



# **Performance Evaluation of PAPR Reduction Techniques in Filter Bank Based Multicarrier Systems in AWGN and Fading Channels**

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**ABSTRACT:** Orthogonal Frequency division Multiplexing (OFDM) is a multicarrier modulation technique which supports high data rate transmission and are widely used in digital audio/ video broadcasting, WiFi and WiMax. It has the ability to cope up with several channel conditions and the insertion of a guard band called cyclic prefix eliminates inter-symbol interference. But this guard band reduces spectral efficiency which can be overcome by using an alternative technique known as Filter bank based multicarrier (FBMC) systems. This system provides better spectral shaping of the sub-bands by using synthesis and analysis filters. One of the problems associated with FBMC system is high Peak to Average Power Ratio (PAPR). The performance of the system is evaluated using Non Linear Companding techniques like A-law and  $\mu$ -law and they are found to be better compared to other techniques. Performance is evaluated using CCDF plot and BER for different channels like AWGN, Rayleigh and Rician channel.

**KEYWORDS:** Companding, Cosine Modulated Filter bank, Filter Bank based Multicarrier (FBMC) Systems, PAPR.

## **I. INTRODUCTION**

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multicarrier modulation scheme, which is widely used in audio or video broadcasting, Wi-Fi etc. It consists of multiple subcarriers and these subcarriers are orthogonally spaced to avoid inter-carrier interference. The advantages of OFDM are the ability to cope with several channel conditions, robust against narrow band channel interference, robust against fading caused by multipath channels etc. It also uses a guard band called cyclic prefix to eliminate inter-symbol interference. But this guard band reduces the bandwidth efficiency and power efficiency, which is the main drawback of OFDM [1]. The individual subcarriers can be completely separated and demodulated by FFT at the receiver when there is no inter-symbol interference.

Filter bank based multicarrier (FBMC) system is an alternative technique to overcome the limitations in OFDM. The main characteristics of FBMC system is that it provides better spectral shaping of the sub bands which in turn simplifies the equalization of inter-carrier and inter-symbol interference when no cyclic prefix is used. This system is developed from the multirate filter banks in transmultiplexer configuration [2]. The IFFT and FFT block in OFDM systems are replaced by synthesis and analysis filters of filter bank theory in FBMC system. Cosine Modulated Filter banks (CMFB) uses filters of larger length and provides smaller side lobes and better stop band attenuation than Discrete Fourier transform (DFT) based systems [3]. The analysis and synthesis filters are cosine modulated versions of low pass prototype filter and therefore the design of entire filter bank reduces to the design of a single prototype filter. Here the prototype filter for CMFB is designed using window method with Kaiser Window as the window function. Kaiser window offers a simple means of designing low pass FIR filters and can have a direct control over the stop-band attenuation of the prototype filter [4].

A major issue that limits the performance of the FBMC system is its high Peak to Average Power Ratio (PAPR). It is the result of the superposition of large number of subcarriers which constructively add up to form high peak power. The main problem is that practical transmission systems are peak power limited which cause spectral widening of the

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

transmitted signal and requires the use of amplifiers and ADC/DAC with large dynamic range. The power efficiency of the amplifier depends on its linear dynamic range and efficiency decreases with increase in dynamic range. Therefore it is necessary for the communication system employing FBMC to reduce the PAPR [5].

A large number of techniques have been reported for reducing the PAPR value and they have many limitations. In this paper, Non Linear Companding techniques like A- law and  $\mu$ - law Companding techniques used for reducing PAPR value. In this technique, the Companding transform is applied at the transmitter side to attenuate large peaks and expand smaller amplitudes of FBMC signal, thereby reducing the PAPR [6]. At the receiver decomanding is performed by using inverse companding function to recover the original FBMC signal. It is found to be a very effective technique to reduce the PAPR of FBMC signal and also the complexity is very low [7].

