



Takagi-Sugeno Fuzzy Control for Nonlinear Systems

Rajesh Tanna¹, Alok Kanti Deb²

Assistant Professor, Dept. of EEE, Vignan Institute of Information Technology, Andhra Pradesh, India¹

Associate Professor, Dept. of EEE, IIT Kharagpur, West Bengal, India²

ABSTRACT: This paper gives position and tracking control of Nonlinear System. Many methods are available to design controllers for nonlinear systems. But In this frame work, a nonlinear system is represented as the fuzzy average of local linear models which are popularly known as Takagi-Sugeno (T-S) Fuzzy Model. Given a nonlinear system in terms of a T-S Fuzzy Model, based on the T-S fuzzy model design a controller for actual systems. The control scheme can be designed using traditional control techniques such as Linear Matrix Inequality (LMI) technique. Moreover, the overall fuzzy scheme guarantees that all signals involved are bounded and the output of the closed-loop system will asymptotically track the desired output trajectory. In this Paper the algorithm has been applied to Ball and Beam System with two nonlinearities. The simulations are performed on the Ball and Beam systems and the effectiveness of the proposed method is demonstrated and robustness also verified.

KEYWORDS: PDC Control, LMI Technique.

I. INTRODUCTION

This Paper contains the design and implementation of controller for nonlinear systems. The design of controllers is mainly based on fuzzy logic concept. Control of nonlinear multivariable systems is a very challenging area. The problem becomes more complicated when the dynamics of the systems are too complex for mathematical modelling. Fuzzy logic, introduced by Zadeh, has been successfully applied to many engineering fields, especially in the area of control engineering. Based on fuzzy logic, fuzzy controllers convert the linguistic control strategy into an automatic control strategy. Fuzzy system provides an effective approach to handle nonlinear systems, especially in the presence of incomplete knowledge of the plant or the situation where precise control action is unavailable. The most advantageous property is that fuzzy controllers do not rely on the exact mathematical models of the processes, as in the case of conventional control schemes. It appears that fuzzy controllers yield results superior to those obtained by conventional control methods when dealing with complex ill-defined processes that cannot be precisely described by mathematical formulae. The formulation of fuzzy control is heuristic. The Takagi-Sugeno (T-S) model accurately describes the dynamics of the system within some operating range. There are many works in literature that are concerned with the stability analysis of T-S fuzzy model. The plant is first modelled by a T-S fuzzy model with a weighted sum of a set of linear state equations, and then a stable fuzzy controller is designed based on this fuzzy plant model by using the linear matrix inequality (LMI) optimization technique.

For the state feedback case, various design conditions based on LMIs can be derived via the design technique of parallel distributed compensation (PDC). The resulting state feedback gains are linearly dependent on the normalized membership functions of the considered fuzzy systems. Most of the previous works are based on the assumption that the states are available online, but some states of the system may not be measurable in general. Hence fuzzy controllers can be designed based on fuzzy observers, which estimate the unmeasured states of the system. But the estimation of all the state variables may not be possible at all times and also it increases the complexity of the closed loop system, which is undesirable. So the concept of output feedback control was introduced, which did not use the fuzzy observer. In this paper, we have also discussed the design of dynamic.



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II. TAKAGI SUGENO FUZZY CONTROL DESIGN

Many physical systems are very complex in practice so that rigorous mathematical models can be very difficult to obtain. However, many of these systems can be expressed in some form of mathematical model locally or as an aggregation of a set of mathematical models. Takagi and Sugeno have proposed a fuzzy model to describe the complex dynamic model to represent a complex Single-input multi output system, which includes both local analytic linear models and fuzzy membership functions. Specifically, the continuous-time Takagi–Sugeno fuzzy dynamic model is described by fuzzy IF–THEN rules, which locally represent linear input–output relations of nonlinear systems[1][4]. Let us consider general nonlinear dynamical systems in continuous time, described by

$$\dot{x} = f(x, u) \tag{1}$$

Where x is n - dimensional state vector
 u is m -dimensional input vector

The above system can be effectively modeled by fuzzy merging of equivalent linear systems in different operating regions using T-S fuzzy model. A T-S fuzzy model is composed of r rules, where the j^{th} rule has the following form[1][4]:

Rule i : IF x_1 is F_1^i and x_2 is F_2^i and x_n is F_n^i THEN $\dot{x} = A_i x + B_i u$

Where F_j^i is j^{th} fuzzy term of the i^{th} rule; $j=1, 2, 3, \dots, n$

$$x = [x_1, x_2, x_3, \dots, x_n]^T, i = 1, 2, \dots, r$$

Let $\mu_i = \prod_{j=1}^n \mu_j^i(x_j)$ Where $\mu_j^i(x_j)$ is the membership function of the i^{th} rule of j^{th} fuzzy term F_j^i .

