ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

Analysis of Bifurcation in Quadratic Buck Converter

B.Dani Raja[1], Dr.A.Kavitha[2], M. K.Kavitha[3]

PG Scholar, Department of EEE, Anna University, Chennai, India[1]

Assistant Professor (SG), Department of EEE, Anna University, Chennai, India[2]

Research Scholar, Department of EEE, Anna University, Chennai, India[3]

ABSTRACT: This paper presents the analysis of a bifurcation in quadratic buck converter. This class of converter offers very wide range of DC voltage conversion ratio enabling higher switching frequencies. In this paper, analysis of the quadratic buck converter in discontinuous conduction mode (DCM) is presented and corresponding simulation results obtained using MATLAB software. Pulse train control quadratic buck converter operated in DCM is analysed for bifurcation. Non linear dynamics of DCM quadratic buck converter under different input voltage and load conditions are examined and the simulation results are given.

KEYWORDS*:* Quadratic buck converter, Pulse train control, Bifurcation, Non linear dynamics.

I. INTRODUCTION

DC-DC converters continue to evolve as an indispensable part of source in many electronic circuits. The switching frequency of DC-DC converters can be suitably selected to offer the desired output voltage, and the switches allowed to turn ON and OFF in tune with the design specifications. However in real time applications, the supply voltages and the loads usually vary, necessitating a suitable corrective control action to stabilize the system in the desired operating state.

A very low voltage output from a conventional DC/DCbuck converter demands for a very small switching duty ratio. But there is a limit on the minimum on time of an active switch and this may lead to an implementation problem. In cascade configuration, the conversion ratio is increased by the order of number of stages of converters. However, one of the main disadvantages of cascaded converters is that the overall efficiency is reduced by losses in switching devices. In the case of the quadratic buck converter, the voltage conversion ratio has a quadratic dependence on the duty ratio and is electrically equivalent to two buck converters connected in cascade with only one active switch.The behaviours of the converter operating in discontinuous conduction mode (DCM) have been reported here.

Pulse train (PT) control technique, which realizes the output voltage regulation of switching dc–dc converters by applying high-power control pulses or low-power control pulses with discrete duty ratios. In VMPT controller, the duty ratios of high-power control pulse and low-power control pulse are preset. This paper exhibits idea of voltage mode pulse train control applied to the quadratic buck converter for the bifurcation analysis.

Section I deals with the introduction to non linear dynamics and the basics of quadratic buck converter. The objective of the project is also highlighted in this section.SecionII presents the analysis of quadratic buck converter with its various mode of operation in discontinuous conduction mode (DCM). The state space equations for the respective modes are also derived. Section III explains the voltage mode pulse train control applied to the quadratic buck converter for the voltage regulation. The control strategy for generation of the pulses corresponding to the required output power is also presented. Section IV shows the simulated results of the quadratic buck converter in discontinuous conduction mode. It also includes the waveform for different resistance load with the VMPT controller. Section V summarizes the conclusion.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

II. OPERATION OF QUADRATIC BUCK CONVERTER IN DCM

During the last few years, a great number of applications for quadratic buck converters (QBCs) have been reported. In a QBC with a single switch, the dc conversion ratio has a quadratic dependence of the duty ratio. Its dynamical behaviour is similar to two cascaded buck converters but using one active switching device.

A. Modes of Operation in Discontinuous Conduction Mode:

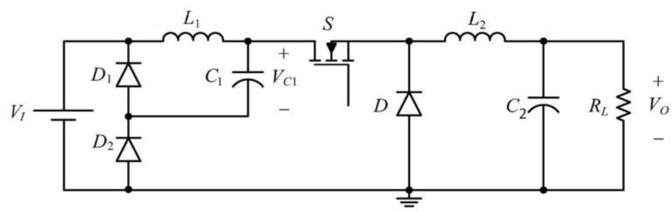

Fig.1 circuit diagram of quadratic buck converter

The circuit diagram of a single-switch quadratic buck converter is shown in Fig 1. The converter consists of two LC filters, one active switch (Q) and three passive switches (D_1, D_2, D. If the converter is operated at fixed switching frequency, the state-space averaged model of the quadratic buck converter can be described as d(t) is the duty ratio, v_1 is the supply voltage, L_1 and L_2 are the inductances of the first and second stages of the converter, C_1 and C_2 are the capacitances of the first and second stages of the converter, R is the loading resistance, i_{L1} and i_{L2}) are the currents of inductors L_1 and L_2, and V_{C1} and V_{C2} are the voltages across the capacitors C_1 and C_2 respectively. When the QBC is operated in discontinuous conduction mode (DCM) it exhibits three modes. The first two modes are similar to the continuous conduction mode. In mode III only the output capacitor feeds the load.

