ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 2, February 2014

Development of a Novel ECG signal Denoising System Using Extended Kalman Filter

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ABSTRACT - ECG signal plays a crucial role in diagnosis of a Varity of diseases. At the time of diagnosis the proper information from the ECG signals helps to make a proper and efficient diagnosis for the patient. Most often it is found that treatment of the patient suffers due to improper information of ECG signals. The cause behind this problem is the noise added in the ECG signals at the time of signal acquisition. Hence to overcome this problem efficient denoising of ECG signals is required. This paper presents efficient denoising scheme for electrocardiogram (ECG) signals based on extended Kalman filter (EKF) structure. The basic idea is to overcome the disadvantages of conventional techniques like median filter by utilizing the adaptive nature of EKF structure. For the comparative analysis this paper deploys three important parameters; mean square error (MSE), Peak signal to noise ratio (PSNR), and most importantly RR interval estimation. On the basis of the three parameters a comparative analysis has been presented to explore efficient denoising capability of EKF over median filter. The results obtained indicated that EKF provides very less MSE and very high PSNR as compare to median filter. On the other side the estimated RR interval obtained using EKF is the closest match with original signal RR intervals, while median filter provides so many RR intervals, which are not even presents in the original signal.

Keywords - Denoising, ECG dynamical model (EDM), extended Kalman filter (EKF), hidden state variables, lossy compression.

I. INTRODUCTION

ECG recordings obtained by a noninvasive technique is a harmless, safe, and quick method of cardiovascular diagnosis. The accuracy and content of information extracted from a recording require proper characterization of waveform morphologies, which, in turn, require the preservation of the phase and amplitude important clinical features and high attenuation of noise. ECG signals are usually corrupted with unwanted interference such as muscle noise, electrode artifacts, line noise, and respiration. Several techniques have been proposed to extract the ECG components contaminated with the background noise and allow the measurement of subtle features in the ECG signal.

One of the common approaches is the adaptive filter architecture, which has been used for the noise cancellation of ECGs containing baseline wander, electromyogram (EMG) noise, and motion artifacts [2], [3]. Statistical techniques such as principal component analysis [4], independent component analysis [5], [6], and neural networks [7] have also been used to extract a noise-free signal from the noisy ECG. Over the past several years, methods based on the wavelet transform (WT) have also received a great deal of attention for the denoising of signals that possess multi resolution characteristics such as the ECG [8]–[13].

On the other hand, a synthetic model has been proposed for generating artificial ECGs, which has unified the morphology and pulse timing in a single nonlinear dynamic model [16]. Concerning the simplicity and flexibility of this model, it can be easily used as a base for ECG processing, as demonstrated by Clifford *et al.* [17], where the use of the model to filter, compress, and classify the ECG was first proposed. This approach was based on the least squares error (LSE) optimization. The model may be further used in dynamic adaptive filter, such as the *Kalman Filter (KF).* Sameni *et al.* proposed the use of a KF framework to update the model on a beat-to-beat basis in order to filter noisy ECGs [18]–[21]. The polar form of the dynamical equations was also used for Kalman-based ECG denoising [20].

II. DYNAMICAL SYSTEM AND EXTENDED KALMAN FILTER

Let us consider nonlinear dynamical systems of the following form:

$$x_{t+1} = f_t(x_t) + w_t \qquad \ldots(1)$$

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International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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$y_t = g_t(x_t) + v_t$ …(2)

Where, $x_t : \varepsilon R^k$ and $y_t : \varepsilon R^p$ are the state and output, respectively, at time t. The noise terms, w_t and v_t, are zero-mean normally-distributed random variables with covariance matrices Q_t and R_t, respectively. In general, the state update function $f_t : R^k \rightarrow R^k$, the state-to-output mapping function $g_t : R^k \rightarrow R^p$, and the covariance matrices of the noise variables can all vary with time. The initial state, x_1, is normally-distributed with mean π_1 and variance V_1.

