18) in (15), we have

$$Y\left[2^q(2p+1)\right] = \sum_{n=1}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]\sum_{j=1}^{n} A_{nj}\sin^{2j-1}\theta_{qp} \tag{19}$$

$$= \sum_{j=1}^{\frac{N}{2^{q+1}}}\sum_{n=j}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]A_{nj}\sin^{2j-1}\theta_{qp} \tag{20}$$

$$\Rightarrow Y\left[2^q(2p+1)\right] = \sum_{j=1}^{\frac{N}{2^{q+1}}} T_q^p(j)\sin^{2j-1}\theta_{qp} \tag{21}$$

Where

$$T_q^p(j) = \sum_{n=j}^{\frac{N}{2^{q+1}}}\left[x_q(n) + (-1)^{2^{q}(2p+1)+1} x_q\left(\frac{N}{2^q}+1-n\right)\right]A_{nj} \tag{22}$$

The $(N-1)$ output components $Y(1), Y(2), \ldots, Y(N-1)$ of DST-II can be realized by recursive filter using (21) and (22). The N^{th} output component $Y(N)$ can be found out using (5).

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III.EXAMPLE FOR REALIZATION OF DST-II OF LENGTH N = 8

To clarify the proposed radix-2 algorithm, the DST-II of length $N = 8 = 2^3$ is taken. As $m = 3$, the 7 output components $Y(1), Y(2), Y(3), Y(4), Y(5), Y(6), Y(7)$ are divided into following 3 groups according to (3).

$$Y(2p+1), Y[2(2p+1)] \; and \; Y[2^2(2p+1)]$$

The number of output components in the groups is 2^r, where r = 2, 1& 0. So, the number of output components in first, second and third group respectively is 4, 2 & 1.
The grouping of seven output components is done as given below.

- q = 0 for the first group, $Y(2p+1)$. Then, p = 0,1,2,3 as per (4). This group contains four output components $Y(1)$, $Y(3)$, $Y(5)$ and $Y(7)$.
- q = 1 for the second group, $Y[2(2p+1)]$. Then, p = 0, 1 as per (4). This group contains two output components $Y(2)$ and $Y(6)$.
- q = 2 for the third group, $Y[2^2(2p+1)]$. Then, p = 0 as per (4). This group contains one output component $Y(4)$.

Let us now find out the expansion of

$$\sin\left[(2n-1)\theta_{qp}\right] \quad \text{for } n = 1, 2, 3 \;\&\; 4.$$

i) For $n = 1$,

$$\sin\left[(2n-1)\theta_{qp}\right] = \sin\theta_{qp} = A_{11}\sin\theta_{qp} = \sum_{j=1}^{j=n=1} A_{1j}\sin\theta_{qp}$$

Where $A_{11} = 1$

ii) For $n = 2$,

$$\sin\left[(2n-1)\theta_{qp}\right] = \sin 3\theta_{qp} = 3\sin\theta_{qp} - 4\sin^3\theta_{qp} = A_{21}\sin\theta_{qp} + A_{22}\sin^3\theta_{qp}$$

$$= \sum_{j=1}^{j=n=2} A_{2j}\sin^{2j-1}\theta_{qp}$$

Where $A_{21} = 3$ and $A_{22} = -4$

iii) For $n = 3$,

$$\sin\left[(2n-1)\theta_{qp}\right] = \sin 5\theta_{qp} = 5\sin\theta_{qp} - 20\sin^3\theta_{qp} + 16\sin^5\theta_{qp}$$

$$= \sum_{j=1}^{j=n=3} A_{3j}\sin^{2j-1}\theta_{qp}$$

Where $A_{31} = 5$, $A_{32} = -20$ and $A_{33} = 16$

iv) For $n = 4$,

$$\sin\left[(2n-1)\theta_{qp}\right] = \sin 7\theta_{qp} = 7\sin\theta_{qp} - 56\sin^3\theta_{qp} + 112\sin^5\theta_{qp} - 64\sin^7\theta_{qp}$$

$$= \sum_{j=1}^{j=n=4} A_{4j}\sin^{2j-1}\theta_{qp}$$

Where $A_{41} = 7$, $A_{42} = -56$, $A_{43} = 112$ and $A_{44} = -64$

The A_{nj}s in the above 4 cases can be arranged in a 4×4 matrix.

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -4 & 0 & 0 \\ 5 & -20 & 16 & 0 \\ 7 & -56 & 112 & -64 \end{bmatrix} \quad (23)$$

From the above 4 cases, we conclude in general that

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$$\sin\left[(2n-1)\theta_{qp}\right] = \sum_{j=1}^{n} A_{nj}\sin^{2j-1}\theta_{qp}$$

$T_q^p(j)$ given by (22) is now solved for q = 0, 1, 2.

