ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

An ISO 3297: 2007 Certified Organization Vol. 4, Special Issue 1, March 2015

National Conference on Recent Advances in Electrical & Electronics Engineering (NCREEE 2015)

Organized by

Dept. of EEE, Mar Baselios Institute of Technology & Science (MBITS), Kothamangalam, Kerala-686693, India

On 26th & 27th March 2015

Tracking of Maneuvering Targets using Particle Filter

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ABSTRACT: Target Tracking has become a challenging task in many fields of science. The tracking problem involves estimating moving target's states using noisy measurements obtained at a single observation point. Particle filter offers a general solution for such problems. This paper presents the application of particle filtering technique to a target tracking example, in which a radar sends a signal towards a maneuvering target (aircraft) and estimates the state (position and velocity) of the target using the observations (Range and Bearing angle). This paper also deals with the analysis of the effect of number of particles in the performance of Particle Filter algorithm. Simulation is done in the MATLAB. From the analysis, it can be concluded that increase in the number of particles will give more optimal solution for the tracking problem.

KEYWORDS: Target Tracking, Dynamic state space models, Particle Filter, Nonlinear Systems, Non-Gaussian Systems, Monte Carlo Simulations and Relative RMSE.

I.INTRODUCTION

Nonlinear filtering problems arise in many fields including statistical signal processing[10], economics, statistics, and engineering such as communications, radar tracking[18], sonar ranging, and satellite navigation[17]. The problem consists of estimating a possibly dynamic state of a nonlinear stochastic system, based on a set of noisy measurements. Many of these problems can be written in the form of Dynamic State Space (DSS) models. The model can be either linear or nonlinear. Sequential Bayesian estimation techniques, such as Kalman Filtering[19], Extended Kalman Filtering[21], and Particle Filtering[20], are widely used to estimate parameters of dynamic state-space models. Kalman filters (KFs) can provide optimal parameter estimates for linear systems in additive Gaussian noise [1]. When the systems are nonlinear and non-Gaussian, Particle Filters (PFs) yield more accurate estimation results than extended KFs. Particle filtering is a powerful, emerging methodology with a wide range of applications in science and engineering. Researchers from a variety of fields ranging from signal processing to statistics and econometrics use particle filters because of its potential for coping with difficult nonlinear and/or non-Gaussian noise problems. They are based on the idea of approximating the probability density functions (PDFs) of the state of a dynamic model by random samples (particles) with associated weights and propagating them across iterations based on a probabilistic model of the state update and the measurements.

This paper outlines the analysis of the effect of number of particles in the performance of Particle Filter algorithm by considering a typical target tracking example in which the position and velocity of an aircraft is estimated using a 2D constant acceleration model. In this case, range and bearing angle are the noisy measurements applied to the filter.

II.PARTICLE FILTER ALGORITHM

The dynamic estimation problem[3] assumes two fundamental mathematical models:the state dynamics and the measurement equation. The dynamics model describes how the state vector evolves with time and isassumed to be of the form

$$x_k = f_{k-1}(x_{k-1}, u_k) \text{ , for} k > 0 \quad (2a)$$

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Here x_k is the state vector to be estimated, k denotes the time step and f_{k-1} is aknown possibly non-linear function. u_k is a white noise sequence, usually referredto as the process, system or driving noise. The pdf ofu_k is also assumed known. Note that (2a) denotes a first order Markov process, and an equivalent probabilistic description of the state evolution is $(x_k|x_{k-1})$, which is sometimes called the transition density. For the special case when $f$is linear and u is Gaussian, the transition density $p(x_k|x_{k-1})$ is also Gaussian.

The measurement model relates the received measurements to the state vector:

$$z_k = h_k(x_k, w_k) \text{ , for } k > 0 \tag{2b}$$

where z_k is the vector of received measurements at time step k, h_k is the knownmeasurement function and $w_k$is a white noise sequence (the measurement noise or error). Again, the pdf of w_k is assumed known and u_k and w_k are mutuallyindependent. Thus, an equivalent probabilistic model for (2b) is the conditional pdf$p(z_k|x_k)$. For the special case when h_k is linear and w_k is Gaussian, $p(z_k|x_k)$ is alsoGaussian.

The final piece of information to complete the specification of the estimationproblem is the initial conditions. This is the prior pdf $p(x_0)$ of the state vector at time $k = 0$, before any measurements have been received. So, in summary, theprobabilistic description of the problem is: $p(x_0)$, $p(x_k|x_{k-1})$ and $p(z_k|x_k)$.