As in [3], we assume that the parameters of the nonlinear dynamical system, namely f_t, g_t, Q_t, R_t, π_1, and V_1 are known. Whereas the outputs are observed, the state and noise variables are hidden. The Extended Kalman approach approximates the nonlinear system described by (1) and (2) with a linear system using first-order Taylor approximations

$$f_t(x_t) \approx f_t(x_t^t) + A_t(x_t - x_t^t) \quad ...(3)$$
$$g_t(x_t) \approx g_t(x_t^{t-1}) + C_t(x_t - x_t^{t-1}) \quad ...(4)$$

Where,

$$A_t = \frac{\partial f_t(x)}{\partial x} |x = x_t^t$$
$$C_t = \frac{\partial g_t(x)}{\partial x} |x = x_t^{t-1}$$

Note that f_t is linearized around x_t^t, while g_t is linearized around x_t^{t-1} because g_t is involved in generating the output y_t.

Substituting (3) and (4) into (1) and (2), we obtain a linear time-varying system with input-like terms

$$x_{t+1} = (A_t X_t + d_t) + w_t \quad …(5)$$
$$y_t = (C_t X_t + e_t) + v_t \quad …(6)$$

Where,

$$d_t = f_t(x_t^t) - A_t x_t^t \quad …(7)$$
$$e_t = g_t(x_t^{t-1}) - C_t x_t^{t-1} \quad …(8)$$

The goal is to determine $P(x_t |\{y\}_1^t)$ and $P(x_t |\{y\}_1^T)$ for t = 1;………T. These are the solutions to the filtering and smoothing problems, respectively. Both distributions are normally-distributed for the linearized system, so it suffices to find the mean and variance of each distribution.

We will use the same notation as in [26]. We will simply state the final result or omit certain steps for derivations that correspond exactly to those found in [26].

Forward Recursions: Filtering

For the linearized system described by (5) and (6), $P(x_t |\{y\}_1^t)$ is a normal distribution. We seek its mean x_t^t and varianceV_t^t.

$$\log P(x_t |\{y\}_1^t) = \log P(y_t | x_t) + \log P(x_t | \{y_t\}_1^{t-1}) + \cdots$$
$$= -\frac{1}{2}(y_t - C_t x_t - e_t)' R_t^{-1}(y_t - C_t x_t - e_t) - \frac{1}{2}(x_t - x_t^{t-1})'(V_1^{t-1})^{-1}(x_t - x_t^{t-1}) + \cdots$$
$$= -\frac{1}{2} x_t' (C_t' R_t^{-1} C_t + (V_1^{t-1})^{-1} x_t + x_t' (C_t' R_t^{-1} y_t - C_t' R_t^{-1} e_t + (V_t^{t-1})^{-1} x_t^{t-1}) + \cdots \quad …(9)$$

Using the Matrix Inversion Lemma,

$$V_t^t = (C_t' R_t^{-1} C_t + (V_t^{t-1})^{-1}$$
$$= V_t^{t-1} - K_t C_t V_t^{t-1} \quad …(10)$$

Where, $K_t = V_t^{t-1} C_t^{t-1} (R_t + V_t^{t-1} C_t^{t-1})^{-1}$

To find the time update for the variance, we use the fact that Ax_{t-1} and w_{t-1} are independent and treat d_{t-1} as a constant

$$V_t^{t-1} = \mathrm{Var}(A_{t-1} x_{t-1} + d_{t-1} |\{y\}_1^{t-1}) + \quad \mathrm{Var}(d_{t-1} |\{y\}_1^{t-1})$$
$$= A_{t-1} V_{t-1}^{t-1} V A_{t-1}' + Q_{t-1} \quad …(12)$$

As in [3], we will use the matrix identity

$$(I - (A + B)^{-1} A) B^{-1} = (A + B)^{-1} \quad …(13)$$

ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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Vol. 3, Issue 2, February 2014

