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Mathematical Model for Auto Tuning of PID Controller

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ABSTRACT: This new Mathematical Model for Auto Tuning of PID controller is composed of modeling of closed loop system, modeling of the process and Tuning formulas in terms of the relative damping of the transient response to set point changes. In this paper we are interested with the response of a PID Tuned system (Linear or Non-Linear) which has been subjected to step input .In conventional PID Tuning process, initially the process curve is analyzed with the help of Runge-kutta Numerical analysis method , then parameters like (Kc , Ti and Td) are determined using Ziegler Nichols Method and finally we got a PID response curve. However this conventional method suffers from the disadvantage of factors like Peak-Overshoot, Rise Time, steady State Error and Settling Time etc. In order to improve one factor other one has to be compromised. So we need to develop a Mathematical Model that can overcome this difficulties as well as takes care of all the above mentioned factors very efficiently. This introduces the concept of Auto Tuning method in which all factors as well as parameters are adjusted and determined and compared with the results of conventional PID Tuning. To start with this Method, firstly the response curve of a Linear or Non-Linear processes are divided into four different regions and it is based on the value of the Error and the change in error which occurs in the four different regions. As we see that in the first and fourth region, error is positive , while the change in error is negative and positive respectively. Similarly in the second and third region, error is negative and the change in error is negative and positive respectively. The system to be undergone Auto Tuning may be considered as a Linear or Non-Linear complex differential equation of order one or two.

KEYWORDS: PID , Auto Tuning , PID controller, Runge Kutta, Ziegler Nichols, Peak-Overshoot, Settling Time, Rise Time, Numerical analysis

I. INTRODUCTION

The Mathematical Model of Auto Tuning Method has been derived by considering the concept of an operation of control valve. As we know that as error gets more positive more control-action would require, similarly as it gets less positive less control-action would be needed. Again following the same process when error changes to more negative then a lesser control-action would come into play and when it gets less negative a lower control-action would be required. In this way. the conceptual logic behind the Auto Tuning method has been formulated. In this paper four different zones have been considered and keeping these zones in mind corresponding control-action formulas have been as shown in fig-1.

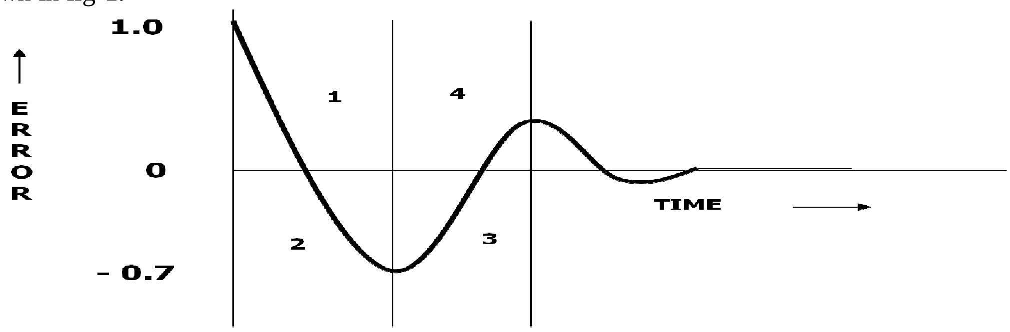

Fig-1

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In this paper we have discussed about Modeling of Auto Tuning Method. Auto Tuning is a process in which the characteristics of a process parameters like Peak-Overshoot, Settling Time, Rise Time, Steady-State Error etc. are properly adjusted in order to get a good response characteristics. Thus , during Auto Tuning Method these parameters get their value decreased or become more accurate compare to the previous process characteristics. It is a simple and effective method for tuning of PID(Proportional-Integral-derivative) controller based on closed loop control valve logic. Since a proposed closed loop controller uses conventional algorithm and output membership functions, this controller can be considered as a conventional PID controller whose parameters are tuned as per the requirement of the system based on a mathematical model. The performance comparison of conventional PID with Auto tuned PID type controllers has been done in terms of several performance measures such as Peak-Overshoot, Settling Time ,Rise time for a step input signal. Simulation results show the effectiveness and robustness of the proposed Auto Tuning mechanism. A simulation analysis of a wide range of Linear and Non Linear process is carried out and a comparison result is obtained based on it.