The fuzzy model around this operating point is constructed as the weighted average of the local models and has the form [4]

$$\dot{x} = \frac{1}{\sum_{i=1}^r \mu_i} \sum_{i=1}^r \mu_i (A_i x + B_i u) \tag{2}$$

The T-S model representation of the nonlinear system is more informative in terms of local dynamics. The fuzzy system can be rewritten as

$$\dot{x} = \sum_{i=1}^r \sigma_i (A_i x + B_i u) \tag{3}$$

Where $\sigma_i = \frac{\mu_i}{\sum_{i=1}^r \mu_i}$ and $\sum_{i=1}^r \sigma_i = 1$

Once a nonlinear system is identified in terms of a T-S fuzzy model, the following model can be applied to compute a suitable controller for the system.

1. STABILITY ANALYSIS OF THE NONLINEAR SYSTEM :

To design any controller first we have to consider stability. It means of checking the stability is necessary for any system. The lyapunov method is a primary method of testing the stability of nonlinear systems or the linear systems with uncertainty. In general, there are two ways to check the stability of a system:

- i. Lyapunov Direct Method
- ii. Indirect Method

Lyapunov direct method involves finding a Lyapunov function for a system. If such a function exists, then the system is stable.

A Lyapunov function is defined as a scalar function $V(X)$ that satisfies the two conditions:

- i. $V(X)$ is a positive definite function
- ii. $\dot{V}(X)$ is a negative definite function



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Lyapunov function is a function in time, whose scalar value is always positive, with a negative derivative. It must converge to the point where $V(X) = \dot{V}(X) = 0$ as $t \rightarrow \infty$. In general, $V(X)$ is a quadratic function of $X(t)$, $V(0) = 0$ when $X = 0$. This means that system states must converge to origin, in the absence of input. This is basically same as asymptotic stability.

2. LINEAR MATRIX INEQUALITY TECHNIQUE:

In this design technique, first a set of inequalities is formed base on the Lyapunov sufficient condition to ensure stability of the T-S fuzzy model using a parallel distributor fuzzy compensator and the compensator gains are found by solving the set of inequalities. In this LMI technique one can solve feasibility problems and generalized Eigen value minimization problems. Basic LMI form as

$$A_0 + A_1 X + A_2 X + \dots + A_n X < 0 \quad \text{Where } X\text{-matrix vector}$$

Theorem1: The unforced continuous T-S fuzzy system is described by the following equation [1][3][4]

$$\dot{x} = \sum_{i=1}^r \sigma_i A_i x(t) \quad (4)$$

The above system is asymptotically stable if there exists a common positive definite matrix P such that

$$A_i^T P + P A_i < 0 \quad (5)$$

For $i=1,2,3,\dots,r$

The above inequality gives a sufficient condition for ensuring stability of above system

3. PARALLEL DISTRIBUTED COMPENSATOR :

In parallel distributed compensator the control in each fuzzy region is designed based on the corresponding linear subsystem of the T-S fuzzy model. The designed fuzzy controller uses the same fuzzy sets of the model. For the fuzzy model (3), a fuzzy regulator can be designed as follows [1][4]

Regulator rule j : IF $x_1(t)$ is F_1^j and $x_2(t)$ is F_2^j and $x_n(t)$ is F_n^j THEN

$$u(t) = -K_j x(t) \quad j=1, 2, 3, \dots, r$$

The overall fuzzy regulator is represented by

$$u(t) = -\sum_{j=1}^r \sigma_j K_j x(t) \quad (6)$$

The design problem simplifies to designing the local feedback gains K_j while establishing the stability of the overall system using the control input $u(t)$ substituting the control law (6) in (3), the closed loop system becomes

$$\dot{x}(t) = \sum_{i=1}^r \sigma_i (A_i - B_i \sum_{j=1}^r \sigma_j K_j) x(t) \quad (7)$$

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r \sigma_i \sigma_j (A_i - B_i K_j) x(t)$$