Applying (13) and substituting the definition of e_t from (8),

$$x_t^t = V_t^t(C_t' R_t^{-1}(y_t - e_t) + (V_t^{t-1})^{-1} x_t^t)$$
$$= K_t(y_t - e_t) + (I - K_t C_t)x_t^{t-1}$$
$$= x_t^{t-1} + K_t(y_t - g_t)(x_t^{t-1}). \quad \ldots(14)$$

The time update for the mean can be found by conditioning on x_{t-1} and substituting the definition of d_t from (7)

$$x_t^{t-1} = E_{xt-1}(E(x_t|x_{t-1,}\{y\}_1^{t-1})|\{y\}_1^{t-1})$$
$$= E_{xt-1}(A_{t-1}x_{t-1} + d_{t-1}|\{y\}_1^{t-1})$$
$$= A_{t-1}x_{t-1}^{t-1} + d_{t-1}$$
$$= f_{t-1}(x_{t-1}^{t-1}). \quad \ldots(15)$$

The recursions start with x_t^0 and $x_t^{t-1} = V_1$. Equations (10), (11), (12), (14), and (15) together form the Extended Kalman filter forward recursions, as shown in [24], [25].

The time update equations can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems as shown below in Figure 2-1.

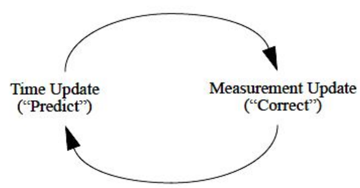

Figure (2.1) the ongoing discrete Kalman filter cycle.

The time update projects the current state estimate ahead in time. The measurement update adjusts the projected estimate by an actual measurement at that time. The specific equations for the time and measurement updates are presented below in Table 1 and Table 2.

Table 1: Discrete Kalman filter time update equations.

$$x_{t+1} = (A_t X_t + d_t)$$
$$\log P(x_t|\{y\}_1^t) = \log P(y_t|x_t) + \log P(x_t|\{y_t\}_1^{t-1}) + \cdots$$

Table 2: Discrete Kalman filter measurement update equations.

$$K_t = V_t^{t-1} C_t^{t-1}(R_t + V_t^{t-1} C_t^{t-1})^{-1}$$
$$x_t^t = x_t^{t-1} + K_t(y_t - g_t)(x_t^{t-1}).$$

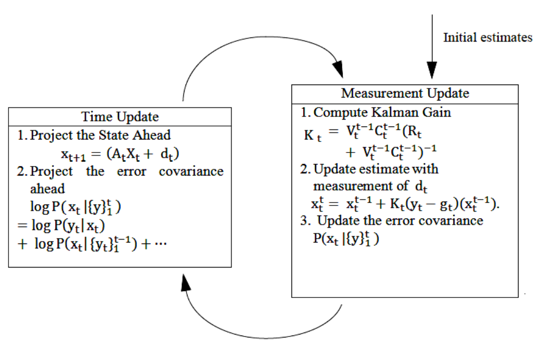

Figure (2.2) a complete picture of the operation of the Kalman filter, combining the high-level diagram of Figure 2.1 with the equations from table 1 and table 2

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Vol. 3, Issue 2, February 2014

III. METHODOLOGY

The proposed work of this paper, deals with the implementation of extended Kalman filter (EKF) for the efficient ECG signals denoising and RR interval estimation in comparison with available median filter. Figure (3.1) shows the flow chart representation of proposed work.