II. LITERACY SURVEY

A. FORMULATION OF AUTO TUNING METHOD USED FOR MATHEMATICAL MODELING

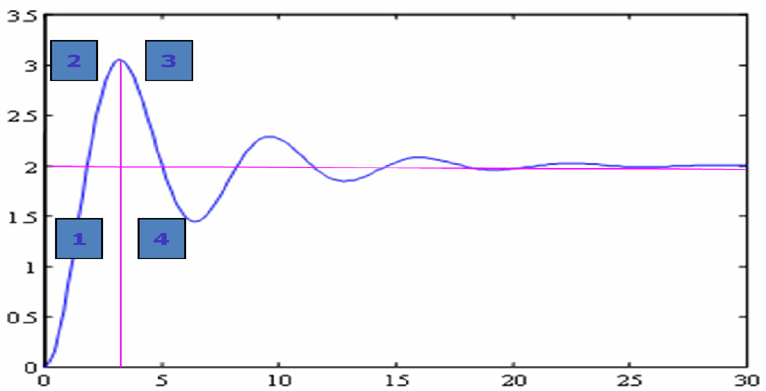

Fig-2

MATHEMATICAL EQUATION USED u=u (1+a/b)

Where, U=Control Action, a And b are Scaling Factors

Scaling factor b is considered as 0.05(constant) in the four different regions

B. Algorithm Developed For 1st Region (e=+VE, Δe=-VE) e=error and Δe =change in error

a=
- 0.1 , e <0.1
- 0.3 , 0.1 < e < 0.3
- 0.5 , 0.3 < e < 0.5 && -0.04 < Δe <-0.001
- 0.7 , 0.5 < e < 0.75
- 0.9 , e >0.75

C. Algorithm Developed For 2nd Region (e=-VE, Δe=-VE)

a=
- -0.1 , e> -0.1
- -0.3 , -0.3 < e <-0.1
- -0.5 , -0.5 < e < -0.3 && -0.04 < Δe < -0.001
- -0.7 , -0.75 < e < -0.5
- -0.9 , e < -0.75

when the response curve approaches the step input signal in the corresponding four different regions, the error and the change in error becomes infinitesimally small, so the control action must be selected in a way such that it does not affect in other different regions.

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D. Algorithm Developed For Neutral Zone

```
if(Δe >= -0.001 &&  Δe <=0.0)  {
if(e >=0.0  &&  e<=0.001)
a=0.0;                              }
if(Δe >=0.0 && Δe <= 0.001)   {
if(e >= -0.001 && e <= 0.0)
a=0.0;                         }
```

E. Algorithm Developed For 3rd Region (e=-VE, Δe=+VE)

$$a=\begin{cases} -0.1\ , & e > -0.1 \\ -0.3\ , & -0.3 < e < -0.1 \\ -0.5\ , & -0.5 < e < -0.3 \quad \&\& \quad +0.001 < \Delta e < 0.02 \\ -0.7\ , & -0.75 < e < -0.5 \\ -0.9\ , & e < -0.75 \end{cases}$$

F. Algorithm Developed For 4th Region (e=+VE, Δe=+VE)

$$a=\begin{cases} 0.1\ , & e < 0.1 \\ 0.3\ , & 0.1 < e < 0.3 \\ 0.5\ , & 0.3 < e < 0.5 \quad \&\& \quad 0.001 < \Delta e < 0.02 \\ 0.7\ , & 0.5 < e < 0.75 \\ 0.9\ , & e > 0.75 \end{cases}$$

Thus from the above mentioned mathematical formulation the scaling factor a depends on the magnitude of the control action needed. In other words for higher control action scaling factor ”a “ increases and for lower control action scaling factor “a “ decreases. Generally “a” ranges from 0.1 to 0.9 for the four different regions and scaling factor ” b “ will remain constant.

III. RESULTS AND INTERPRETATIONS

Some typical differential equations undergone for Auto Tuning are-

2ND ORDER LINEAR DIFFERENTIAL EQUATIONS:-

1. $d^2y/dx^2-0.8(dy/dx)+0.5x=0$
2. $d^2y/dx^2-0.4(dy/dx)-7=0$
3. $d^2y/dx^2-0.2(dy/dx)=0$

2ND ORDER NON-LINEAR DIFFERENTIAL EQUATIONS:-

1. $d^2y/dx^2-0.6y(dy/dx)-1=0$
2. $d^2y/dx^2-0.05(dy/dx)^3-9=0$

Results obtained from the Mathematical Model of Auto Tuning are compared with the conventional PID Tuning method and are shown in terms of Steady State Error and Peak Overshoot.

Process/system	Auto Tuning		PID Tuning	
Linear	Steady State error	Peak Overshoot	Steady State error	Peak Overshoot
1	-0.00071	1.117263	-0.00166	1.44822
2	0.00526	1.19567	0.01458	1.43684
3	0.00000	1.55499	0.01389	1.72716
Non Linear				
1	0.00053	1.34515	0.0039	1.44575
2	0.00017	1.10654	0.01071	1.21148

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For Analysis Purpose we have used C programming language and graphical plots are done with the help of MS-Excel. From the above figures it is seen that good response characteristics can be obtained by Auto Tuning method and this Mathematical Model is applicable to Linear as well as Non Linear system.

Some Typical Response curves we got from Auto Tuned Method and Compare them with the PID tuned Response curve in case of a Non-Linear system $d^2y/dx^2-0.6*y*dy/dx-1=0$ and Linear system $d^2y/dx^2-0.2*dy/dx=0$ as shown below in fig 3 and fig 4.