B. Mode I:

The equivalent circuit corresponding to this stage is shown in Fig. 2.During this interval, diode D_2 and switch S are ON, whereas the diodes D_1 and D are OFF. The input voltage source V_1 appears in series with the inductors L_1 and L_2.

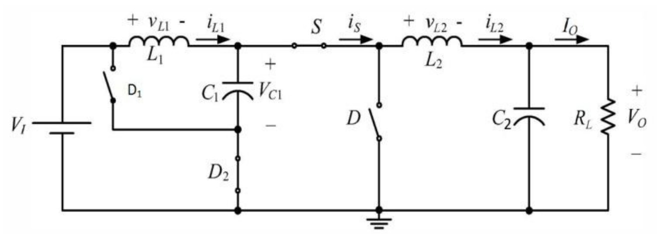

Fig 2 circuit diagram for mode I

The corresponding state equations are as follows.

$$\frac{di_{L1}}{dt} = \frac{-V_{C1}}{L_1} + \frac{V_1}{L_1} \qquad (1)$$

$$\frac{di_{L2}}{dt} = \frac{-V_{C2}}{L_2} + \frac{V_{C1}}{L_2} \qquad (2)$$

$$\frac{dV_{C1}}{dt} = \frac{iL_1}{C_1} - \frac{i_{L2}}{C_1} \qquad (3)$$

$$\frac{dV_{C2}}{dt} = \frac{i_{L2}}{C_2} - \frac{V_{C2}}{RC_2} \qquad (4)$$

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

C. Mode II:

The equivalent circuit corresponding to this stage is shown in Fig.3.During this time interval, the switch S and diode D_2 are turned OFF whereas diodes D_1 and D are ON. The inductor L_l begins to discharge through the capacitor C_1 and L_2 discharge through C_2.

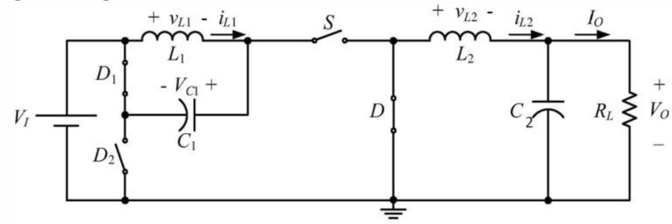

Fig 3 circuit diagram for mode II

The corresponding state equations are as follows.

$$\frac{di_{L1}}{dt} = \frac{-V_{C1}}{L_1} \tag{5}$$

$$\frac{di_{L2}}{dt} = \frac{-V_{C2}}{L_2} \tag{6}$$

$$\frac{dV_{C1}}{dt} = \frac{i_{L1}}{C_1} \tag{7}$$

$$\frac{dV_{C2}}{dt} = \frac{i_{L2}}{C_2} - \frac{V_{C2}}{RC_2} \tag{8}$$

D. Mode III:

The equivalent circuit corresponding to this stage is shown in Fig.4. In this stage all the energy stored in C_2 was transferred to the load. With this, the diode D blocks and output capacitor C_2 feeds the load.

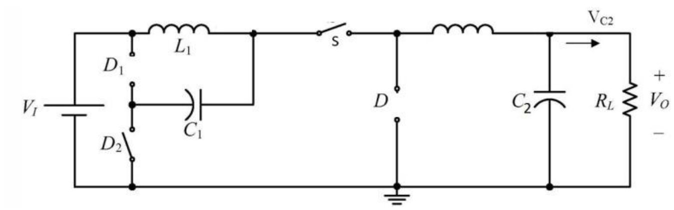

Fig 4 circuit diagram for mode III

The corresponding state equation is as follows.