As already indicated, in the Bayesian approach one attempts to construct the posterior pdf of the state vector x_k given all the available information. This posteriorpdf at time step k is written $p(x_k|Z_k)$, where Z_k denotes the set of all measurementsreceived up to and including z_k: $Z_k = \{ z_i, i = 1,\ldots, k\}$. The formal Bayesian recursive filter consists of a prediction and an update operation. The prediction operationpropagates the posterior pdf of the state vector from time step $k-1$ forwards totime step k. Suppose that $p(x_{k-1}|Z_{k-1})$ is available, then $p(x_k|Z_{k-1})$, the prior pdfof the state vector at time step $k > 0$ may be obtained via the dynamics model (thetransition density):

$$p(x_k|Z_{k-1}) = \int p\,(x_k|x_{k-1})p(x_{k-1}|Z_{k-1})\,dx_{k-1} \tag{2c}$$

This is called as the Chapman-Kolmogorov equation.

The prior probability density function may be updated to incorporate the new measurements z_k to givethe required posterior pdf at time step $k > 0$:

$$p(x_k|Z_k) = (p(z_k|x_k)p(x_k|Z_{k-1}))/p(z_k|Z_{k-1}) \tag{2d}$$

This is Bayes rule, where the normalising denominator is given by $p(z_k|Z_{k-1}) = \int p(z_k|x_k)\,p(x_k|Z_{k-1})\,dx_k$. The measurement model regarded as a function of $x_k$with z_k given is the measurement likelihood. The relations (2c) and (2d) define theformal Bayesian recursive filter with initial condition given by the specified prior pdf$p(x_0|Z_0) = p(x_0)$(where Z_0 is interpreted as the empty set). If (2c) is substitutedinto (2d), the prediction and update may be written concisely as a single expression.

The relations (2c) and (2d) define a very general but formal (or conceptual) solutionto the recursive estimation problem. Only in some special cases can an exact, closed formalgorithm be obtained from this general result. By far the most important of these special cases is the linear-Gaussian (L-G) model: if $p(x_0)$, $p(x_k|x_{k-1})$ and $p(z_k|x_k)$ are all Gaussian, thenthe posterior density remains Gaussian [13] and (2c) and (2d) reduce to the standard Kalman filter (which recursively specifies the mean and covariance of the posteriorGaussian). Furthermore, for non-linear/non-Gaussian problems, the first recourseis usually to attempt to force the problem into an L-G framework by linearisation.This leads to the extended Kalman filter (EKF) and its many variants. For mildlynon-linear problems, this is often a successful method and many real systems operateentirely satisfactorily using Extended Kalman Filters. However, with increasingly severe departuresfrom the L-G situation, this type of approximation becomes stressed to the point of filter divergence. For such grossly non-linear problems, the particle filter may be an attractive option.

The most fundamental particle filter may be viewed as a direct mechanisation of the formal Bayesian filter.

Suppose that a set of M random samples from the posterior pdf $p(x_{k-1}|Z_{k-1})(k > 0)$ is available. We denote these particles by $\{x_{k-1}^{i*}\}_{i=1}^{M}$.

The prediction phase of the basic algorithm consists of passing each of thesesamples from time step $k-1$ through the system model (2a) to generate a set of priorsamples at time step k. These prior samples are written $\{x_k^i\}_{i=1}^{M}$, where

$$x_k^i = f_{k-1}(x_{k-1}^{i*}, u_k^i) \tag{2e}$$

and u_k^i is a (independent) sample drawn from the pdf of the system noise. Thisstraightforward and intuitively reasonable procedure produces a set of samples orparticles from the prior pdf $p(x_k|Z_{k-1})$.

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To update the prior samples in the light of measurement z_k, a weight $\tilde{w}_k^i$ iscalculated for each particle. This calculated weight is the measurement likelihood evaluatedat the value of the prior sample: $\tilde{w}_k^i = p\left(z_k \middle| x_k^i\right)$. The weights are then normalizedso they sum to unity: $w_k^i = \tilde{w}_k^i / \sum_{j=1}^{M} \tilde{w}_k^j$and the prior particles are resampled(with replacement) according to these normalized weights to produce a new set ofparticles:$\{x_k^{i*}\}_{i=1}^{M}$ such that $\Pr\{x_k^{i*} = x_k^j\}=w_k^j$ for all i, j.In other words, a member of the set of prior samples is chosen with a probabilityequal to its normalised weight, and is repeated M times to build upthe new set $\{x_k^{i*}\}_{i=1}^{M}$.