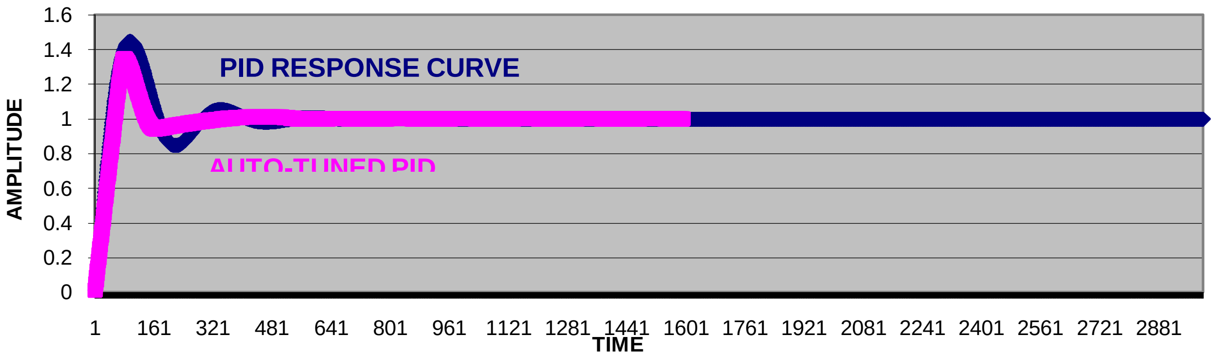

Fig-3 Non-Linear system

Incase of Non-Linear system, we find that after implementing the Auto-Tuning Algorithm both the Steady State Error and the Peak Overshoot decreases as compare to the PID Tuning.Although Rise-Time remains same in both the Tuning Processes.

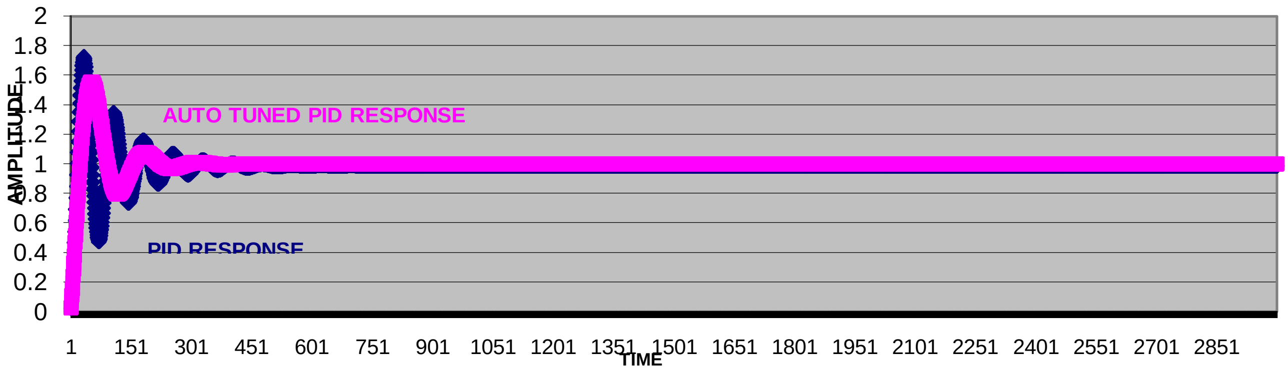

Fig-4 Linear system

Incase of Linear system, we find that after implementing the Auto-Tuning Algorithm both the Steady State Error and the Peak Overshoot decreases as compare to the PID Tuning. Although Rise-Time remains same in both the Tuning Processes.

IV. FUTURE-SCOPE

1. The project has a great future scope since it deals with both linear and non-linear second Order process. At the same time it can easily computes different characteristics of the process with having different control-actions given at a time. This project is also applicable for other higher order complex differential equations.

2. Regarding the development of Response Characteristics of a physical second order system, which I am hopeful can be done with the help of Interfacing with hardware circuitry. So in future, if I get a chance I will certainly want to work on that considering this particular Auto Tuning method

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V. CONCLUSION

Runge-Kutta Method is applicable to both Linear and Non-Linear system. For Getting Good Response characteristics Modified ZN method or Other Auto-tuning Methods should be applied. Sustained Oscillation in some Second Order Processes are not applicable, so in that case Tuning of controller is done by closed loop Damped Oscillation Method of Harriot is used.Tuning Parameters should be calculated based on values of Ultimate-gain(Ku) and Ultimate – period(Pu).During ZN Tuning Process, set Kc as per the Process requirement. Also Increase Kc in small counts. Second Order Non-Linear System has got relatively higher settling time. This AutoTuning Method is applicable to both Linear and Non-Linear 2^{nd} order system.

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