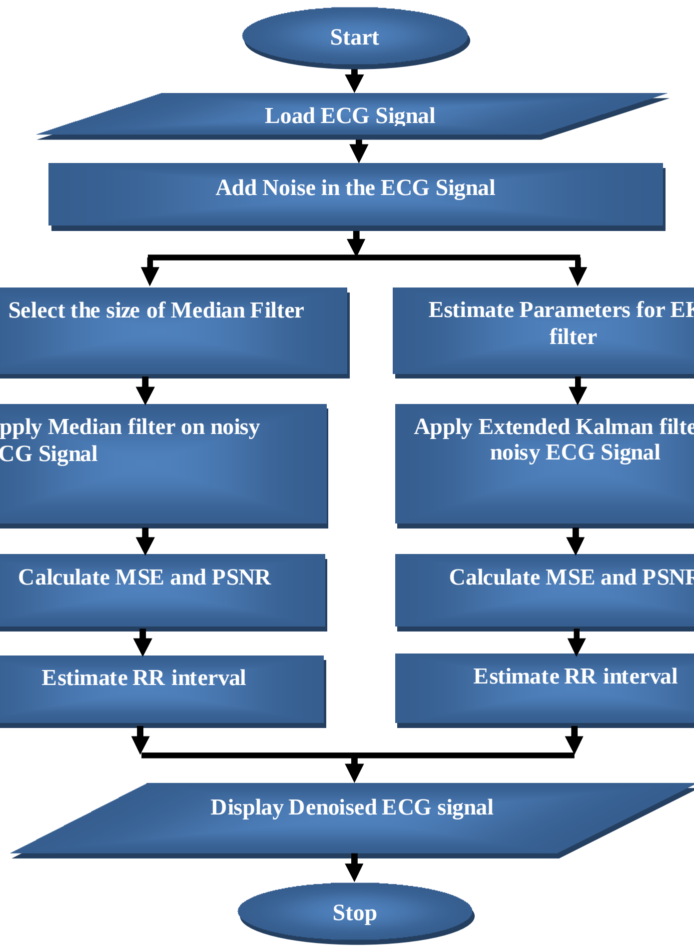

Figure (3.1) Flow chart of the proposed work

IV. RESULTS AND DISCUSSIONS

The proposed work has been successfully implemented in the MATLAB. This section presents the results obtained and comparative analysis with median filer and extended kalman filter. For the testing phase, we have used MIT-ANSI ECG data base. Figure (4.1) to figure (4.3) shows the results obtained after ECG signal denoising using EKF and median filter. For example figure (4.1a) shows original ECG signal, figure (4.1b) shows Noisy ECG signal to be filter, figure (4.1c) shows resultant image after median filtering and figure (4.1d) shows resultant image after EKF filtering.

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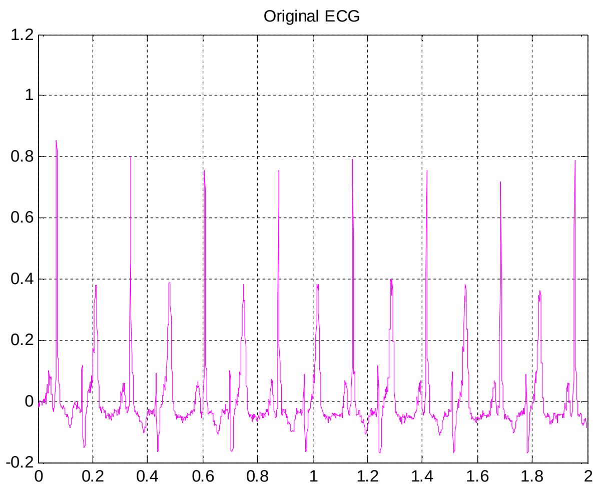

Figure (4.1a)

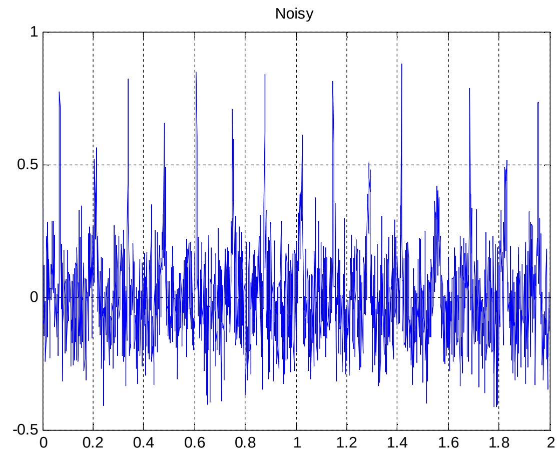

Figure (4.1b)