The rest of the paper is organized as follows: Section II gives an overview of the FBMC system. Section III and IV describes the design of Prototype filter and Cosine Modulated Filter bank. Section V and VI describes about PAPR and its reduction techniques. Section VII focuses on the simulation scenario. The simulation results are described in Section VIII. Section IX concludes the paper.

## II.OVERVIEW OF FBMC SYSTEM

A general overview of FBMC system model is shown in Fig. 1. The filter banks are employed in transmultiplexer configuration. It consists of symbol mapper and an upsampler followed by synthesis filters at the transmitter end. The synthesis filter performs the frequency division multiplexing of complex data symbols into parallel subcarriers. The analysis filter at the receiver end separates the data from each carrier. The synthesis and analysis filters are cosine modulated versions of low pass prototype filter. The prototype filter is designed using Kaiser Window with cut off frequency  $\pi/8$  and enough stop band attenuation.

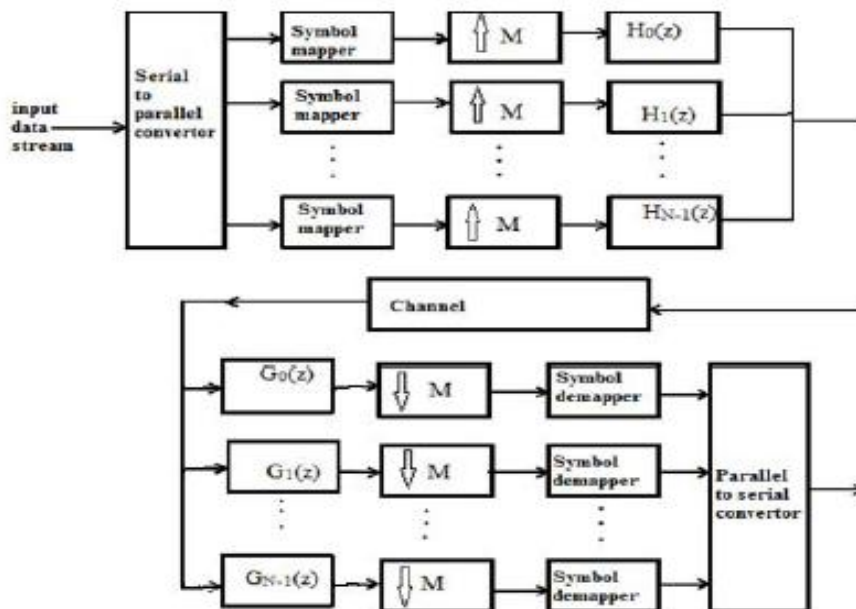


Fig. 1 FBMC System Model

A 4- channel Cosine Modulated filter bank is considered here. At the transmitter, the input data stream is converted into its parallel form with the help of a serial to parallel converter. It is then mapped into symbols by symbol mapper. Mapping is done with the help of Pulse Amplitude modulation (PAM). The modulated symbols are upsampled by a factor of M to achieve maximum data rate. The sampling factor (M) and number of channels must be same to avoid



# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

redundancy. The upsampled data is then passed through the synthesis filter and the composite signal is transmitted over a common channel. Here the performance is evaluated in AWGN, Rayleigh and Rician channel.

At the receiver, inverse operation takes place. The composite signal gets separated into N channels using analysis filters. The analysis filter is followed by a downsampler to obtain the original sampling rate. They are then demapped and converted back to serial form.

### III. DESIGN OF PROTOTYPE FILTER

The prototype filter for CMFB is a symmetric linear phase low pass filter, designed using window method with Kaiser Window as the window function. Prototype filter can be designed such that the FBMC system guarantees perfect reconstruction property. The bandwidth of the filter is  $\pi/2M$ , where M is the number of channels. Let  $h(n)$  be the impulse response of the filter defines as:

$$h(n) = h_i(n) \times v(n) \quad (1)$$

where  $v(n)$  be the window function which is zero for  $|n| > \frac{N}{2}$  and  $h_i(n)$  be the ideal impulse response with cut off frequency  $w_c$  given by:

$$h_i(n) = \frac{w_c}{\pi} \left( \frac{\sin w_c n}{w_c(n)} \right), -\infty \leq n \leq \infty \quad (2)$$

