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Statistical Study of Least Mean Square and Normalised Least Mean Square Algorithms

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ABSTRACT: This paper studies the stochastic behavior of the LMS and NLMS algorithms for a system identification framework when the input signal is a cyclostationary white Gaussian process. The input cyclostationary signal is simulated by a white Gaussian random process with sporadically time varying power. Mathematical models are derived for the mean and mean square-deviation (MSD) conduct of the adaptive weights with the input cyclostationarity. These models are also employed to the non-stationary system with a random walk variation of optimal weights. Monte Carlo simulations of the two algorithms provides strong support for the theory. Finally, the functioning of the two algorithms is compared for a variety of assumptions.

KEYWORDS: System Identification, Adaptive filter, LMS algorithms, NLMS algorithms, analysis.

I.INTRODUCTION

A significant aspect of adaptive filter functioning is the ability to track the time fluctuations of the fundamental signal statistics. The standard analytical model accepts the input signal is stationary. However, a non-stationary signal model can be allowed by a random walk model for the optimum weights. The form of the mean-square error performance surface remains unchanged while the surface actuates in the weight space overtime. This model puts up the conditions for the adaptive algorithm to get over the optimum solution. Alternatively, the input signal can be modeled as a cyclostationary process in many practical applications. In these cases, the form of the performance surface is periodic with the same period as the input autocorrelation matrix. This performance surface contortion affects the adaptive filter intersection and is freelance of changes in the optimum weights. This transient functioning surface deformation can be modeled by standard analytical models. However, it is still worthy to represent the adaptive functioning with non-stationary inputs.

This type of analysis is fundamentally absent from the technical literature. A begin analysis of the Least-Mean Square behavior for cyclostationary inputs examined only its convergence in the mean. The peculiar case of a pulsed variation of the input power and a linear combiner structure has latterly been analyzed for both LMS and NLMS algorithms. An analysis of the Least Mean Fourth (LMF) algorithm behavior for non-stationary inputs has been recently introduced. The analytical model deduced for the LMF behavior was effectual only for a specific form of the input auto-correlation matrix, and can't be easily lengthened to a general time-varying input statistics. Also, as the LMF weight update equation is a part of a higher power of the estimation error, the statistical supposals used are inevitably different from those desired for the analysis of the LMS and NLMS algorithms. Hence, the comparative analyses of the behaviors of the LMS and NLMS algorithms under cyclostationary inputs cannot be inferred from the analysis and new models must be plagiarized.

Adaptive solutions implying cyclostationary signals are used for many application areas. Particularly, communication, radar, and sonar systems extensively need such solutions, as several man-made signals come across in these areas have parameters that vary periodically with time. Studied adaptive beam forming algorithms for applications where inputs are cyclostationary. Projected an adaptive minimum variance equalizer that feats the cyclostationarity attributes of the inter-symbol and adjacent channel interferences. Objected, a gradient based adaptive beam forming algorithm that exploits the cyclostationarity of the input signal. Employed adaptive filtering to educe cyclostationary interference from speech signals. The reader is directed before a survey on the significance of cyclostationary signals in several areas, letting in

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communications, channel identification and equalization. Therefore, a statistical analysis of adaptive algorithms under cyclostationary inputs could have a significant effect on a wide variety of areas demanding cyclostationary processes.

The adaptive filter behavior analysis for cyclostationary inputs quite difficult because of the trouble of modeling the input cyclostationarity in a mathematically dealable way. Thus, comparatively simple models are required from which gives the algorithm behavior for inputs with time varying statistics.

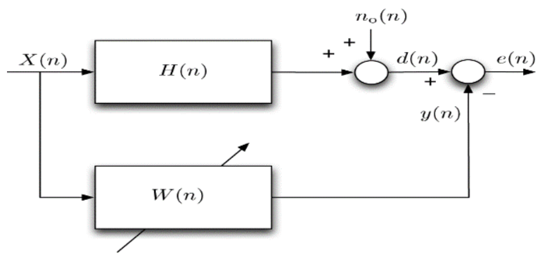

Fig.1. System identification framework.