We contend that the new sets of particles are samples of therequired pdf $p(x_k|Z_k)$ and so a cycle of the algorithm is complete.

This simple algorithm is often known as the Sampling Importance Resampling (SIR) filter and it was introduced in 1992 [1] where it called the bootstrap filter. Itwas independently proposed by a number of other research groups including Kitagawa [14] as a Monte Carlo filter and Isard and Blake [15] as the CONDENSATIONAlgorithm.

III.TARGET TRACKING EXAMPLE

The performance of particle filter is analysed by considering a typical maneuvering target tracking example in which the position and velocity of an aircraft is estimated using a 2D constant acceleration model. In this case the range and bearing angle are the measurements which are applied to the filter. From the dynamic state space model, it can be seen that the model has linear state equation and nonlinear measurement equation.

The dynamic state space model of the above tracking example is given below

$$x_k = \begin{pmatrix} 1 & 0 & T & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & T & 0 & T^2/2 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} x_{k-1} + u_k \quad (3a)$$

$$y_k = \begin{pmatrix} r \\ \varphi \end{pmatrix} = \begin{pmatrix} \sqrt{(p_x^2 + p_y^2} \\ arctan(p_y|p_x) \end{pmatrix} + e_k \quad (3b)$$

where the state vector $x_k = [p_x\, p_y\ \ v_x\, v_y\, a_x a_y]^T$i.e. position, velocity and acceleration. We have discarded the height component, since a level flight is considered. The sample time is taken as constant and denoted by T=1sec.The measurement noise e_k is Gaussian with zero mean and covariances R=Diag(50,2). The process noisesu_k are assumed Gaussian with zero mean and covariances Q=Diag (50, 50,0.01,0.01,0.01,0.01).

Algorithm: Particle Filter for the above aircraft tracking model (Model with nonlinear measurement equation).

1) Particle Generation
   For $i = 1,2 \ldots, M$

(a) Generate M random numbers $u_k^{(i)} \sim N(0, \sigma_u^2)$

(b) Particle Computation $x_k^{(i)} = F x_{k-1}^{(i)} + u_k^{(i)}$

2) Weight Computation

$$\tilde{w}_{k1}^{(i)} = N\left(r - \sqrt{p_x^{(i)^2} + p_y^{(i)^2}}, \sigma_r^2\right) \quad (3c)$$

$$\tilde{w}_{k2}^{(i)} = N\left(\varphi - \operatorname{atan}\left(\frac{p_y^{(i)}}{p_x^{(i)}}\right), \sigma_\varphi^2\right) \quad (3d)$$

$$\tilde{w}_k^{(i)} = (\tilde{w}_{k1}^{(i)} + \tilde{w}_{k2}^{(i)})/2 \quad (3e)$$

3) Weight Normalization

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$$w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{j=1}^{M} \tilde{w}_k^{(j)}}, j = 1,2 \ldots, M \quad (3f)$$

4) Output

$$x_k = \sum_{i=1}^{M} w_k^{(i)} x_k^{(i)} \quad (3g)$$

5) Resampling

$$\left\{\tilde{x}_k^{(i)}, \frac{1}{M}\right\}_{i=1}^{M} \approx \left\{x_k^{(i)}, w_k^{(i)}\right\}_{i=1}^{M} \quad (3h)$$

6) $k=k+1$, go to step 1(b).