The Kaiser Window is given by:

$$v(n) = \left\{ I_0 \left[ \beta \sqrt{1 - (n/0.5N)^2} \right] / I_0(\beta) \right\}, -\frac{N}{2} \leq n \leq \frac{N}{2} \quad (3)$$

Where  $I_0(x)$  represents the modified zeroth order Bessel function of the first kind which can be computed using the expression:

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[ \frac{(0.5x)^k}{k!} \right]^2 \quad (4)$$

The parameter  $\beta$  can be adjusted to obtain the desired stop band attenuation and is given by:

$$\beta = \begin{cases} 0.1102(A_s - 8.7), \dots \text{if } A_s > 50 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21), \dots 21 < A_s < 50 \\ 0, \dots \text{if } A_s < 21 \end{cases} \quad (5)$$

The order N is adjusted to satisfy the requirement on  $\Delta f$  and is given by:

$$N = \frac{A_s - 7.95}{14.36 \Delta f} \quad (6)$$

Thus the design of Kaiser Window is spanned by three parameters  $w_c$ ,  $N$  and  $\beta$ . This technique has the advantage of having direct control over stopband attenuation of the prototype filter and thereby controlling stopband attenuation of analysis and synthesis filters. Thus Kaiser Window offers a simple method of designing linear phase FIR filters.



# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

## IV. DESIGN OF COSINE MODULATED FILTER BANK

Cosine Modulated Filter banks (CMFB) are real coefficient filter banks derived from a single prototype filter using cosine modulation. The impulse response of synthesis filter  $f_k(n)$  and the impulse response of the analysis filter  $h_k(n)$  can be expressed as:

$$f_k(n) = 2h_p(n) \cos(2k+1) \frac{\pi}{2M} \left( n - \frac{N}{2} \right) + \phi_k \quad (7)$$

$$h_k(n) = 2h_p(n) \cos(2k+1) \frac{\pi}{2M} \left( n - \frac{N}{2} \right) - \phi_k \quad (8)$$

Where,  $h_p(n)$  denotes the impulse response of symmetric linear phase prototype filter. To be free from phase distortion, analysis filters are chosen such that

$$h_k(n) = f_k(N-1-n) \begin{cases} 0 \leq k \leq M-1 \\ 0 \leq n \leq N-1 \end{cases} \quad (9)$$

$\phi_k$  must be chosen such that  $\phi_k = (-1)^n \frac{\pi}{4}$ ,  $k = 0, 1, 2, \dots, M-1$ ,  $M$  is the number of channels and  $N$  is the length of the prototype filter. Thus a nearly perfect reconstruction filter bank can be obtained by satisfying the above conditions.

## V. PEAK TO AVERAGE POWER RATIO

Peak to Average Power Ratio is one of the main drawbacks of every multicarrier system. Similar to OFDM, FBMC system also suffers from the problem of high PAPR. It is defined as the ratio of maximum signal power to the average signal power. For a continuous time signal PAPR is evaluated by the equation:

$$PAPR = \frac{\max_{0 \leq t \leq NT} |x(t)|^2}{\frac{1}{NT} \int_0^{NT} |x(t)|^2 dt} \quad (10)$$

Where,  $|x(t)|$  denotes the amplitude of the FBMC signal. With high PAPR, high power amplifiers are required in the transmitter side to obtain sufficient transmission power, which is expensive. Also it demands the use of DAC with large dynamic range in order to accommodate large peaks of the multicarrier modulation signal. Therefore the best solution is to reduce PAPR before the signals are transmitted through highpower amplifiers and DAC. The primary goal of PAPR reduction technique is to reduce the factor  $\max |x(t)|$ . The PAPR of a system can be studied using Complementary Cumulative Distribution Function (CCDF). It is the probability that PAPR exceeds a particular threshold value and is given by:

$$CCDF = \Pr(\text{probability}(PAPR > P_0)) \quad (11)$$

Where,  $P_0$  is the threshold.