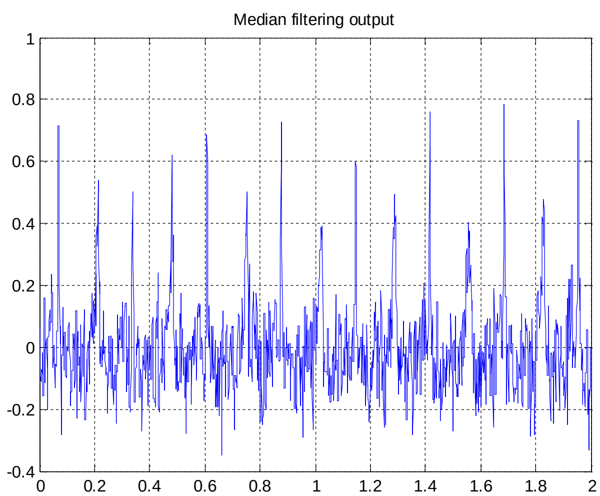

Figure (4.1c)

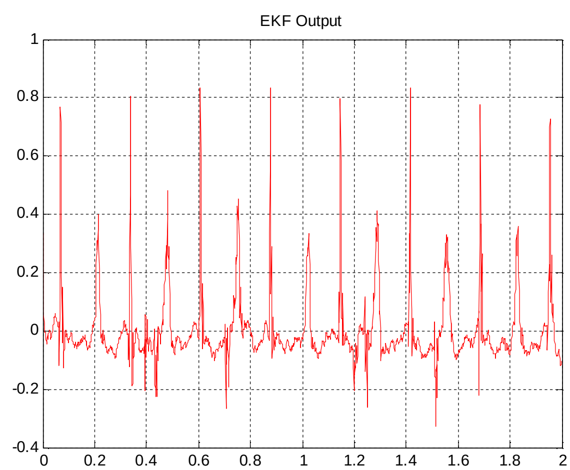

Figure (4.1d)

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ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

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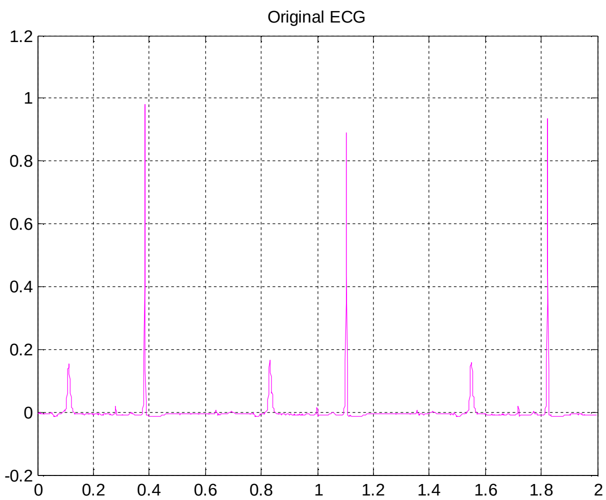

Figure (4.2a)

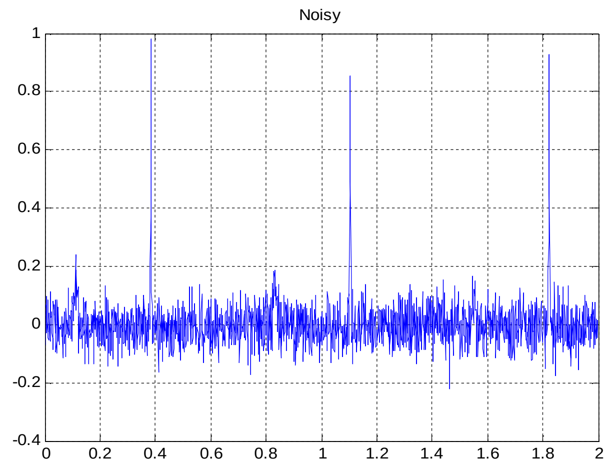

Figure (4.2b)