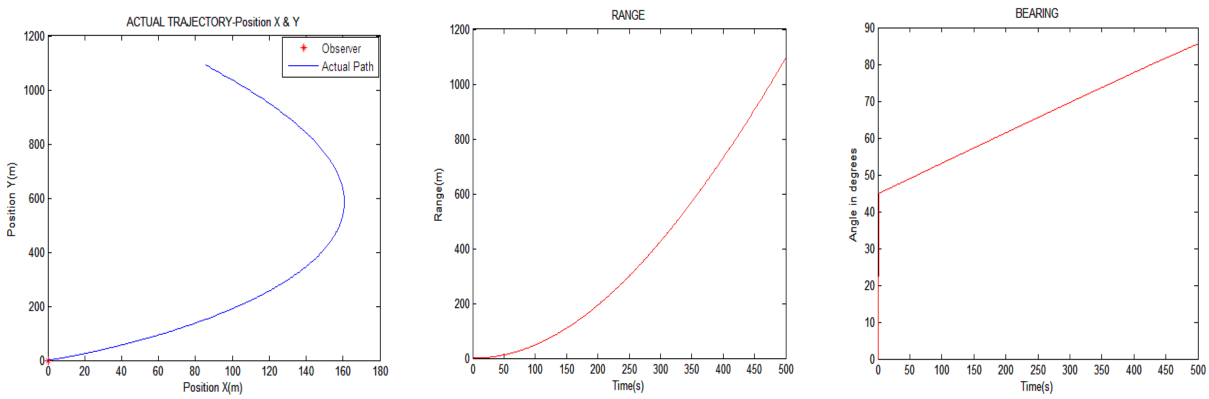
IV.SIMULATION RESULTS

In this section particle filter will be analyzed in an extensive Monte Carlo (MC) simulations using the model described in (3a) and (3b). The simulation for the tracking of the aircraft using particle filter is done using MATLAB.The main purpose of this simulation is to compare the performance of standard particle filter under different scenarios. A target trajectory and associated measurements (range and bearing angle) have been generated according to equations (3a) and (3b) for 500 time instants. The parameter values are given in table below.

Table I:Parameter Values

Parameter	Value
Number of Monte Carlo Simulations	100
Initial Position $[p_x \; p_y]$(m)	[0 0]
Constant Acceleration(m/s^2)	0.01
Initial state Covariance Q	Diag (50, 50, 0.01, 0.01, 0.01, 0.01)
Measurement noise Covariance R	Diag (50, 1)

The trajectory of aircraft that are used for simulation is shown in Figure 1(a). The range and bearing are measured from the generated trajectory.



(a) (b) (c)

Figure 1: (a) shows the Maneuvering trajectory generated. (b) and (c) shows the measured range and bearing from the generated trajectory.

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An ISO 3297: 2007 Certified Organization *Vol. 4, Special Issue 1, March 2015*

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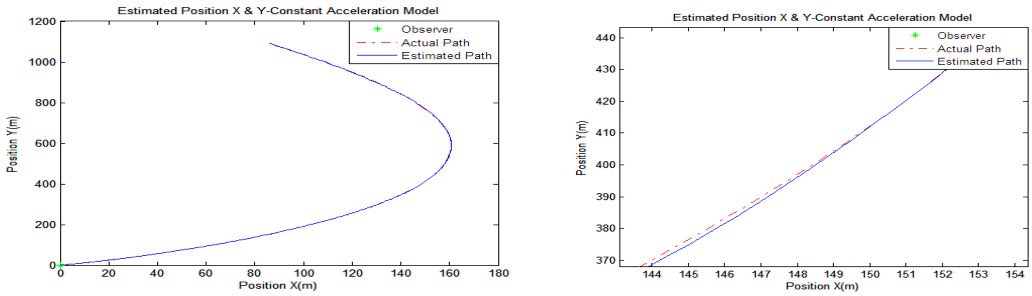
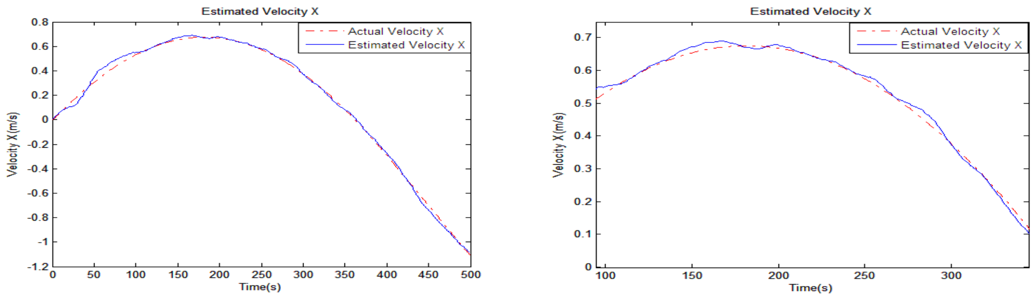
Estimation of Position and Velocity using Particle Filter for Maneuvering trajectory with N=1000 particles are shown in Figure 2, 3 and 4.