## VI. NON LINEAR COMPANDING TRANSFORM

Non linear Companding transform is one of the most attractive schemes in conventional OFDM system due to its good PAPR reduction, better BER performance, low computational complexity etc. Companding transform technique performs compression at the transmitter end after synthesis filter and expansion at the receiver end before the signal is fed to analysis filter. It compand original signals using a strict monotonically increasing function. Therefore



# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

companded signal at the transmitter can be correctly recovered at the receiver using an inverse transform function. There are two types of Non linear Companding: A-law Companding and  $\mu$ - law Companding.

## 6.1 A- Law Companding

A- law encoding at the transmitter for a given input  $x$  is given as:

$$F(x) = \text{sgn}(x) \begin{cases} \frac{A|x|}{1 + \ln(x)}, |x| < \frac{1}{A} \\ \frac{1 + \ln(A|x|)}{1 + \ln(A)}, \frac{1}{A} \leq |x| \end{cases} \quad (12)$$

Where,  $A$  denotes the compression parameter. The value of  $A$  is chosen to be 50. At the receiver,  $A$  law expansion is given as:

$$F^{-1}(y) = \text{sgn}(y) \begin{cases} \frac{|y|(1 + \ln(A))}{A}, |y| < \frac{1}{1 + \ln(A)} \\ \frac{\exp(|y|(1 + \ln(A)) - 1)}{A}, \frac{1}{1 + \ln(A)} \leq |y| \leq 1 \end{cases} \quad (13)$$

## 6.2 $\mu$ - Law Companding

$\mu$  - law companding is simple and an effective technique to reduce PAPR compared to  $A$  - law Companding. It also provides a slightly larger dynamic range without any distortion for small signals. The signal after companding for a given input  $x$  is

$$F(x) = \text{sgn}(x) \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)}, -1 \leq x \leq 1 \quad (14)$$

The inverse operation is carried out at the receiver after analysis filter bank.  $\mu$  - law expansion at the receiver is given as:

$$F^{-1}(y) = \text{sgn}(y) \left( \frac{1}{\mu} \right) \left( (1 + \mu)^{|y|} - 1 \right), -1 \leq y \leq 1 \quad (15)$$

where  $y$  denotes the received signal and the range of the function is from -1 to 1. Although both  $A$  - law and  $\mu$  - law Companders are simple, the performance of two companders for different values of compressing factors are evaluated and their performance are shown in simulation results.

## VILSIMULATION SCENARIO

The simulation has been done on Matlab. A 4- channel Cosine Modulated Filter bank has been assumed for an FBMC system with sampling factor of 4. The Prototype filter for the analysis and synthesis filters are designed using Kaiser Window method. The prototype filter controls the phase and amplitude distortions in the subchannels and the interference between the subchannel. The simulation results show the variation of Peak to Average Power ratio for different values of Companding factors and BER performance of Non linear Companding transform techniques. The parameter settings for the simulations have been summarized in Table 1.

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

Table 1. Simulation Parameters

Parameter	Value
Number of data bits	10 <sup>4</sup>
Number of channels	4
Sampling factor	4
Modulation Technique	PAM
Channels used	AWGN, Rayleigh, Rician
Stop band attenuation	160
Transition bandwidth	0.06

## VIII. RESULT AND DISCUSSION

An FBMC system is designed using the simulation parameters as given in Table 1. Initially the bits are mapped into symbols using PAM modulation and modulated signals are fed to synthesis filter bank. The resulting filter bank based multicarrier signal is transmitted over an AWGN, Rayleigh or Rician channel. It is then followed by an analysis filter bank and symbol de-mapping at the receiver side. PAPR Performance is evaluated using CCDF plot for the following reduction techniques, A- law companding,  $\mu$ - law companding, and Raised Cosine Pulse shaping technique.