$$\frac{dV_{C2}}{dt} = -\frac{V_{C2}}{RC_2} \tag{9}$$

III.VOLTAGE-MODE PULSE TRAIN CONTROL OF QUADRATIC BUCK CONVERTER IN DCM

Many control methods have widespread applications in the closed-loop control of conventional DC/DC buck converters but there are very few discussions on the close loop Control of quadratic buck converters. This may be due to the fact that a quadratic buck converter has more complex non-linearity than a single stage buck converter, which may result with an increased difficulty in designing controller for the quadratic converter. Voltage mode pulse train control QBC offers wide range of voltage regulation.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

E. Voltage Mode Pulse Train Control:

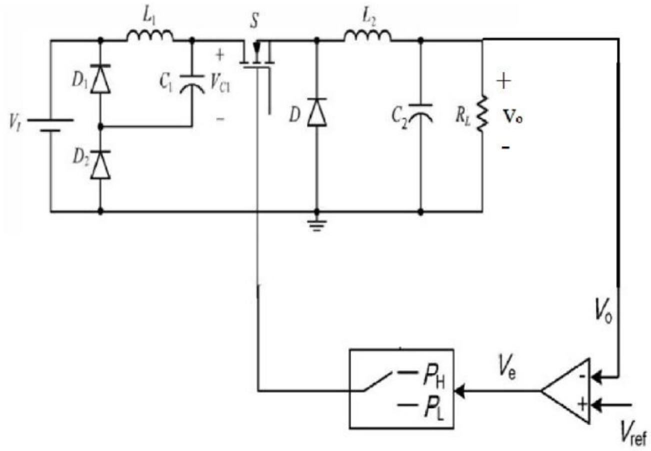

Fig 5 Circuit diagram of VMPT control quadratic buck converter

Pulse train (PT) is a new control technique for switching DC-DC converters, which achieves output voltage regulation by generating a control pulse train constructed by two control pulses with different duty ratio. Pulse train technique has several advantages over conventional PWM control techniques, such as simplicity of design and implementation, and excellent transient performance. Furthermore, pulse train control switching DC-DC converters can operate in fixed switching frequency and achieve soft switching conveniently.

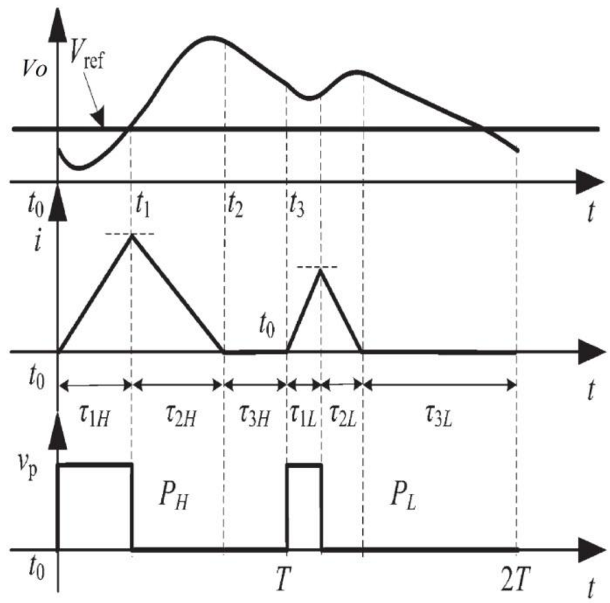

Fig 6 Theoretical waveform of VMPT control

Pulse train control technique realizes the control of switching DC-DC converters by a pulse train constructing by two control pulses with different duty ratio, high power pulse P_H and low power pulse P_L is shown in fig6. At the beginning of each switching cycle, the output voltage V_O is sensed and compared with the desired reference voltage V_{ref}. If V_O is lower than Vref, high power pulse P_H will be generated as the control signal in this switching cycle. On the other hand, if V_O is higher than Vref, low power pulse P_L, which has a shorter on time compared with P_H, will be generated.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

Obviously, the converter will deliver more power to the load during high power pulse cycle than during low power pulse cycle.

According to the principle described above, pulse train control method realizes output voltage regulation of switching DC-DC converter by adjusting the combination of control pulse of P_H and P_L. In steady state, control pulse train constructed by proper combination of P_H and P_L in sequential switching cycles is generated for the control of switching DC-DC converters. The pulse train in the sequential switching cycles composes a new repetition cycle. The on times of P_H and P_L are different, but the switching periods *T* are the same, therefore, the switching frequency of the converter is constant.