Theorem 2: The closed fuzzy system is globally is asymptotically stable if there exists a common P for all the subsystems which satisfies the following Lyapunov inequalities[4]:

$$G_{ii}^T P + P G_{ii} < 0 \quad (8)$$

$$\left(\frac{G_{ij} + G_{ji}}{2} \right)^T P + P \left(\frac{G_{ij} + G_{ji}}{2} \right) \leq 0, \quad i < j \quad \text{Where } i, j=1,2,\dots,r \quad (9)$$

Where $G_{ii} = A_i - B_i K_i$, $G_{ij} = A_i - B_i K_j$,

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Pre-multiplying and post-multiplying of both sides of inequalities in (8 & 9) by P^{-1} and using the following change of variables: $X = P^{-1}, M_i = K_i X$

We obtain the following LMI equation[1][3][4]:

$$\begin{aligned}
 & -XA_i^T - A_i X + M_i^T B_i^T + B_i M_i > 0 \\
 & -XA_i^T - A_i X - XA_j^T - A_j X + M_j^T B_i^T + B_i M_j \\
 & \quad + M_i^T B_j^T + B_j M_i \geq 0, i < j
 \end{aligned}
 \tag{10}$$

We have shown that the stability analysis of the fuzzy control system is reduced to a problem of finding a common P . If r is large, it might be difficult to find a common P satisfying the conditions of Theorem 2. In this subsection, we will derive new stability conditions by relaxing the conditions of Theorems 2. Theorems 3 contain the relaxed conditions.

Theorem 3: The continuous fuzzy control system described by (7) is asymptotically stable in the large if there exists a common positive definite matrix P and a common positive semi definite matrix Q such that

$$\begin{aligned}
 & -XA_i^T - A_i X + M_i^T B_i^T + B_i M_i - (s-1)Y > 0 \\
 & 2Y - XA_i^T - A_i X - XA_j^T - A_j X + M_j^T B_i^T + B_i M_j \\
 & \quad + M_i^T B_j^T + B_j M_i \geq 0, i < j
 \end{aligned}
 \tag{11}$$

where, $Y = XQX$

III. MODEL AND DYNAMICS

BALL AND BEAM SYSTEM : In Fig. 1 shows the mathematical modelling of Ball and Beam system. The aim is to design a control system to track the ball to a commanded position. When coupled to the SRV02 plant, the DC motor will drive the beam such that the motor angle controls the tilt angle of the beam. The ball then travels along the length of the beam.

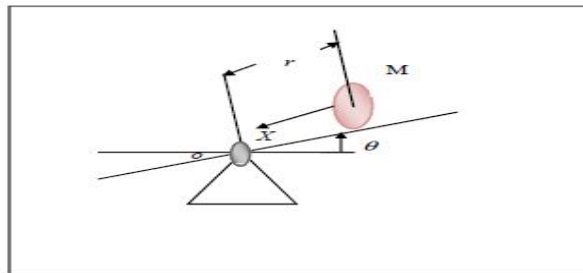


Fig. 1 Mathematical Modelling of Ball and Beam system

Dynamic equation of Ball and Beam Systems is

$$\begin{aligned}
 \dot{r} &= B(r\dot{\theta}^2 - g \sin \theta) \\
 \dot{\theta} &= u(t)
 \end{aligned}
 \tag{12}$$

where, r = ball position ; θ = beam angle

\dot{r} = ball velocity ; $\dot{\theta}$ = beam velocity

The State space representation of ball and beam system



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$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= B(x_1 x_4^2 - g \sin x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u \end{aligned} \tag{13}$$

$$\text{where, } B = \frac{M}{\left(M + \frac{J_b}{R^2}\right)}$$

Table. 1 The physical parameters of the Ball and Beam system

Parameter	Value
mass of the Ball, M	0.05 Kg
radius of the Ball, R	0.01m
Gravity, g	9.81 m / sec^2
moment of inertia of the Ball, J_b	$2 \times 10^{-6} \text{ Kg-m}^2$

IV. DESIGN OF CONTROLLER

T-S Fuzzy Control With Sector Nonlinearity Method:

The nonlinear functions of the equation (13) can be chosen as:

$$\begin{aligned} f_1(x) &= \frac{-B g \sin(x_3)}{x_3} \\ f_2(x) &= B x_1 x_4 \end{aligned} \tag{14}$$

Which depend on the states of the system and these functions can be taken as premise variables for the T-S fuzzy model. Hence, the ball and beam dynamics can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & f_1 & f_2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \tag{15}$$

The operating region of the T-S fuzzy model can be selected as

$$\begin{aligned} x_1(t) &\in [-1, 1]; x_2(t) \in [-1, 1] \\ x_3(t) &\in [-1, 1]; x_4(t) \in [-5, 5] \end{aligned}$$

For the given bounds on system states, therefore obtained the maximum and minimum value of all the functions.