The filter banks are derived from a single Prototype filter. The Prototype filter is designed such that the system satisfies perfect reconstruction or near perfect reconstruction property. Fig. 2 shows the magnitude response of Prototype filter with stop band attenuation of 160dB.

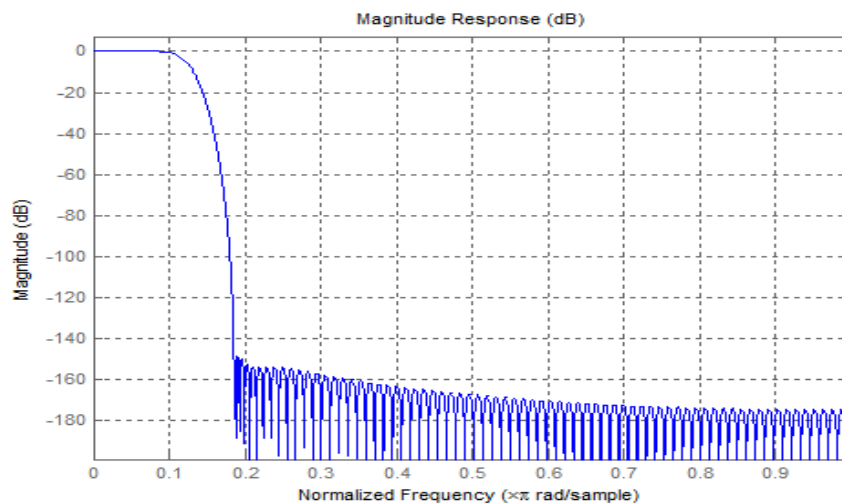


Fig. 2 Magnitude Response of Prototype Filter

All subchannel filters in synthesis and analysis filter bank are formed by cosine modulation of a single valued linear phase FIR prototype filter. Cosine Modulated filter bank uses filters of larger length and provides lower side lobes, thereby reducing the interference between subcarriers. Fig. 3 shows the response of Cosine Modulated Filter bank.

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

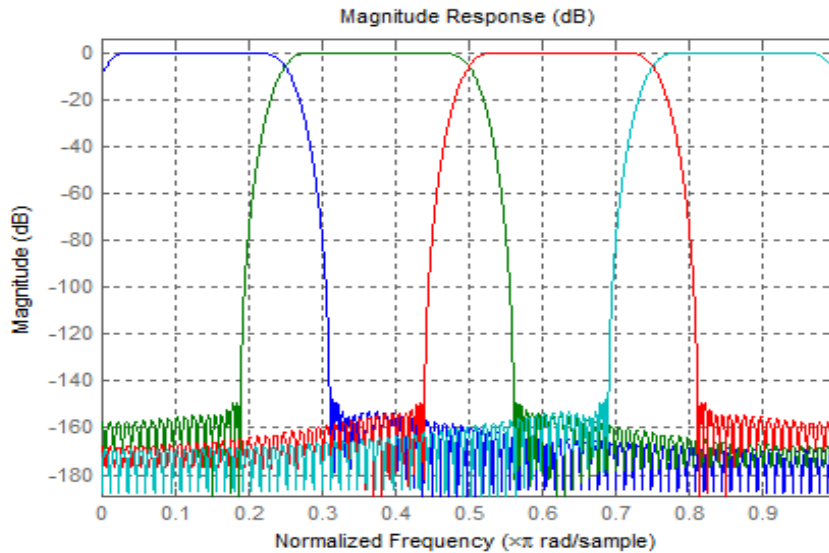


Fig. 3 Cosine Modulated Filter Bank

The main drawback of FBMC system is high PAPR. The PAPR of a system can be studied using Complementary Cumulative Distribution function. Fig. 4 shows the CCDF plot obtained for different values of  $A$  in  $A$ -law Companding.  $A$  can have the values of 20, 50 or 80. When compression factor  $A = 20$ , PAPR value obtained is around 15.133 dB and for  $A = 80$ , it is 14.73 dB. It can be observed that when the value of  $A$  becomes 50, better reduction in PAPR of about 13.53 dB is obtained compared to other two values.

