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Design and Optimization of Fractional Order Proportional Integral Derivative Controllers for Hard- Disk Drive Servo Systems

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ABSTRACT: Hard-disk drives (HDD's) are widely used data storage medium for computers and many other data processing devices. The servo mechanism in HDD plays an important role in high performance requirements of HDD's. Gain variations present in hard ware components of HDD's affects the consistent track following performances. So the servo system needs to improve in track following and robustness performances. Integer order controllers were not found robust against gain variations. In this brief fractional order proportional integral derivative (FO-PID) controllers are proposed and designed for a hard disk drive model .In order to achieve desired performances, certain tuning constraints are imposed for the design purpose. Simulation results are provided to show the benefits of methods presented and also to compare the performances with integer order PID (IO-PID) controller. From the results it can be observed that the FO-PID controller outperforms the IO-PID controller.

KEYWORDS: Fractional order proportional integral derivative (FO-PID), Hard- disk drive (HDD), Integer order proportional integral derivative (IO-PID), Robustness, Servo system

I. INTRODUCTION

In the modern era of digital technology Hard -Disk Drives (HDD) are the most preferred data storage device due to its larger data storage capacity, high data transfer rate, high areal density [1]. In HDD's data or information is stored on several rotating disks using a read/write head assembly. The servo mechanism in HDD is a control system that provides high precision control of output of the system [2]. The servo system consists of a Voice Coil Motor (VCM) actuator which is used to position the read/write head form one track to desired track (track seeking) and to follow the desired track (track following) [3]. Fig.1 shows a typical HDD system with VCM actuator.

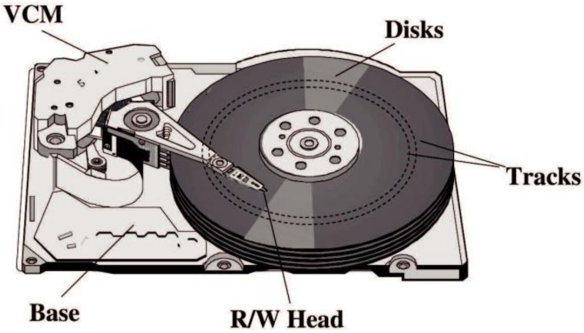

Fig.1 HDD system with VCM actuator

To meet the increasing demand for high speed data transfer and larger storage capacity, the servo system needs to have better track following and robustness performances. To solve the problem, several integer order controllers PID controllers [4], are proposed for the track following mode. But these were not found robust against loop gain variations. Then several control approaches such as Linear Quadratic Gaussian (LQG) control, adaptive control, H_∞ control, robust control have been discussed in [5]-[8]. In [9], [10] automatic gain control methods were developed to minimize the effect of loop gain variations. But gain variations resulting from environmental changes cannot be handled. More control approaches need to be conducted to improve the performances of HDD's under gain variations.

Recently, fractional order (FO) controllers based on fractional order calculus (FOC) is becoming a hot topic in control engineering and technology [11], [12].In the past the FO controllers were not used because of unfamiliarity with the use of FO parameters, and the absence of necessary computational power. But the fractional order differential equations could well define the dynamic processes and so the computational progresses can be made easier. It is shown that with the introduction of fractional order parameters, it is possible to improve the performance of integer order controllers [13]. Among the FO controllers, the FO- PID control is the most widely used technique in real industries [14].

In this brief FO- PID controller is proposed and designed for an HDD model with VCM actuator. The three design specifications to be met are the phase margin, gain cross over frequency and gain variation robustness. The proposed method uses Astrom and Hagglund PID [16] tuning method to obtain integer order parameters K_p, K_i and K_d. The fractional order parameters λ and μ are solved following the three tuning specifications. The parameters are then optimized using ISE criterion.

The rest of this brief is organized as follows. In section II the HDD- actuator system modelling is presented. The basic concepts of fractional order calculus and control are given in section III. In section IV the tuning process for FO-PID controllers is given. In section V the implementation details of FOPID controllers on the HDD model are given. In section VI simulation results are shown for the validation of the method discussed. Conclusions are given in section VII.