$f_1(x) \in [-5.896, -7.0071]$; $f_2(x) \in [3.5715, -3.5715]$ Hence the T-S fuzzy model for the ball and beam system can be given as equation (16).



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$$\dot{x} = \sum_{j=1}^4 \sigma_j (A_j x + B_j u) \quad (16)$$

here,

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & f_{1\max} & f_{2\max} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & f_{1\max} & f_{2\min} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly for A_3, A_4

The membership functions of fuzzy rules are shown (17).

$$\text{ex: } f_1(x(t)) = \frac{\sin x_3}{x_3} = M_{11} f_{1\max} + M_{12} f_{1\min} \quad (17)$$

$$M_1 + M_2 = 1$$

By solving the equation (17), obtain the membership functions

$$M_{11} = \frac{f_1(x(t)) - f_{1\min}}{f_{1\max} - f_{1\min}}; M_{12} = \frac{f_{1\max} - f_1(x(t))}{f_{1\max} - f_{1\min}} = 1 - M_1$$

Similarly for $f_2(x(t))$

The Normalized membership functions are

$$M = M_{11}(f_1(x))M_{21}(f_2(x)) + M_{11}(f_1(x))M_{22}(f_2(x)) + M_{12}(f_1(x))M_{21}(f_2(x)) + M_{12}(f_1(x))M_{22}(f_2(x)) \quad (18)$$

$$\sigma_1(f(x)) = \frac{M_{11}(f_1(x))M_{21}(f_2(x))}{M}$$

$$\sigma_2(f(x)) = \frac{M_{11}(f_1(x))M_{22}(f_2(x))}{M}$$

$$\sigma_3(f(x)) = \frac{M_{12}(f_1(x))M_{21}(f_2(x))}{M} \quad (19)$$

$$\sigma_4(f(x)) = \frac{M_{12}(f_1(x))M_{22}(f_2(x))}{M}$$

The T-S fuzzy model of the equation (16) is derived. Based on the T-S fuzzy model, design a controller for actual system. By solving the LMI constraints given in (11) in MATLAB, the controller gains are found.

Assume $S=2$,

$$Q = 10^3 * \begin{bmatrix} 0.0005 & 0.0013 & -0.0085 & -0.0005 \\ 0.0013 & 0.0033 & -0.0219 & -0.0014 \\ -0.0085 & -0.0219 & 0.1450 & 0.0089 \\ -0.0005 & -0.0014 & 0.0089 & 0.0006 \end{bmatrix}$$

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$$P = 10^4 * \begin{bmatrix} 0.0002 & 0.0004 & -0.0023 & -0.0001 \\ 0.0004 & 0.0010 & -0.0057 & -0.0003 \\ -0.0023 & -0.0057 & 0.0368 & 0.0020 \\ -0.0001 & -0.0003 & 0.0020 & 0.0002 \end{bmatrix}$$

$$K_1 = [-1.87 \ -6.11 \ 43.51 \ 1.72]$$

$$K_2 = [-4.30 \ -14.49 \ 96.52 \ 3.76]$$

$$K_3 = [-4.37 \ -14.73 \ 98.05 \ 3.82]$$

$$K_4 = [-6.73 \ -22.86 \ 149.5271 \ 5.809]$$

(20)

Overall control signal is

$$u(t) = - \sum_{j=1}^4 \sigma_j K_j x(t) \quad (21)$$

Simulation Results of Ball and Beam Systems Nonlinearity as a membership function :