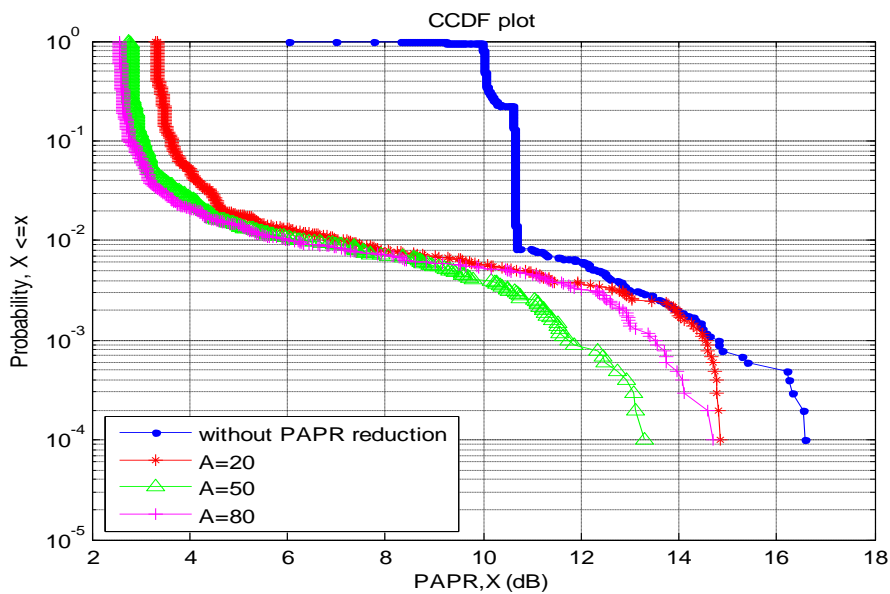


Fig. 4 CCDF Plot showing variation of PAPR for different values of  $A$ .

Also FBMC system with  $\mu$ -law companding for different values of  $\mu$  parameter is simulated. It is observed that the value of PAPR varies for different values of  $\mu$ , such as  $\mu = 50, 100$ , or 200. As the value of  $\mu$  increases, the value of PAPR decreases. When  $\mu = 50$ , the value of PAPR obtained is 13.46 dB and for  $\mu = 100$ , it is 12.69 dB. The better

# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

reduced value of PAPR is obtained when  $\mu = 200$  and is found to be 12.39 dB. This variation can be easily demonstrated by the CCDF plot as shown in Fig. 5.

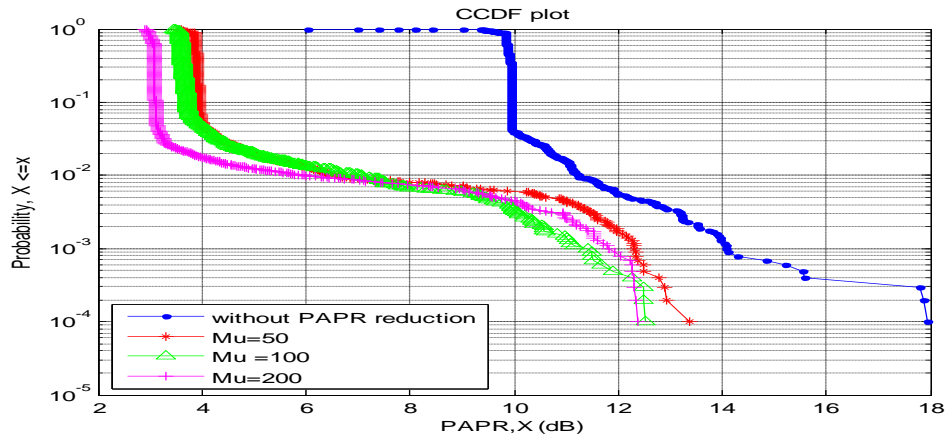


Fig. 5 CCDF Plot showing variation of PAPR for different values of  $\mu$

Fig. 6 shows the CCDF plot showing variation of PAPR for FBMC system with without reduction techniques. It is clear from the plot that  $\mu$  law companding provides better PAPR reduction capability than  $A$  law companding.