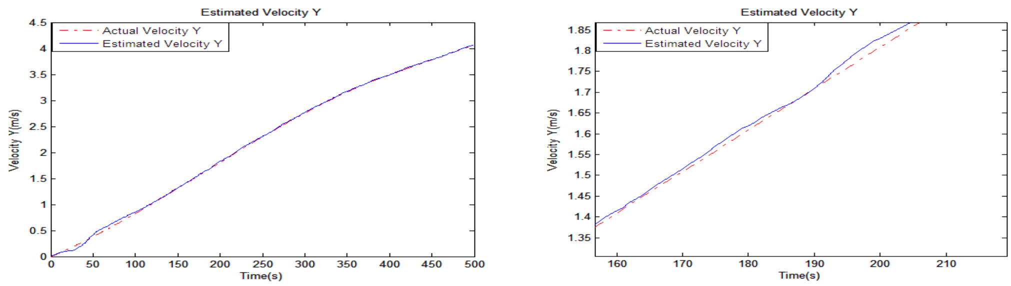
(a) (b)

Figure 2:(a) Estimation of Position of Maneuvering trajectory using 100 Monte Carlo simulations with N=1000 particles for complete time samples. (b) Zoomed version.



(a) (b)

Figure 3:(a) Estimation of Velocity X of Maneuvering trajectory using 100 Monte Carlo simulations with N=1000 particles for complete time samples. (b) Zoomed version.



(a) (b)

Figure 4:(a) Estimation of Velocity Y of Maneuvering trajectory using 100 Monte Carlo simulations with N=1000 particles for complete time samples. (b) Zoomed version.

In order to analyse the effect of number of particles in the performance of Particle Filter, algorithm is executed for five different numbers of particles M=100, 300, 500, 700, 1000 and compared the Relative RMSE (Root Mean Square Error).Here the Relative RMSE is given by:

$$Relative\ RMSE = \frac{\sqrt{\sum_{n=1}^{N} \frac{(x_k^{Obs} - x_k^{Act})^2}{N}}}{x_k^{Act}} \quad (4a)$$

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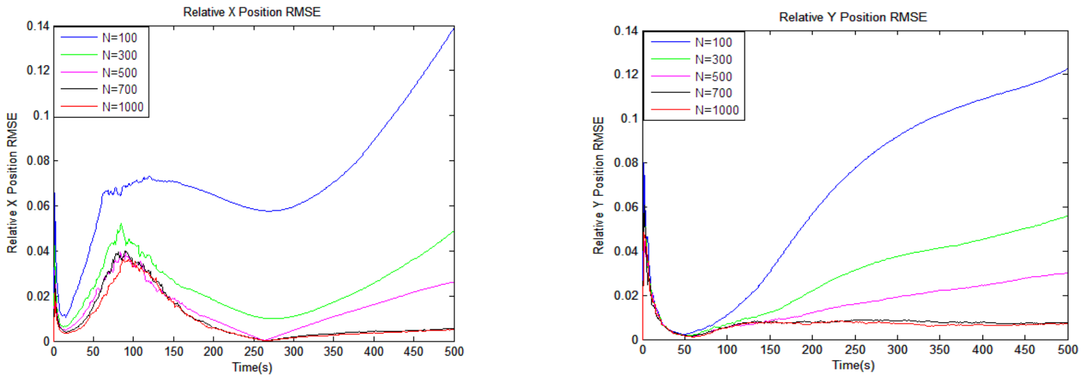
where $x_k^{Obs}$is observed values and $x_k^{Act}$is modelled values of trajectory. N is the Number of Monte Carlo Simulations The Relative RMSEof Position X and Position Y are plotted in Figure 5.



(a) (b)

Figure 5:Position RMSE for the target tracking problem using 100 Monte Carlo Simulation with N=100, 300, 500, 700, 1000 particles.(a)Relative X Position RMSE. (b) Relative Y Position RMSE.

The above figure 5 shows the relative RMSE of Position X and Y is less for the Particle Filter algorithm with N=1000 particles. As the number of particles increases, the relative RMSE of Position X and Y are decreases. It means that the performance of Particle filter algorithm improves with the increase in the number of particles.

V.CONCLUSION

In this paper, we verified particle filtering technique for a maneuvering target tracking example using a linear state model and a non-linear measurement model with additive white Gaussian noise in MATLAB. The effect of number of particles in the performance of Particle Filter algorithm is also analyzed. The results obtained from thesimulation clearly indicate that increase in the number of particles will give more optimal solution for the tracking problem.