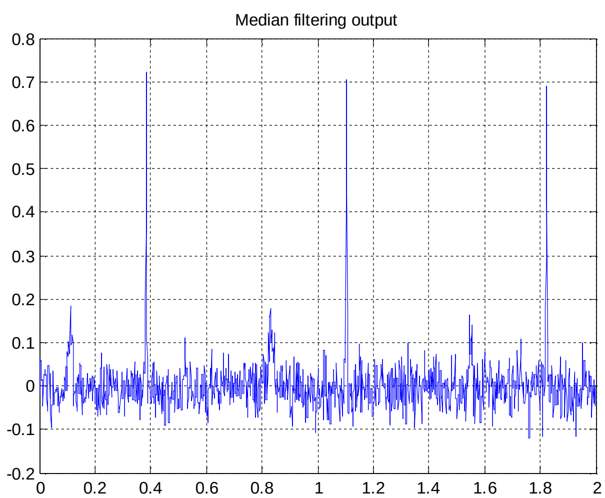

Figure (4.2c)

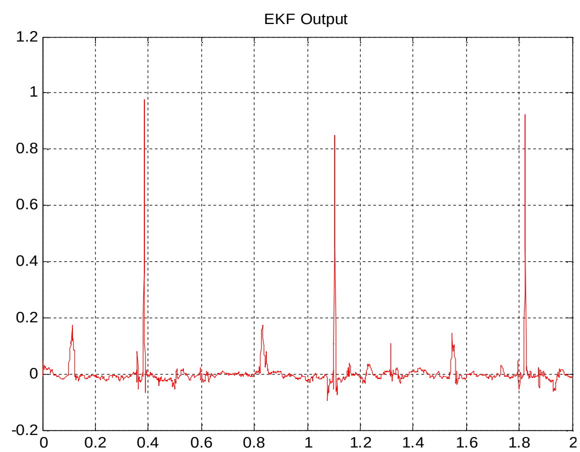

Figure (4.2d)

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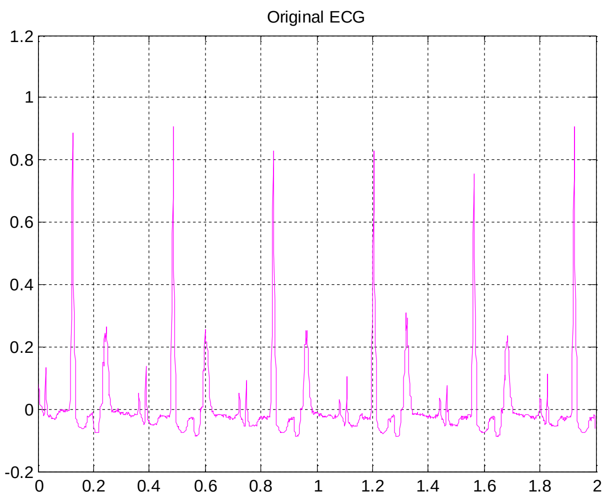

Figure (4.3a)

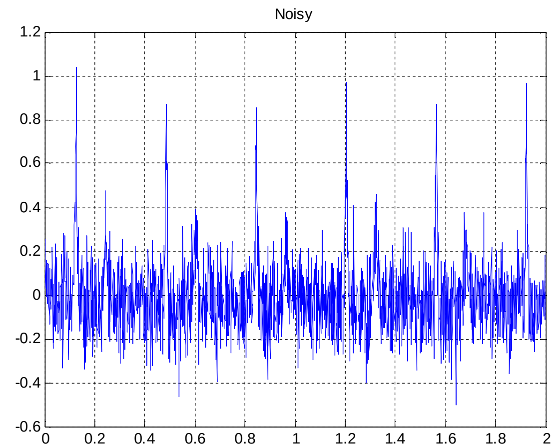

Figure (4.3b)