This paper introduces random analyses of the Least Mean Square(LMS)and the Normalized Least Mean Square(NLMS) algorithms with particular cyclostationary input signals and an unknown system in a system identification framework. The input cyclostationary signal is patterned by a white Gaussian random action with periodically time-varying power. These models are used to examine the adaptive filter functioning for input signals with sinusoidal and pulsed power variations and a transversal filter structure. The study in[15],[16]is shown to be a specific case of the new results. The events of fast, moderate and slow power variations are considered. Mathematical models are deduced for the mean and mean-square-deviation(MSD) doings of the adaptive weights with these input cyclostationarities. These models are deduced via extension of well-known results for the LMS and NLMS algorithms to the cyclostationary case. These models are also implemented to the non- stationary channel with a random walk variation of the optimal weights. Simulation results appears excellent agreement with the theoretically prefigured behaviors, confirming the utility of the analytical model to study the adaptive filter behavior.

II. PROBLEM DEFINITION

A. System Identification and the Markov Channel Model

This paper deals with the system identification model given in fig 1. The N-dimensional input vector to the adaptive filter tap weights is given by

$$\mathbf{X(N)=[\ x(n),x(n-1),\ldots\ldots,\ x(n-N+1)]^T}$$

Where T indicates the transpose. The noticed noise n_0 (n) is considered as zero-mean white Gaussian noise with variance σ_0^2 and independed of X(n). The standard random walk model for unknown channel is given by

$$\mathbf{H(n-1)=H(n)+Q(n)}$$

Where H(n) is channel response and Q(n) is white Gaussian vector with zero mean and covariance matrix is $E[Q(n)Q^T(n)]=\sigma_q^2 I$, whre I is identity matrix. The vector sequence Q(n) is independent of both X(n) and n_0 (n).

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B .Independence Theory and Performance Measure

The independence theory of adaptive filtering assumes that weights are statically independent of input vector at time n. The use of this assumption considerably simplifies the statistical analysis of adaptive filter applications. The MSD is given by

$$\mathbf{MSD(n)= E[(W(n)\text{-}H(n))\,(W(n)\text{-}H(n))^{T}\,]=Tr[K_{vv}(n)]}$$

W(n) is the weight vector of adaptive filter at time n, Tr[B] is the trace of matrix B and

$$\mathbf{K_{VV}\,[n]=E[(W(n)\text{-}H(n))\,(W(n)\text{-}H(n))^{T}\,]}$$

C. Cyclostationary Input Signal Model

A Cyclostationary random process y(t) is defined as

$$\mathbf{E[y(t_1+T)]=E[y(t_1)]}$$

$$\mathbf{E[y(t_1+T)(t_2+T)]=E[y(t_1)y(t_2)]}$$

For all t_1 and t_2 and where T is the period. In the present paper it is assumed that X(n) is a zero mean white Gaussian vector with time-varying variance

$$\mathbf{R_x(n)= E[X(n)X^{T}\,(n)]\ =diag\ [}[\sigma_x^2(n), \sigma_x^2(n-1), \ldots\ldots, \sigma_x^2(n-N+1)]$$

Where σ_x^2 (n) is periodic with period T. Hence, X(n) is a discrete time wide sense cyclostationary process. Although this model is not general, it defines a non-trivial model. It allows the input to display a simple type of cyclostationarity which can be used to model more complex time varying statically properties of the inputs. More importantly, the behavior of the LMS and NLMS algorithms can be accurately analyzed for this inputs signal as will be shown subsequently. Two simple models for $\sigma_x^2(n)$ are considered here: a sinusoidal power time variation

$$\boldsymbol{\sigma_x^2(n) = \beta(1+\sin(\omega_0(n))}$$

For $\beta > 0, \omega_0 > 0$

The sinusoidal variation model can be used to study the algorithm behavior for different speeds of input power variation with bounded maximum power.