II. DYNAMIC MODELLING OF HDD ACTUATOR

VCM actuator is the torque producing component in HDD. The most commonly used actuator is the rotary actuator. For the movement of the actuator, it needs to be powered by an amplifier. Current amplifiers are commonly used due to its high impedance. If *u(s)* is the input to the current amplifier and *y(s)* is the position output of read/write head in form of track widths, the transfer of HDD actuator with current amplifier can be represented as:

$$\frac{y(s)}{u(s)} = \frac{k_a k_y}{s^2} \qquad (1)$$

However, the actuator structures are not perfectly rigid and contain several flexible modes that can cause resonances and vibrations. So the effects of such modes cannot be neglected. More accurate model is obtained by considering the resonance effects. The overall accurate model is given as:

$$P(s) = \frac{k_a k_y}{s^2} \frac{(a\omega_n s + b\omega_n{}^2)}{s^2 + 2\zeta\omega_n s + \omega_n{}^2} \qquad (2)$$

where, k_a is the acceleration constant, k_y is the position measurement gain. ω_n is the resonant frequency, a and b are the resonant coupling coefficients. The parameters for the plant model are given in Table I.

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TABLE I

PLANT MODEL PARAMETERS

Parameter	Description	Value
J	Moment of Inertia	0.2
k_t	Torque constant	20
k_y	Position measurement gain	10000
ω_n	Resonant Frequency	80
a	Coupling coefficient	0.0032
b	Coupling coefficient	0.25

Then the plant model described in (2) can be written as

$$P(s) = \frac{1.287e8s+8.085e8}{s^4+502.7s^3+2.527e5s^2} \qquad 3$$

III. FUNDAMENTALS OF FRACTIONAL ORDER CALCULUS AND CONTROL

A. Fractional order calculus

The fractional order calculus (FOC) constitutes the branch of mathematics dealing with differentiation and integration under an arbitrary order of the operation. FOC has wide spread applications in the fields of electronics, robotics, control theory and signal processing etc. There are different definitions of fractional order differentiations and integrations. Well established definition is the Caputo definition [15]. The definitions will be summarized as:

Caputo's definition for fractional order integration is:

$$ {}_0\mathrm{D}_t^{\gamma} y(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{\gamma+1}} d\tau \qquad 4$$

where, γ<0

Similarly definition for fractional order differentiation is

$$ {}_0\mathrm{D}_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau \qquad 5$$

where, α=m+ γ, m is an integer.

B. Fractional order control

The integro differential equation for FO- PID controller is given by:

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t) \qquad 6$$

Applying Laplace transform

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$$C(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} \qquad 7$$

When $\lambda = ⍰ = 1$, an IO- PID controller of the form (8) is obtained.

$$C_1(s) = K_p + \frac{K_i}{s} + K_d s \qquad 8$$

When λ=1, μ=0 and μ=1, λ=0 respectively, conventional PI and PD controllers can be obtained.

IV. TUNING OF FRACTIONAL ORDER PID CONTROLLER

In this section the tuning methods for FO-PID controllers are discussed. The design specifications to be met are given as follows:

1. Phase margin specification

$$< (C(j\omega_{gc})P(j\omega_{gc}) = \varphi_m - \pi \qquad 9$$

where φ_m is the desire phase margin and ω_{gc} is the gain cross over frequency.

2. Gain cross over frequency specifications

$$\left|\left(C(j\omega_{gc})P(j\omega_{gc})\right)\right| = 1 \qquad 10$$

3. Robustness to gain variations

$$\frac{d<\left(C(j\omega_{gc})P(j\omega_{gc})\right)}{d\omega_{gc}} = 0 \qquad 11$$

i.e., the derivative of the phase of the open-loop system with respect to the frequency is forced to be zero at the gain crossover frequency so that the closed-loop system is robust to gain variations, and therefore the time responses of the systems are almost invariant.

We assume that the gain cross over frequency ω_{gc} is provided by the designer and the desired phase margin φ_m is also specified. Based on the tuning specifications discussed above, the following equations can be obtained. For the controller of the form (7) and the plant model described in (3), we can write

From specification 1

$$< \left(C(j\omega_{gc})P(j\omega_{gc})\right) = tan^{-1}\frac{B_1}{A_1} + < P(j\omega_{gc}) = \varphi_m - \pi \qquad 12$$

where,

$$A_1 = k_p + k_i \omega_{gc}{}^{-\lambda} cos\, {}^{\lambda\pi}/_2 + k_d \omega_{gc}{}^{\mu} cos\, {}^{\mu\pi}/_2 \qquad 13$$

$$B_1 = -k_i \omega_{gc}{}^{-\lambda} sin\, {}^{\lambda\pi}/_2 + k_d \omega_{gc}{}^{\mu} sin\, {}^{\mu\pi}/_2 \qquad 14$$