For voltage-mode pulse train control scheme, control pulses P_H and P_L have constant duty ratios D_H and D_L (D_H>D_L) respectively. In the case of high power pulse, the power switch is turned on at the beginning of each switching cycle, t = nT, and turned off at time t = nT+D_HT. It will keep off until the beginning of next switching cycle.

F. Analysis of VMPT Control:

For the VMPT control quadratic buck converter operating in steady state, suppose the number of P_H and P_L in the pulse train repetition cycle is μ_H and μ_L respectively, Average Switch current:

$$I_{SW} = \frac{V_1 - V_{c1}}{2L} D_H^2 T \qquad (10)$$

The energy drawn from the input during this pulse train repetition cycle is

$$E_{inH} = \frac{V_1(V_1 - V_{c1})}{2L} D_H^2 T \qquad (11)$$

$$E_{inL} = \frac{V_1(V_1 - V_{c1})}{2L} D_L^2 T \qquad (12)$$

$$E_{in} = \mu_H E_{inH} + \mu_L E_{inL} \qquad (13)$$

$$E_{in} = V_1 \frac{V_1 - V_{c1}}{2L} T^2 \{\mu_H D_H^2 + \mu_L D_L^2\} \qquad (14)$$

If the efficiency of the converter is η , the output power P can be expressed as

$$P(\mu_H + \mu_L)T = \eta E_{in} \qquad (15)$$

$$P = \eta V_1 T \frac{V_1 - V_{c1}}{2L\,(\mu_H + \mu_L)} \{\mu_H D_H^2 + \mu_L D_L^2\} \qquad (16)$$

The proportion between μ_H and μ_L :

$$\frac{\mu_H}{\mu_L} = \frac{1 - MD_L^2}{MD_H^2 - 1} \qquad (17)$$

where

$$M = \eta V_1 T \frac{V_1 - V_{c1}}{2LP} \qquad (18)$$

Suppose the input voltage of the VMPT control converter is constant, we can conclude that in each pulse train repetition cycle, when higher output power is required, the control strategy will generate more control pulse P_H and less control pulse P_L, and vice versa with lighter load. This conclusion can be well explained from the conversion of energy. If the load of buck converter increases, the converter needs to deliver more energy by using P_H control pulses.

On the other hand, if the load of buck converter is constant, there will be more P_H for the case of lower input voltage, and more P_L for the case of higher input voltage. This can also be well explained from the conversion of energy. We know that when the input voltage of the converter increases, the energy drawn from the input power source in P_H or P_L switching cycle will increase. As the energy consume by load is constant, the number of P_H in a pulse train repetition cycle will be fewer.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

G. Effect of Varying Resistance:

Resistance ranges(Ω)	***Different periodicities And pulse repetition Pattern***	$\frac{?_H}{?_L}$
< 10	*Period-1(1P$_H$)*	
(13.8-15.1)	*Period-2(1P$_H$-1P$_L$)*	*1*
(18.7-19.1)	*Period-3(1P$_H$-2P$_L$)*	*0.5*
>35	*Period-1(1P$_L$)*	

H. Effect of Varying Input Voltage:

Voltage ranges(v)	***Different periodicities and pulse repetition pattern***	$\frac{?_H}{?_L}$
< 24	*Period-1(1P$_H$)*	
(26-27.5)	*Period-2(1P$_H$-1P$_L$)*	*1*
(29.4-30.1)	*Period-3(1P$_H$-2P$_L$)*	*0.5*
>36	*Period-1(1P$_L$)*	

IV .SIMULATION RESULTS

The quadratic buck converter is simulated using MATLAB/SIMULINK with and without controller. When the Voltage mode Pulse train is implemented voltage regulation of the converter is achieved. The bifurcation analysis is done with the variation of load resistance.

A . Specifications:

Circuit components	**Values**
Switching frequency	110 kHz
Input Voltage V_1	24V
Inductance L_1	10 µH
Inductance L_2	10 µH
Capacitance C_1	16 µF
Capacitance C_2	16 µF
Load resistance R	10Ω

Table 1: Component values used in simulation

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

*B. **S**imulation results without controller:*

The quadratic buck converter is simulated without controller. The output voltage, output current, inductor current and capacitor voltages are analysed.. The simulated waveform for the inductor current, capacitor voltage is shown in fig 7.