VI.ACKNOWLEDGEMENT

The authors would like to thank Toc H Institute of Science and Technology for their support.

REFERENCES

[1] N. J. Gordon, D. J. Salmon, and A. F. M. Smith, “Novel approach to nonlinear/non-Gaussian Bayesian state estimation,” in *IEEE Proceedings F in Radar and Signal Processing*, 1992,vol. 140, pp. 107–113.

[2] J. Uk Cho, S. H. Jin, J. E. Byun, H. Kang, “A real-time object tracking system using a particle filter”, *Proc. of Int. Conf. on Intelligent Robots and Systems*, Oct. 9-15, 2006, Beijing, China, pp: 2822-2827.

[3] M.Sanjeev Arulampalam, Simon Maskell, Neil Gordon, and Tim Clapp. ”A tutorial on particle filters for online Nonlinear/Non-Gaussian Bayesian Tracking”, *IEEE Trans. Signal Process*., Vol 50 no.2, pp.174-188. Feb.2002.

[4] A. Doucet, N. de Feritas, and N. Gordon, Eds., Sequential Monte Carlo Mehods in Practice, Springer Verlag,New York, 2001.

[5] A. Doucet, S. J. Godsill, and C. Andrieu, “On sequentialMonte Carlo methods for Bayesian filtering,” *Statisticsand Computing*, pp. 197–208, 2000.

[6] M. Bolic, P. M. Djuric, S. Hong, “Resampling algorithms fordistribution particle filters”, *IEEE Transactions on Signal Processing*, 2003.

[7] J. Uk Cho, S. H. Jin, X. D. Pham, and J. W. Jeon, “Object tracking circuit using particle filter with multiple features”, *SICEICASE Int. Joint Conf*., Oct. 18-21, 2006, Bexco, Busan, Korea, pp: 1431-1436.

[8] Fredrik Gustafsson, “Particle filter theory practice with positioning applications*” IEEE A&E systems magazine*, vol 25, no.7, July 2010.

[9] P. Li, T. Zhang, "Visual Contour Tracking Based on Particle Filters, "*Proceedings of Generative-Model-Based Vision*, Copenhagen, pp. 61-70, June 2002.

[10] Monson H.Hayes “Statistical Digital Signal Processing And Modeling”, John Wiley & Sons,Inc. 1996, 371- 377.

[11] Papoulis,S.Unnikrishna Pillai, “Probability, Random Variables, and Stochastic Processes” Tata McGraw-Hill Education 2002.

[12] J. Miguez, “Analysis of parallelizable resampling algorithms for particle filtering”, *Elsevier Signal Pressing 87*, 2007, pp: 3155-3174.

[13] Y. C. Ho and R. C. K. Lee, “A Bayesian approach to problems in stochasticestimation and control," IEEE Trans. Automatic Control, vol. 9, pp. 333-339,1964.

[14] G. Kitagawa, “Monte Carlo filter and smoother for non-Gaussian non-linearstate space models," Journal of Computational and Graphical Statistics, vol. 5,no. 1, pp. 1-25, 1996.

[15] M. Isard and A. Blake, “CONDENSATION - conditional density propagationfor visual tracking," International Journal of Computer Vision, vol. 29, no. 1, pp.5-28, 1998.

[16] B. Ristic, S. Arulampalam, and N. Gordon, Beyond the Kalman filter: particle filters for tracking applications. Artech House, 2004.

[17] K. R. Britting, Inertial Navigation Systems Analysis. New York:Wiley-Interscience, 1971.

[18] N. Bergman, “Recursive Bayesian estimation: Navigation and trackingapplications,” Dissertation 579, Linköping Univ., Linköping, Sweden,1999.

[19] K.V. Ramachandra, Kalman Filtering Techniques for Radar Tracking,Marcel Dekker, Inc, New York, USA, 2000.

[20] Petar M. Djuri´c, Jayesh H. Kotecha, Jianqui Zhang, Yufei Huang,Tadesse Ghirmai, M´onica F. Bugallo, and Joaqu´in M´iguez, “ParticleFiltering,” Signal Processing Magazine, IEEE, vol.20, Issue: 5 , Sep2003.

[21] A. R. Reshma, Anooja S, Deepa Elizabeth George-Bearing Only Tracking UsingExtended Kalman Filter.International Journal of Advanced Research in Computer andCommunication Engineering Vol. 2, Issue 2, February 2013