1. When $q=0$,

$$\begin{aligned} T_0^p(j) &= \sum_{n=1}^{4}\left[x_0(n) + (-1)^{(2p+1)+1}x_0(8+1-n)\right]A_{nj} \\ &= \left[x_0(1) + (-1)^{2p+2}x_0(8)\right]A_{1j} + \left[x_0(2) + (-1)^{2p+2}x_0(7)\right]A_{2j} \\ &\quad + \left[x_0(3) + (-1)^{2p+2}x_0(6)\right]A_{3j} + \left[x_0(4) + (-1)^{2p+2}x_0(5)\right]A_{4j} \qquad (24) \\ &\quad for\ j = 1,2,3,4 \end{aligned}$$

2. When $q=1$,

$$\begin{aligned} T_1^p(j) &= \sum_{n=1}^{2}\left[x_1(n) + (-1)^{2(2p+1)+1}x_1(4+1-n)\right]A_{nj} \\ &= \left[x_1(1) + (-1)^{4p+3}x_1(4)\right]A_{1j} + \left[x_1(2) + (-1)^{4p+3}x_1(3)\right]A_{2j} \qquad (25) \\ &\quad for\ j = 1,2 \end{aligned}$$

Substituting $k = 2^q(2p+1)$ and N = 8 in (8), we have for q =1 & $n = 1,2,3,4$

$$x_1(1) = \left[x_o(1) + (-1)^{4p+3}x_o(8)\right] \qquad (26)$$

$$x_1(2) = \left[x_o(2) + (-1)^{4p+3}x_o(7)\right] \qquad (27)$$

$$x_1(3) = \left[x_o(3) + (-1)^{4p+3}x_o(6)\right] \qquad (28)$$

$$x_1(4) = \left[x_o(4) + (-1)^{4p+3}x_o(5)\right] \qquad (29)$$

Using (26), (27), (28) and (29) in (25), $T_1^p(j)$ can be expressed in terms of input data sequence.

3. When $q=2$,

$$\begin{aligned} T_2^p(j) &= \sum_{n=1}^{1}\left[x_2(n) + (-1)^{2^2(2p+1)+1}x_2(2+1-n)\right]A_{nj} \\ &= \left[x_2(1) + (-1)^{8p+5}x_2(2)\right]A_{1j} \qquad (30) \\ &\quad for\ j = 1 \end{aligned}$$

Substituting $k = 2^q(2p+1)$ and N = 8 in (11), we have for q =2 & $n = 1,2$

$$x_2(1) = \left[x_1(1) + (-1)^{8p+5}x_1(4)\right] \qquad (31)$$

$$x_2(2) = \left[x_1(2) + (-1)^{8p+5}x_1(3)\right] \qquad (32)$$

Using (26), (27), (28), (29), (31) and (32) in (30), $T_2^p(j)$ can be expressed in terms of input data sequence.
The values of A_{nj} in (24), (25) and (30) are known from the matrix (23).
Substituting N = 8 and q = 0, 1 &2 in (21), the seven output components arranged in 3 groups can be found out as given below.

- When $q=0$,

$$Y\left[(2p+1)\right] = \sum_{j=1}^{4} T_0^p(j)\sin^{2j-1}\theta_{0p}$$

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$$= T_0^p(1)\sin\theta_{0p} + T_0^p(2)\sin^3\theta_{0p} + T_0^p(3)\sin^5\theta_{0p} + T_0^p(4)\sin^7\theta_{0p}$$

$$\Rightarrow Y(2p+1) = \left[\left\{\left(T_0^p(4)\sin^2\theta_{0p} + T_0^p(3)\right)\sin^2\theta_{0p} + T_0^p(2)\right\}\sin^2\theta_{0p} + T_0^p(1)\right]\sin\theta_{0p} \quad (33)$$

$$for\ p = 0,1,2,3$$

The values of $T_0^p(j)$ in the above expression are given by (24). The output components $Y(1)$, $Y(3)$, $Y(5)$ and $Y(7)$ given by (33) can be realized by the recursive filter, shown in Fig.1. The symbol Z^{-1} denotes the delay element.

- When $q = 1$,

$$Y[2(2p+1)] = \sum_{j=1}^{2} T_1^p(j)\sin^{2j-1}\theta_{1p}$$

$$= T_1^p(1)\sin\theta_{1p} + T_1^p(2)\sin^3\theta_{1p}$$

$$\Rightarrow Y[2(2p+1)] = \left[T_1^p(2)\sin^2\theta_{1p} + T_1^p(1)\right]\sin\theta_{1p} \quad (34)$$

$$for\ p = 0,1$$

The values of $T_1^p(j)$ in the above expression are given by (25). The output components $Y(2)$ and $Y(6)$ given by (34) can be realized by the recursive filter, shown in Fig.2.