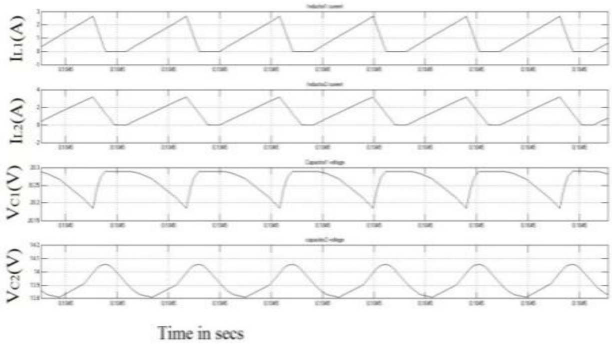

Fig 7 simulated waveform of inductor current and capacitor voltage

C. Simulations results with controller:

i)Varying load resistance:

The quadratic buck converter is analysed by implementing voltage mode pulse train control and the simulated results are shown below.

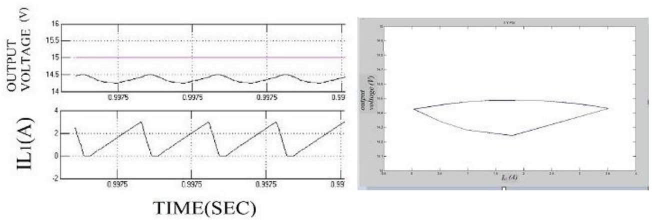

Fig 8. Simulated waveform of inductor current and output voltage for R=8 Ω

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=8Ω is shown in fig8. The control pulses within a pulse train repetition cycle are $1P_H$ in this case. The control pulses within a pulse train repetition cycle are contains only the lower width pulses (P_H) since the output voltage is lesser than that of the reference voltage.

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=14.5Ω is shown in fig 9.

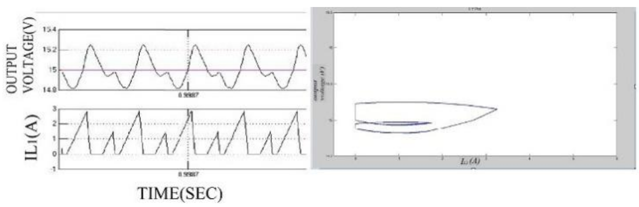

Fig 9 Simulated waveform of inductor current and output voltage for R=14.5Ω

D. Simulations results with controller:

i)Varying load resistance:

The quadratic buck converter is analysed by implementing voltage mode pulse train control and the simulated results are shown below.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

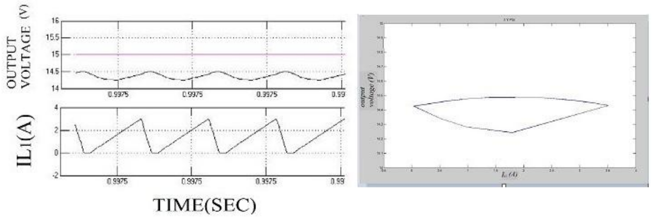

Fig 8. Simulated waveform of inductor current and output voltage for R=8 Ω

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=8Ω is shown in fig8. The control pulses within a pulse train repetition cycle are $1P_H$ in this case. The control pulses within a pulse train repetition cycle are contains only the lower width pulses (P_H) since the output voltage is lesser than that of the reference voltage.

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=14.5Ω is shown in fig 9.

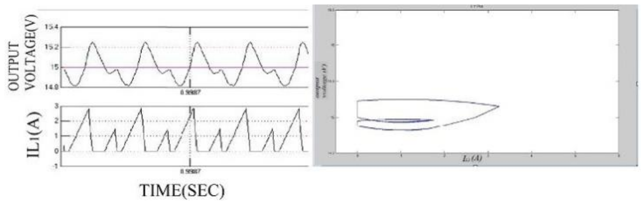

Fig 9 Simulated waveform of inductor current and output voltage for R=14.5Ω

ii)Varying Input Voltage:

The same analysis has been carried out for varying input voltage where the resistance is kept constant. The waveforms for the quadratic buck converter with VMPT control for the input voltages of V_{in}=23V,29.5V are shown in Fig10,Fig11 respectively. The control pulses within a pulse train repetition cycle are $1P_H$,$1P_H$-$2P_L$ in this case.