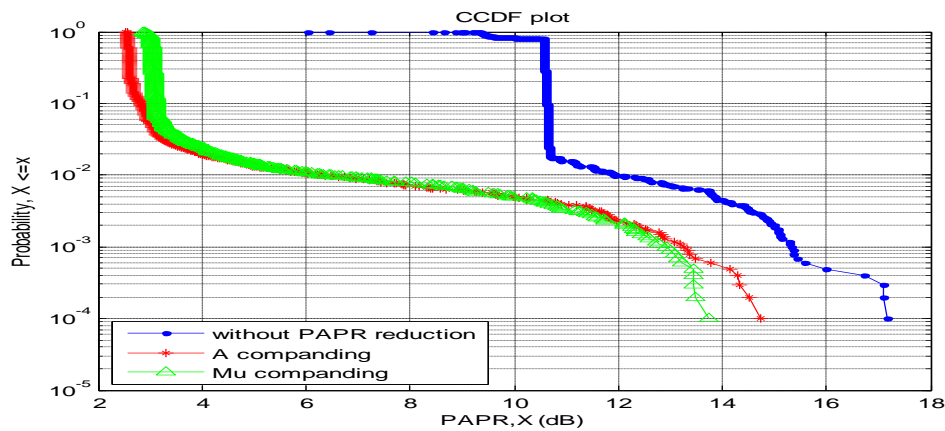


Fig. 6 CCDF Plot showing PAPR distribution

The numerical values obtained for FBMC system with and without reduction techniques have been summarized in Table 2.

Table 2. Numerical values of PAPR

PAPR of FBMC signal= 17.14 dB	
Reduction Technique	Measured PAPR (dB)
$A$ Companding	14.84
$\mu$ Companding	11.65



# International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 7, July 2015

The effectiveness of PAPR reduction technique also depends on BER performance. Here the performance of FBMC system with and without PAPR reduction techniques is evaluated in AWGN, Rayleigh or Riccian channel. Fig. 7 shows the BER performance of FBMC system with and without PAPR reduction technique under AWGN channel.

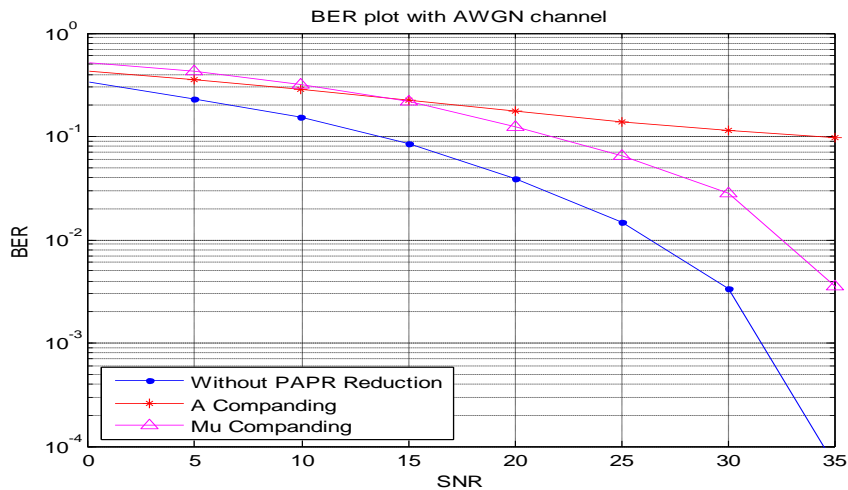


Figure 7. BER Performance under AWGN channel.

Also the BER performance of FBMC system is evaluated under Rayleigh channel and can be observed in Fig. 8. On comparing with AWGN channel, Rayleigh channels offers only less performance.