III STOCHASTIC ANALYSIS OF LMS ALGORITHM

The LMS weight update recursions is

$$\mathbf{W(n+1)=W(n)+\mu e(n)X(n)}$$

Where

$$\mathbf{e(n)=H^{T}\,(n)X(n)+n_o(n)\text{-}W^{T}(n)X(n)}$$

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And the μ is step-size. Defining the weight error vector **V(n)=W(n)-H(n)**

$$\mathbf{V(n+1)=\{ I-\mu X(n)X^T(n)\}V(n)+\mu n_o(n)X(n)-Q(n)}$$

A. LMS Mean Behavior

Ensemble averaging both sides and using independent theory yields

$$\mathbf{E[V(n+1)]=\{i-\mu R_x (n)\}e[V(n)]}$$

The formal solution is

$$\mathbf{E[V(n+1)]=\prod_{i=0}^{n-1}[I - \mu R_x (i)]V(0)}$$

B. LMS MSD Behavior

By using $\mathbf{e(n)=V^T(n)X(n)+n_0(n)}$, post multiplying by its transpose and averaging, yields

$$\mathbf{K_{vv}(n+1)=K_{vv}- \mu[R_x(n)K_{vv}(n)+K_{vv}(n)R_x(n)+ \mu^2E[X(n)X^T(n)K_{vv}(n)\ X(n)X^T(n)]+ \mu^2\sigma_0^2 R_x(n)+\sigma_q^2(n)I}$$

Where the independent theory and the above assumptions $n_0(n)$ and Q(n) are used. For zero-mean Gaussian X(n), the expectation is

$$\mathbf{E[X(n)X^T(n)K_{vv}(n)\ X(n)X^T(n)]=2R_xKvv(n)Rx(n)+Tr[R_x(n)K_{vv}(n)]R_x(n)}$$

In this we discussed about slow, fast and moderate power variations

a. Slow variations

The variations are considered as slow power variations when the length of the filter N is far less than the time period of cyclostationary input signal T. i.e N<<< T

b. *Fast variations*

The variations are considered as slow power variations when the length of the filter N is far greater than the time period of cyclostationary input signal T. i.e N>>>T

c. Moderate Speed Variations

The variations are considered as slow power variations when the length of the filter N is nearly equals to the time period of cyclostationary input signal T. i.e **N ~= T**

IV. STOCHASTIC ANALYSIS OF THE NLMS ALGORITHM

The NLMS weight update recursion is

$$\mathbf{W1(N+1)=W1(N)+}\frac{\mu e_1(n)X(n)}{X^T(n)X(n)}$$

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Where**e_1(n)=H^T(n)X(n)+n_0(n)-w_1^T(n)X(n)**

$$V_1(n+1) = \left\{ \mathbf{I} - \mu \frac{X(n)X^{\mathrm{T}}(n)}{X^{\mathrm{T}}(n)X(n)} \right\} V_1(n) + \mu \frac{n_o(n)X(n)}{X^{\mathrm{T}}(n)X(n)} - Q(n).$$

A. NLMS Mean Behavior

Using independence theory and ensemble averaging on both sides of above equation we get

$$E[V_1(n+1)] = \left\{ \mathbf{I} - \mu E\left[\frac{X(n)X^{\mathrm{T}}(n)}{X^{\mathrm{T}}(n)X(n)} \right] \right\} E[V_1(n)]$$

$$= \left\{ \mathbf{I} - \frac{\mu}{\mathrm{Tr}[\mathbf{R}_X(n)]} \mathbf{R}_X(n) \right\} E[V_1(n)]$$

B. NLMS MSD Behavior

Proceeding in similar manner to the least mean square MSD analysis gives the matrix recursion

$$\begin{aligned}\mathbf{K}_{V_1V_1}(n+1) = E\Bigg\{ &\left[\mathbf{I} - \mu \frac{X(n)X^{\mathrm{T}}(n)}{X^{\mathrm{T}}(n)X(n)} \right] \mathbf{K}_{V_1V_1}(n) \\ &\times \left[\mathbf{I} - \mu \frac{X(n)X^{\mathrm{T}}(n)}{X^{\mathrm{T}}(n)X(n)} \right] \Bigg\} \\ &+ \mu^2 E\left(n_o^2(n)\right) E\left[\frac{X(n)X^{\mathrm{T}}(n)}{[X^{\mathrm{T}}(n)X(n)]^2} \right] \\ &+ \sigma_q^2(n)\mathbf{I}.\end{aligned}$$

a. Slow variations

The variations are considered as slow power variations when the length of the filter N is far less than the time period of cyclostationary input signal T. i.e N<<< T

b. *Fast variations*

The variations are considered as slow power variations when the length of the filter N is far greater than the time period of cyclostationary input signal T. i.e N>>>T

c. *Moderate Speed Variations*

The variations are considered as slow power variations when the length of the filter N is nearly equals to the time period of cyclostationary input signal T. i.e **N ~= T**