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$$< P(j\omega_{gc}) = -\pi - tan^{-1}\left(\frac{1.989436789\,e{-3}\omega_{gc}}{(1-3.957858736\,e{-6}\omega_{gc}{}^{2})}\right) + tan^{-1}(0.159154943\omega_{gc})$$

Here φ_m =71 ° and ω_{gc} =217 rad/sec.
From specification 2

$$\sqrt{A_1{}^2 + B_1{}^2}.\,|P(j\omega_{gc}| = 1 \qquad 16$$

where,

$$|P(j\omega_{gc})| = \frac{3199.999\sqrt{1+(0.159154943\,\omega_{gc})^2}}{\omega_{gc}{}^2\sqrt{(1-3.95785836\,e{-6}\omega_{gc}{}^2)^2+(1.9894368\,e{-3}\omega_{gc})^2}} \qquad 17$$

From specification 3

$$\frac{d}{d\omega_{gc}}\left(tan^{-1}\frac{B_1}{A_1}\right) = -\frac{d}{d\omega_{gc}}(< (P(j\omega_{gc})) \qquad 18$$

The tuning procedures of the FO- PID controllers are summarized as follows:

1. Obtain the values K_p, K_i and K_d from Astrom and Hagglund rule
2. Obtain λ and μ by solving equations (12),(16) and (18)
3. Optimize all parameters with newly obtained values of λ and μ

The objective function is to minimize:

$$ISE = \int_0^{\infty} e^2(t)dt \qquad 19$$

V. FRACTIONAL ORDER PID CONTROLLER APPLIED TO THE HDD MODEL

In this section, the application of proposed method on the HDD model under consideration is given.

Step 1: Determination of PID controller parameters

The initial step is to determine the integer order parameters. Several methods have been discussed in literatures for tuning integer order controllers. Here, the values of integer order parameters are determined from Astrom and Hagglund [16] method, for specified phase margin. Let the transfer function of the IO-PID controller thus obtained be of the form:

$$C_1(s) = 0.33335 + \frac{14.6947}{s} + 0.00189s \qquad 20$$

Where K_p =0.33335, K_i=14.6947, and K_d =0.00189

Step 2: Solving for λ and μ.

The FO parameters λ and μ are solved using ‘fsolve’ function in MATLAB. The transfer function of FO- PID controller is obtained as:

$$C_2(s) = 0.33335 + 14.6947s^{-0.93161} + 0.00189s^{0.83581} \qquad 21$$

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Where the values of λ and μ are 0.93161 and 0.83581 respectively.

Step 3: Optimization of all parameters with new values of λ and μ

Here the values of parameters obtained from step 1 and 2 are optimized to obtain new values of FO- PID parameters. The transfer function of FO- PID controller obtained in the final form as:

$$C(s) = 0.33335 + 14.653s^{-0.9322} + 0.001884s^{0.83572} \quad 22$$

Where the values of K_p, K_i, λ, K_d and μ are 0.33335, 14.653, 0.9322, 0.001884, 0.83572 respectively.

VI. RESULTS AND DISCUSSIONS

In this section the robustnes performances under gain variations of FO-PID controller are evaluated and compared with IO-PID controller.The open loop Bode responses are plotted with $\pm$ 10 % gain variations from the original case. Fig. 3 shows the Open loop Bode responses under gain variations of FO-PID controller.