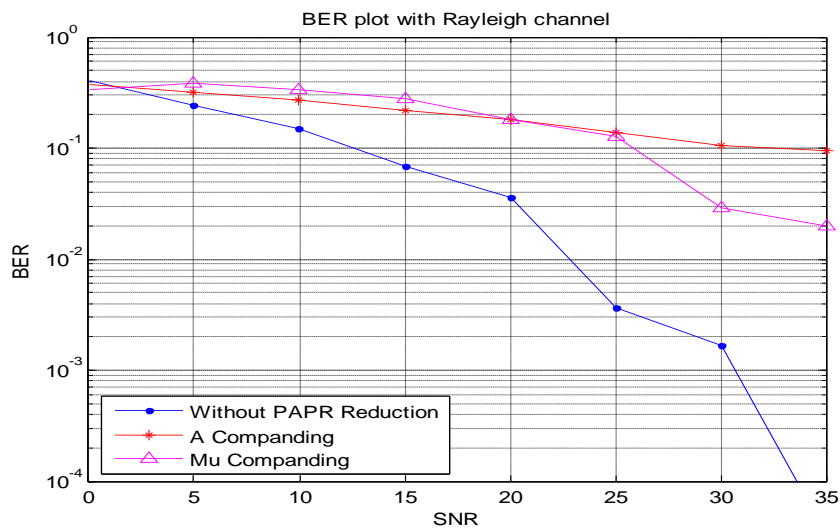


Fig. 8 BER Performance under Rayleigh channel

The BER performance of the FBMC system with and without PAPR reduction technique under Rician channel is also evaluated here and is given in Fig. 9.

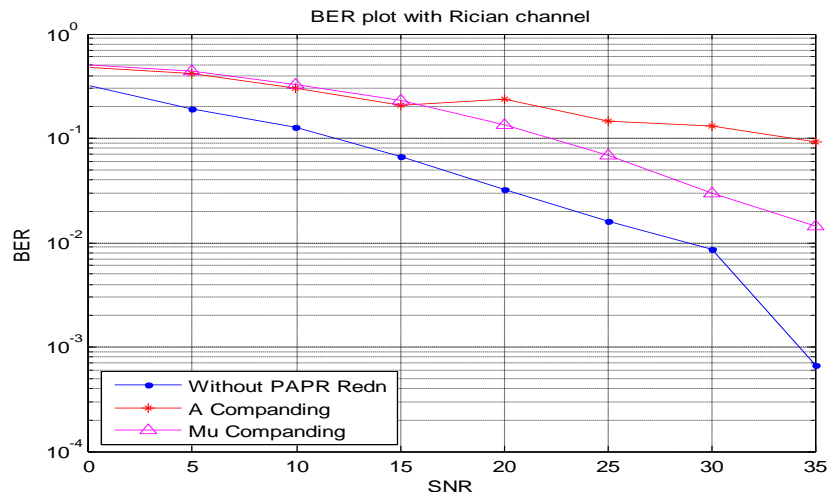


Fig. 9 BER Performance under Rician channel

Of all the BER graphs shown above, as the value of SNR increases, the BER values under AWGN or Rayleigh or Rician channel gets decreased. Also it is clear that AWGN channel provides better performance than Rayleigh and Rician channel. Under the evaluation in all the channels, the BER Performance is better for  $\mu$  law Companding compared to A-law Companding.

## IX.CONCLUSION

In this paper, an FBMC system is designed using cosine modulated filter banks. The Prototype filter for the filter banks are designed using low pass Kaiser Window method. All analysis and synthesis filters are derived from this single Prototype filter. The issue of PAPR is reduced using the techniques like A- law and  $\mu$ - law Companding techniques. These techniques are implemented in a Filter bank based system using Cosine modulated filter banks under different channels like AWGN, Rayleigh and Rician and their BER performance is evaluated. It is found that  $\mu$ - law Companding is a better technique in terms of better BER performance and PAPR reduction capability than A-law Companding.

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