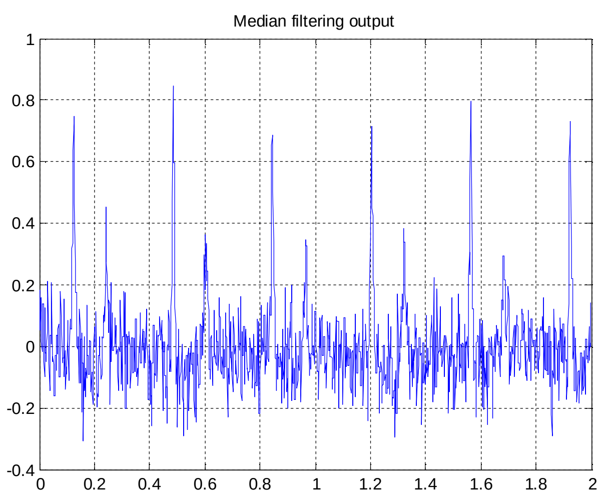

Figure (4.3c)

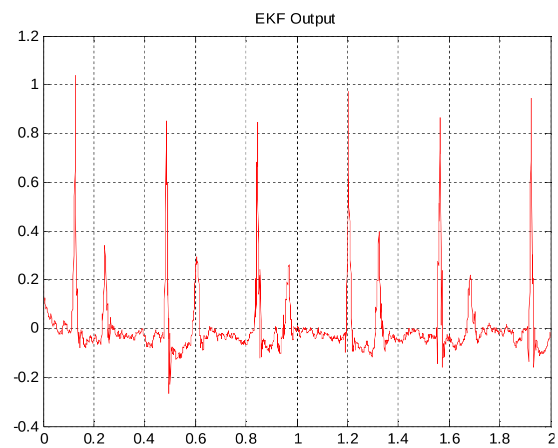

Figure (4.3d)

Table 3

S.No.	Database Signal	RR intervals of original ECG signal	RR intervals Obtained for Median Filter	RR intervals Obtained for EKF Filter	MSE for Median Filter	MSE for EKF Filter	PSNR for Median Filter	PSNR for EKF Filter
		RROS	**RRM**	**RREKF**	**MSEM**	**MSEEKF**	**PSNRM**	**PSNREKF**
1	aami3a	8	58	9	0.8124	0.0027	45.936	73.7539
2	aami3b	3	4	3	0.1357	0.00025	53.5273	84.1015
3	aami3d	6	54	6	0.6424	0.0017	46.6082	75.8117

Table 3 shows the complete comparative analysis of the ECG signal denoising using median filter and EKF filter on the basis of MSE, PSNR and estimated RR intervals. From the Table 3, following things are clearly observable

i. MSE values obtained from EKF filter are much smaller than the available median filter for all the three ECG signals.

ii. PSNR values obtained from EKF filter are much higher than median filter for all the three ECG signals.

iii. The most important parameter is RR interval, from the table it is evident that median is not able to preserve RR interval of original ECG signal during the denoising process, while the EKF filter is found very efficient as far as RR interval preservation is concerned. Hence EKF provides exact estimation of RR interval as in the original ECG signal.

V. CONCLUSIONS

The aim of this paper was to overcome disadvantages of conventional techniques of ECG signal denoising, like median filter by utilizing the adaptive nature of EKF structure. For the comparative analysis, this paper included three important parameters; mean square error (MSE), Peak signal to noise ratio (PSNR), and most importantly RR interval estimation. On the basis of the three parameters a comparative analysis has been done to explore efficient denoising capability of EKF over median filter. The results obtained indicated that EKF provides very less MSE and very high PSNR as compare to median filter. On the other side the estimated RR interval obtained using EKF is the efficient estimation of RR interval as the original signal RR intervals, while median filter provides so many RR intervals, which are not even presents in the original signal.

Hence EKF provides exact estimation of RR interval as in the original ECG signal as well as simultaneously able to highly suppress the noise contents added in the original ECG signal.