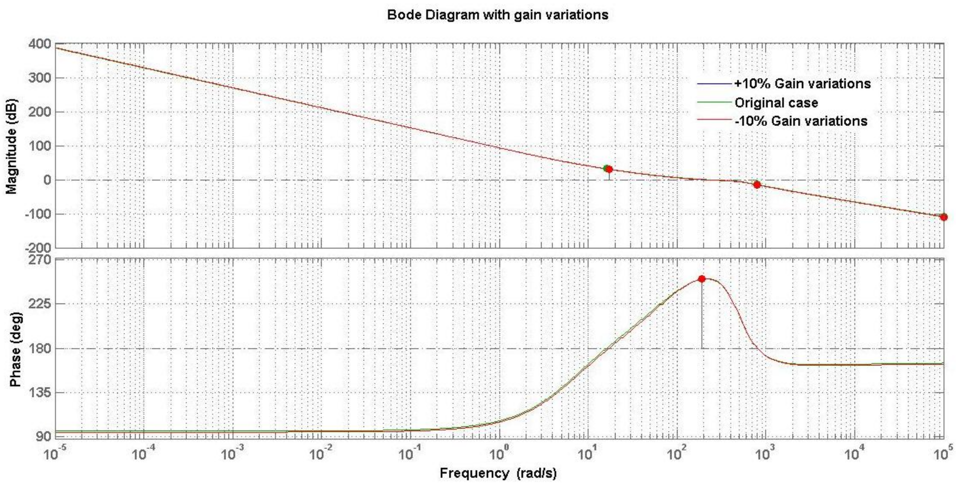

Fig.3 Open loop Bode plot under gain variations with FO-PID controller

In fig. 3, the green line shows the original controller described by (22) .Blue and red line represents +10% and -10 % gain variations from the original. It can be seen that the FO-PID controller well maintained the desired phase margin and gain cross over frequency specifications under gain variations. The phase of the open loop system is with flat phase feature, which obtains robustness to gain variations.

Fig. 4 shows the Open loop Bode responses under gain variations of IO-PID controller described by (20).

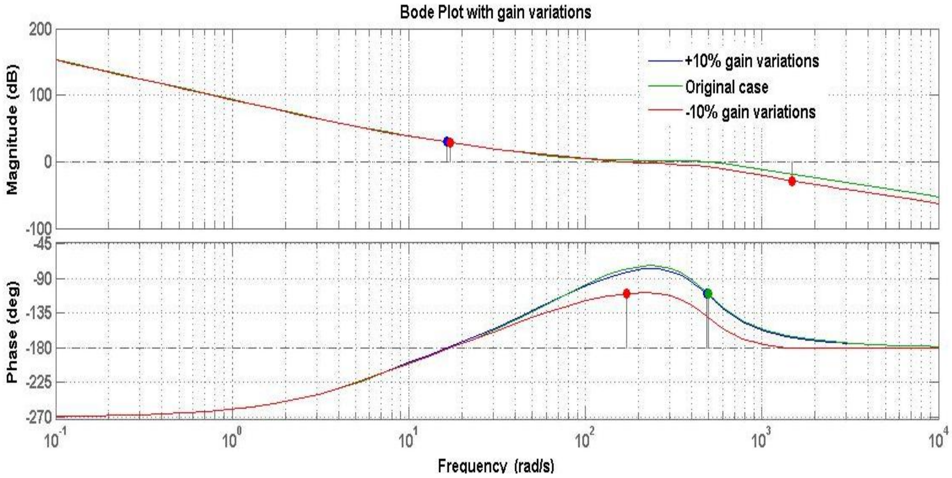

Fig.4 Open loop Bode plot under gain variations with IO-PID controller

From fig.4 we can see that phase margin and gain cross over frequency specifications are not well maintained for the IO-PID controller. So the system designed with IO-PID controller is not so robust against gain variations.

Next,the track following performances of both the controllers is plotted with $\pm$ 10% gain variations from the original value. Fig.5 shows the track following performances under gain variations with FO-PID controllers.