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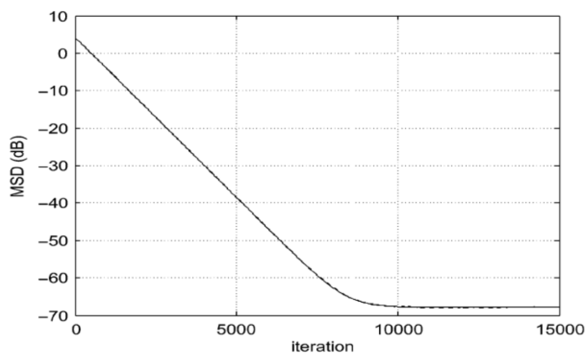
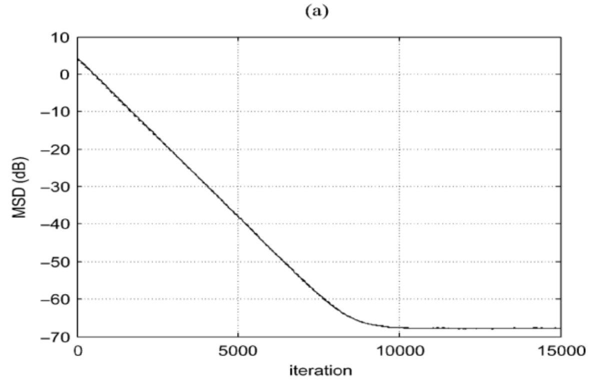
V. MONTE CARLO SIMULATIONS FOR COMPARISION WITH THEORITICAL VALUES

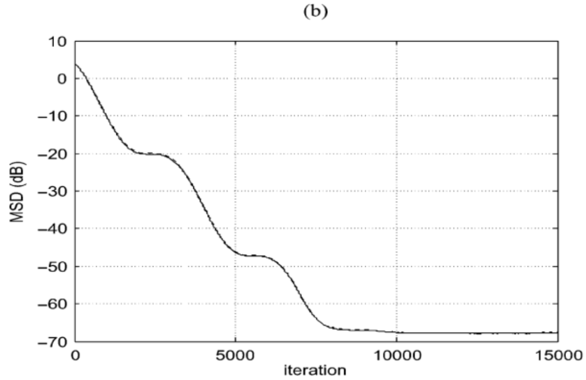
The Monte Carlo simulations of the mean square deviation values are compared with the theoretical mean square deviation values for the sinusoidal power variations for both fixed and time varying channels are presented below.



(a)

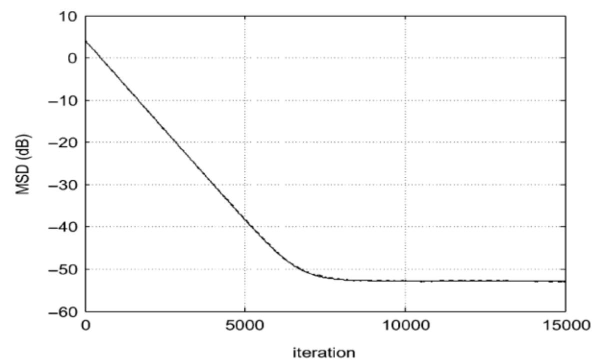


(b)

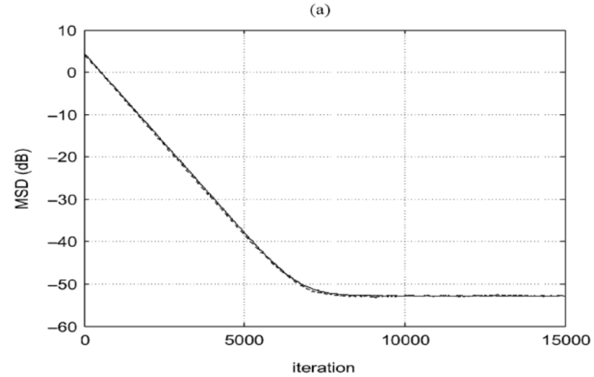


(c)