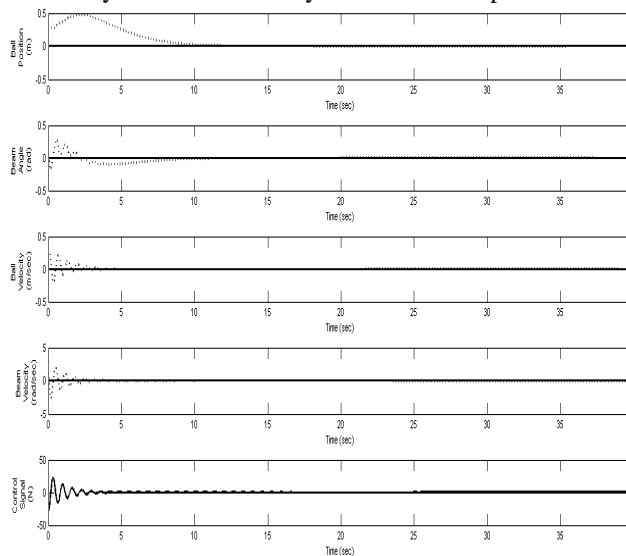


Fig. 2 Simulation Results of Ball and Beam Systems for initial position 0.3m

The Figure 2 shows that the Simulation Results of Ball and Beam Systems for initial position 0.3meters and the settling time is 3.4 seconds (approx). In Figure 3, a small Disturbance in ball and beam system was added to check its robustness due to disturbance input. Figure 3 shows the response of ball and beam system with the initial angle of ball position taken as 0.3 meters and a disturbance of 10 N applied to the ball and beam for 2 sec. The Figure 3 shows that the system is robust to the disturbance and settling time is 5 seconds (approx).

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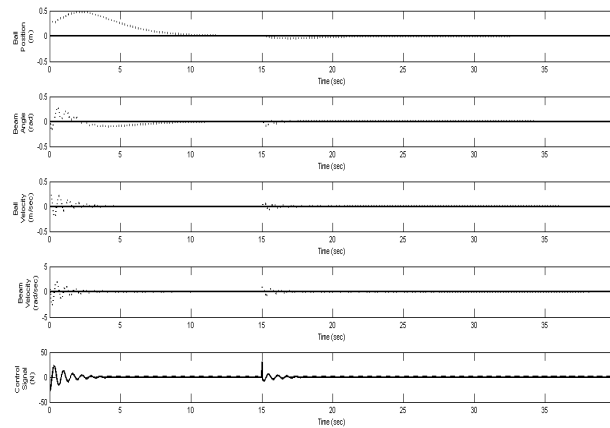


Fig. 3 Simulation Results of Ball and Beam Systems for initial position 0.3m with pulse disturbance of 10 N and width 2 sec applied after 15 sec.

T-S Fuzzy Control With Gaussian Membership Function:

The Ball and Beam system given by equation (13) can be represented as a T-S fuzzy model (22). Hence the T-S fuzzy model for Ball and Beam system can be given as follows:

$$\dot{x} = \sum_{j=1}^3 \sigma_j (A_j x + B_j u) \quad (22)$$

At different operating points the linear models are

Rule 1: If $x(t)$ is around (0,0,0,0) Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & 0 & -7.0073 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

Rule 2: If $x(t)$ is around (0.3,0,0.4,0.2) Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -0.141 & 0 & -6.680 & -0.027 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

similarly for (0.5, 0, 0.6, 0.4).

where the Gaussian membership functions for operating point (0.3,0,0.4,0.2).

$$\begin{aligned} \mu_2^1(x_2) &= \exp(-((0.3) - x_2)^2 / (2 * (0.2)^2)) \\ \mu_2^2(x_2) &= \exp(-((0) - x_2)^2 / (2 * (0.2)^2)) \\ \mu_2^3(x_2) &= \exp(-((0.4) - x_2)^2 / (2 * (0.2)^2)) \\ \mu_2^4(x_2) &= \exp(-((0.2) - x_2)^2 / (2 * (0.2)^2)) \\ \mu_2(x_2) &= \mu_2^1(x_2) * \mu_2^2(x_2) * \mu_2^3(x_2) * \mu_2^4(x_2) \end{aligned}$$

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For similarly μ_1 and μ_3 .

the normalized member ship function is $\sigma_1 = \frac{\mu_1(x_2)}{\mu_1(x_2) + \mu_2(x_2) + \mu_3(x_2)}$. Similarly for σ_2 and σ_3 .

The state space representation of the 4-th order model given by (13) and the T-S fuzzy model of the equation (22) is derived from equation (13). Based on the T-S fuzzy model, design a controller for actual system. By solving the LMI constraints given in (11) in MATLAB, the controller gains are found.