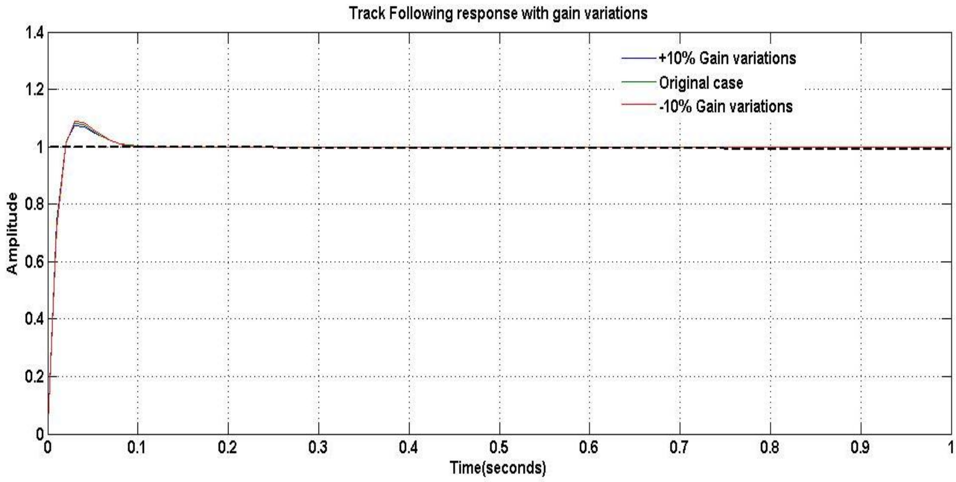

Fig.5 Track following under gain variations with FO-PID controller

In fig. 5, the green line shows the original controller described by (22) .Blue and red line represents +10% and -10 % gain variations from the original.

Next, the track following performances under gain variations of the system with IO-PID controller are plotted and are shown in fig. 6.

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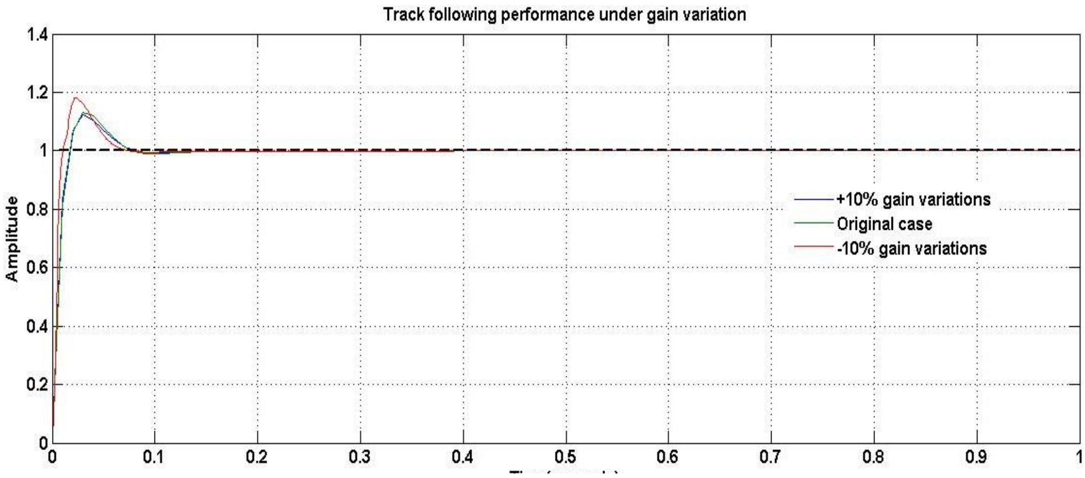

Fig.6 Track following under gain variations with IO-PID controller

From the figure,it is clear that the response characteristics are not uniform in the case of IO-PID controller.The detailed analysis of track following performance characteristics can be inferred from Table II.

TABLE II

TRACK FOLLOWING RESPONSE CHARACTERISTICS UNDER GAIN VARIATIONS

Controller type	90% gain variations		100% gain variations		110% gain variations	
	Peak overshoot %	Settling time (sec)	Peak overshoot %	Settling time (sec)	Peak overshoot %	Settling time (sec)
IO-PID	18.1390	0.0601	13.2063	0.0677	12.2304	0.0677
FO-PID	8.8342	0.0679	8.1719	0.0677	8.0206	0.0670

From the table we can infer that the time response characteristics with the FO-PID controller are almost invariant under gain variations. Isodamping property is exhibited with designed FO-PID controller. More over the response characteristics vary in the range of 37 % to 67 % in the case of IO-PID controller. So the FO-PID controller is more robust to gain variations than IO-PID controller. Therefore consistent track following performance can be achieved under gain variations, by the system when designed with FO-PID controller.

VII. CONCLUSIONS

In this brief Fractional Order- PID controller is proposed and designed for an HDD model. Three tuning constraints are imposed to achieve desired phase margin, gain cross over frequency and robustness. Astrom and Hagglund method was adopted to obtain integer order parameters. The fractional order parameters λ and μ are determined by solving from the tuning specifications. Finally the FO-PID controller parameters are optimized based on ISE criterion. Simulation was done in order to compare and validate the performances of FO-PID controller under gain variations with that of the IO-PID controller. The results showed that the FO-PID controller outperformed the IO-PID controller. Moreover the track following performances could be improved with the FO-PID controller in the HDD servo system.

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