- When $q = 2$,

$$Y\left[2^2(2p+1)\right] = \sum_{j=1}^{1} T_2^p(j)\sin^{2j-1}\theta_{0p}$$

$$\Rightarrow Y\left[2^2(2p+1)\right] = T_2^p(1)\sin\theta_{2p} \quad (35)$$

$$for\ p = 0$$

The above expression gives the output component $Y(4)$. The value of $T_2^p(1)$ in the above expression is given by (30). For N= 8, we have from (17)

$$\theta_{qp} = \frac{2^q(2p+1)\pi}{16} \quad (36)$$

The values of θ_{qp} in (33), (34) and (36) are known from the above expression.

Substituting $N = 8$ in (5), we get

$$Y(8) = \left[x_0(1) - x_0(2)\right] + \left[x_0(3) - x_0(4)\right] + \left[x_0(5) - x_0(6)\right] + \left[x_0(7) - x_0(8)\right] \quad (37)$$

The output component $Y(8)$ given by the above expression is realized by the data flow diagram, shown in Fig.3. The arrow head denotes delay element.

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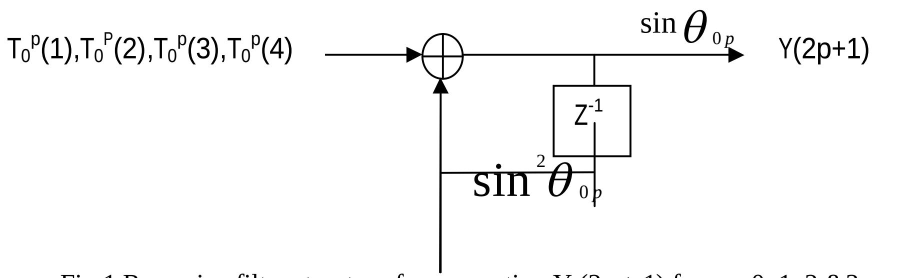

Fig.1.Recursive filter structure for computing Y (2p + 1) for p = 0, 1, 2 &3.

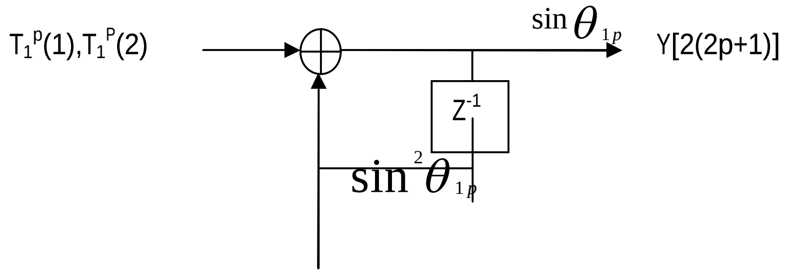

Fig.2.Recursive filter structure for computing Y [2(2p + 1)] for p = 0 &1.

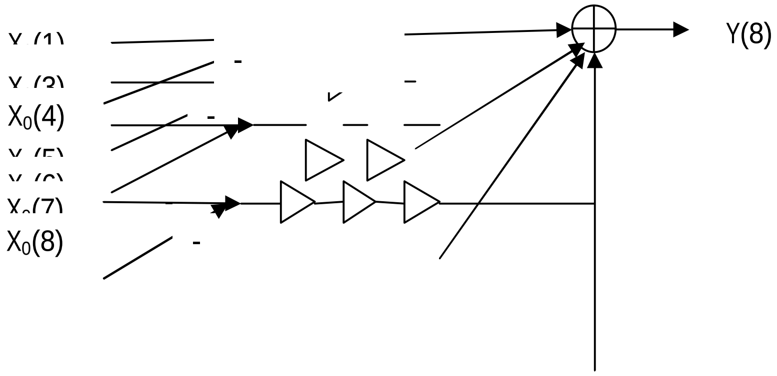

Fig.3. Data flow diagram for computation of Y (8).

V.CONCLUSION

This paper presents radix-2 algorithm for realization of DST-II of length $N = 2^m$ (m = 2, 3, 4,......) . The (N – 1) output components of DST-II are arranged in m groups and are realized using simple recursive filter structure. Data flow diagram for realization of the last N^{th} output component of DST-II is shown. The recursive structures require less memory and are suitable for VLSI implementation.

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