Assume S = 2 ,

$$Q = \begin{bmatrix} 3.4512 & 4.4033 & -18.0213 & -3.6409 \\ 4.4033 & 5.8064 & -23.3553 & -4.7733 \\ -18.0213 & -23.3553 & 97.2653 & 19.5145 \\ -3.6409 & -4.7733 & 19.5145 & 3.9758 \end{bmatrix}$$

$$P = \begin{bmatrix} 1.3506 & 1.0701 & -3.6998 & -0.7731 \\ 1.0701 & 1.9888 & -4.7402 & -1.2302 \\ -3.6998 & -4.7402 & 20.8173 & 4.0254 \\ -0.7731 & -1.2302 & 4.0254 & 1.2116 \end{bmatrix}$$

$$K_1 = [-7.5470 \quad -8.9134 \quad 49.8414 \quad 8.9707]$$

$$K_2 = [-7.7507 \quad -9.2919 \quad 50.7434 \quad 9.2049]$$

$$K_3 = [-7.9005 \quad -9.5705 \quad 51.4074 \quad 9.3772]$$

Hence, the Control Signal is

$$u(t) = -\sum_{j=1}^3 \sigma_j K_j x(t) \tag{23}$$

Simulation Results of Ball and Beam Systems with Gaussian membership function:

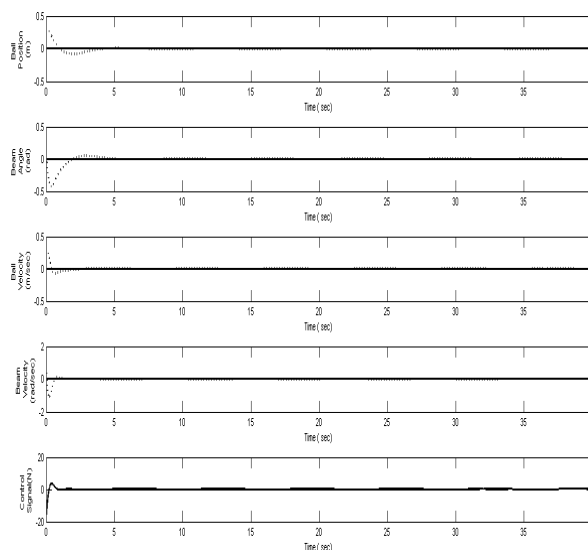


Fig. 4 Simulation Results of Ball and Beam Systems for initial position 0.3m

The Figure 4 shows that the Simulation Results of Ball and Beam Systems for initial position 0.3meters and the settling time is 2.5 seconds (approx). In Figure 5 , a small Disturbance in ball and beam system was added to check its robustness due to disturbance input. Figure 5 shows the response of ball and beam system with the initial angle of ball



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position taken as 0.3 meters and a disturbance of 10 N applied to the ball and beam for 2 sec. The Figure 5 shows that the system is robust to the disturbance and settling time is 5 seconds (approx).

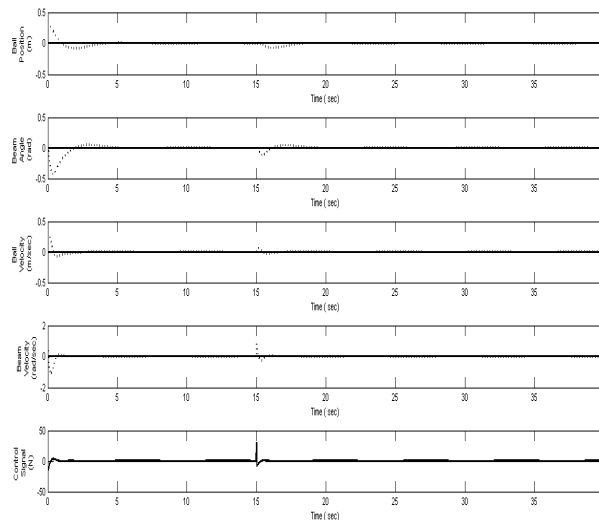


Fig. 5 Simulation Results of Ball and Beam Systems for initial position 0.3m with pulse disturbance of 10 N and width 2 sec applied after 15 sec.

Table. 2: Integral Square Error (ISE) for T-S Fuzzy Control With Gaussian Membership function and With Sector Nonlinearity

	T-S Fuzzy Control With Gaussian Membership Function(GMF)	T-S Fuzzy Control With Sector Nonlinearity
Simulation	0.132	0.109

V. CONCLUSION

The Algorithm of T-S Fuzzy has been successfully applied to the Ball and Beam systems . The robustness of controller was tested by giving a small disturbance and by parameter variation. Also a Comparison between T-S Fuzzy Controller with GMF and with sector nonlinearity is performed which shows that ISE value of T-S fuzzy control with sector nonlinearity is lower then T-S Fuzzy Controller with GMF. But the T-S Fuzzy Controller with GMF of the Inverted Pendulum system with less amount of control input as compared to T-S fuzzy control with sector nonlinearity.

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