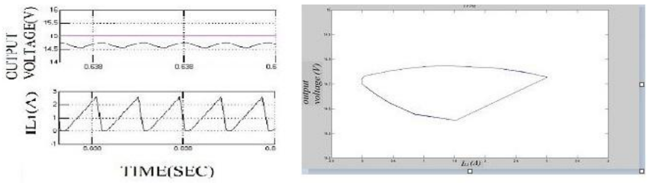

Fig10 Simulated waveform of inductor current and output voltage for Vin=23V

.

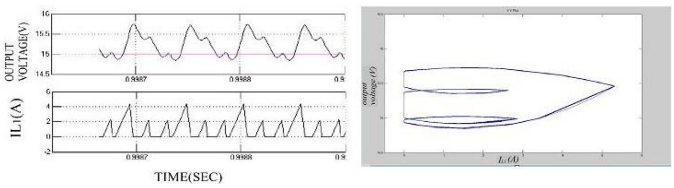

Fig11 Simulated waveform of inductor current and output voltage for Vin=29.5V

V. CONCLUSION

The steady-state analysis of the quadratic buck converter in discontinuous conduction mode is performed. Using MATLAB simulation, the converter is analysed for the given set of specifications.

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 5, May 2015

Pulse train is a new nonlinear control technique for switching DC-DC converters. The control strategy, operation and advantages of pulse train method are discussed. Based on comprehensive analysis and comparison on voltage-mode buck converter in DCM, some useful control regulations are obtained. The operation states of converter under different load resistance and input conditions are studied. Simulation and experimental results are presented to verify theory analysis results.

VI. ACKNOWLEDGMENT

The initiation and the rudimentary concept of this paper are ascribable to my guide **Dr.A.Kavitha,** Assistant Professor (SG), DEEE, Anna University, without whose invaluable guideline, patient and constant encouragement nothing would have been materialized. I owe my gratitude to her.

REFERENCES

[1] X.L. Wei1, K.M. Tsang and W.L. Chan(2008), "Non-linear PWM control of single-switch quadratic buck converters using internal model", IET Power Electronics Received on 15th June 2008 Revised on 26th November 2008 doi: 10.1049/iet-pel.2008.0170.

[2] Agasthya Ayachit and Marian K. Kazimierczuk(2013), "Steady-State Analysis of PWM Quadratic Buck Converter in CCM"IEEE 978-1-4799-0066-4/13.

[3] Oliver Woywode, Jens Weber, Henry Güldner,Alexander L. Baranovski, and Wolfgang Schwarz(2003), "Bifurcation and Statistical Analysis of DC–DC Converters", IEEE transactions on circuits and systems-I: fundamental theory and applications, vol. 50, no. 8, august 2003.

[4] M. Subashini, C. Kamalakannan,and Dr. M. Ramaswamy(2011), " Experimental Analysis of Nonlinear Dynamics of DC-DC Converters", 2011 IEEE PES Innovative Smart Grid Technologies.

[5] Ming Qin, Jianping Xu, Guohua Zhou, Qingbo Mu(2009), "Analysis and Comparison of Voltage-Mode and Current-Mode Pulse Train Control Buck Converter"IEEE 978-1-4244-2800-7/09.

[6] L. Nousiainen and T. Suntio(2011), "Dynamic Characteristics of Current-Fed Semi-Quadratic Buck-Boost Converter in Photovoltaic Applications,IEEE 978-1-4577-0541-0/11.

[7] Ketsuda Karaket and Chanin Bunlaksananusorn(2011), "Modeling of a Quadratic Buck Converter", The 8th Electrical Engineering/ Electronics, Computer, Telecommunications and Information Technology (ECTI) Association of Thailand - Conference 2011.

[8] Morales-saldan˜a j.a., leyva-ramos j., carbajal-gutie´ rrez e.e.'Modeling of switch-mode dc-dc cascade converters',IEEE Trans. Aerosp. Electron. Syst., 2002, 38, (1),pp. 295–299.

[9] Matsuo h., harada k 'The cascade connection of switching regulators', IEEE Trans. Ind. Appl., 1976, 12, (2), pp. 192–198.

[10] M.Telefus, A.Shteynberg, M.Ferdowsi, et al, "Pulse Train control technique for flyback converter," IEEE Transactions on Power Electronics, Vol. 19, pp.757-764, May. 2004.