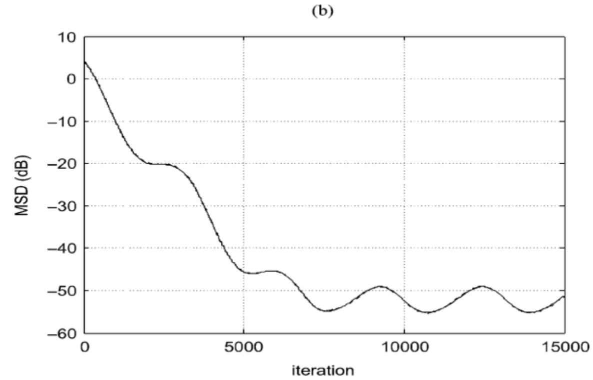
Fig.: For LMS fixed channel (a) fast variations, (b) moderate variations and (c) Slow variations



(a)



(b)



(c)

Fig.: For LMS time-varying channel (a) fast variations, (b) moderate variations and (c) Slow variations

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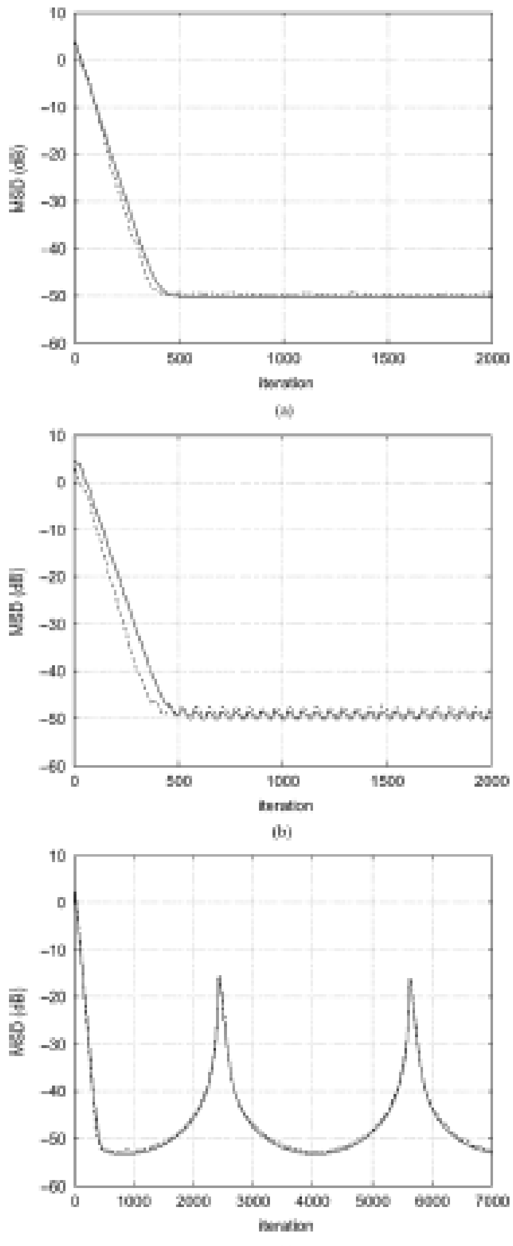

Fig.: For NLMS fixed channel (a) fast variations, (b) moderate variations and (c) Slow variations

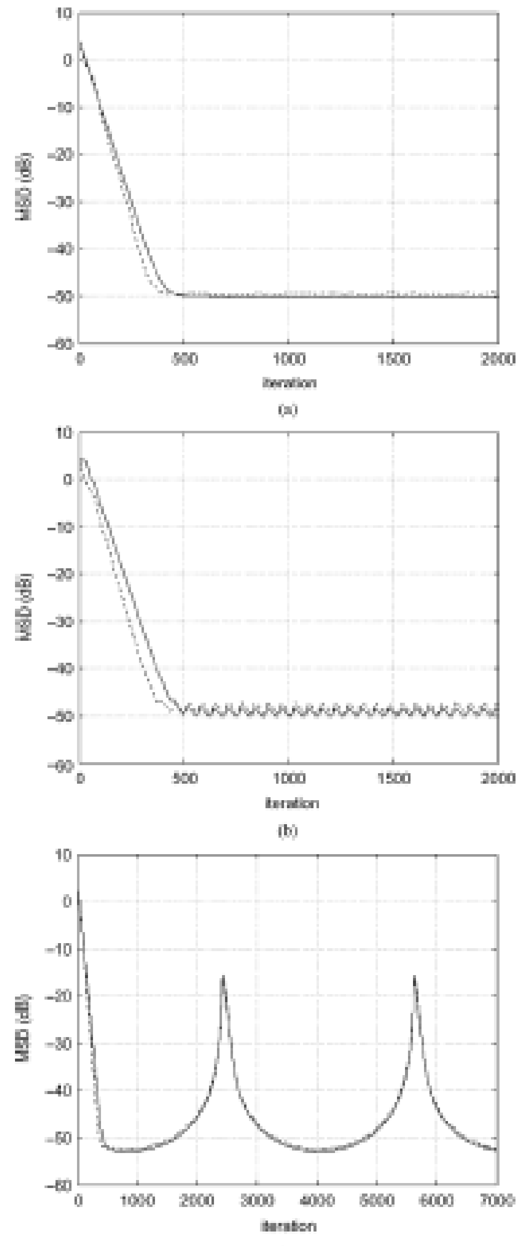

Fig.: For NLMS time-varying channel (a) fast variations, (b) moderate variations and (c) Slow variations

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VI. COMPARISION BETWEEN LMS AND NLMS ALGORITHMS

S. N0	SIMILARITIES	DIFFERENCES
1.	For periodic input power variations, the mean square deviation converges to a period sequence with period same as input.	For slow input power variations, the transient NLMS MSD doesn't depend on rate of variation of input power while LMS MSD does.
2.	Neither transient nor steady-state performance is affected by rapid input power variations.	For a fixed plant with slow input power variations, the steady-state LMS MSD has negligible time-variations while NLMS MSD has significant time-variations**.**

VII. CONCLUSIONS

The study of adaptive filters with non-stationary inputs is a very complex subject. In this the performances of the two algorithms are compared and the NLMS algorithm is chosen on the basis of stability, transient response and steady-state behavior. The results of this paper suggest that the NLMS algorithm (with regularization) can be used effectively with cyclostationary inputs such as voice data. Indeed, this is precisely the behavior that is observed, for instance, with most voice-band echo cancellers.

REFERENCES

[1]. S.Haykin, *Adaptive Filter Theory*, 4theded. Englewood Cliffs, NJ, USA:Prentice-Hall,2002.
[2]. A.H.Sayed, *Fundamentals of Adaptive Filtering*. NewYork, NY, USA:Wiley-Interscience,2003.
[3]. B.Widrow,J.M.McCool,M.G.Larimore,andC.R.Johnson,Jr,"Stationaryandnon-stationarylearning characteristics of the LMS adaptive filter," *Proc.IEEE*,vol.64,no.8,pp.1151–1162,Aug.1976
[4]. S.Marcos and O.Macchi ,"Tracking capability of the least mean square algorithm Application to an asynchronous echo canceller ," *IEEE Trans.Acoust., Speech, Signal Process.*, vol.ASSP-35, no.11,pp. 1570–1578,Nov.1987.
[5]. E.Eweda," Comparison of RLS, LMS ,and signal algorithms for tracking randomly time-varying channels,"*IEEETrans.SignalProcess*.vol.42,no.11,pp.2937–2944,Nov. 1994.
[6]. J.C.M Bermudez and N.J. Bershad, "Transient and tracking performance analysis of the quantized LMS algorithm for time-varying system identification," *IEEE Trans.Signal Process.*, vol.44, no.8, pp.1990–1997, Aug.1996.