REFERENCES

[1] N. V. Thakor and Y.-S. Zhu, “Applications of adaptive filtering to ECG analysis: Noise cancellation and arrhythmia detection,” *IEEE Trans. Biomed. Eng.*, vol. 38, no. 8, pp. 785–794, Aug. 1991.

[2] P. Laguna, R. Jane, O. Meste, P. W. Poon, P. Caminal, H. Rix, and N. V. Thakor, “Adaptive filter for event related bioelectric signals using an impulse correlated reference input: Comparison with signal averaging techniques,” *IEEE Trans. Biomed. Eng.*, vol. 39, no. 10, pp. 1032–1044, Oct. 1992.

[3] G. B. Moody and R. G. Mark, “QRS morphology representation and noise estimation using the Karhunen-Lo‘eve transform,” in *Proc. Comput. Cardiology*, Jerusalem, Israel, Sep. 1989, pp. 269–272.

[4] A. K. Barros, A. Mansour, and N. Ohnishi, “Removing artifacts from electrocardiographic signals using independent components analysis,” *Neurocomputing*, vol. 22, no. 1–3, pp. 173–186, 1998. T. He, G. D. Clifford, and L. Tarassenko, “Application of independent component analysis in removing artefacts from the electrocardiogram,” *Neural Comput. Appl.*, vol. 15, pp. 105–116, 2006.

[5] G. D. Clifford and L. Tarassenko, “One-pass training of optimal architecture auto-associative neural network for detecting ectopic beats,” *Electron. Lett.*, vol. 37, no. 18, pp. 1126–1127, 2001.

[6] K.Daqrouq, “ECG baselinewander reduction using discretewavelet transform,” *Asian J. Inf. Technol.*, vol. 4, no. 11, pp. 989–995, 2005.

[7] H. A. Kestler,M. Haschka,W. Kratz *et al.*, “De-noising of high-resolution ECG signals by combining the discrete wavelet transform with the Wiener filter,” in *Proc. Comput. Cardiology*, Cleveland, OH, Sep. 1998, pp. 233– 236.

[8] D. L. Donoho, “De-noising by soft-thresholding,” *IEEE Trans. Inf. Theory* vol. 41, no. 3, pp. 613–627, May 1995.

[9] M. Popescu, P. Cristea, and A. Bezerianos, “High resolution ECG filtering using adaptive BSayesian wavelet shrinkage,” in *Proc. Comput. Cardiology*, Cleveland, OH, Sep. 1998, pp. 401–404.

[10] P. M. G. A. Da Silva and J. P. M. De´ Sa, “ECG noise filtering using wavelets with soft-thresholding methods,” in *Proc. Comput. Cardiology*, Hannover, Germany, Sep. 1999, pp. 535–538.

[11] O. Sayadi and M. B. Shamsollahi, “Multiadaptive bionic wavelet transform: Application to ECG denoising and baseline wandering reduction,” *EURASIP J. Adv. Signal Process.*, vol. 2007, pp. 1–11, 2007.

[12] S. M. S. Jalaleddine, C. G. Hutcgens, R. D. Strattan, and W. A. Coberly, “ECG data compression techniques—A unified approach,” *IEEE Trans. Biomed. Eng.*, vol. 37, no. 4, pp. 329–343, Apr. 1990.

[13] M. Blanco-Velasco, F. Cruz-Roldan, F. Lopez-Ferreras, A. Bravo-Santos, and D. Martinez-Munoz, “A low complexity algorithm for ECG signal compression,” *Med. Eng. Phys.*, vol. 26, no. 7, pp. 553–568, Sep. 2004.

[14] P. E. McSharry, G. D. Clifford, L. Tarassenko, and L. A. Smith, “A dynamic model for generating synthetic electrocardiogram signals,” *IEEE Trans. Biomed. Eng.*, vol. 50, no. 3, pp. 289–294, Mar. 2003.

[15] G. D. Clifford, A. Shoeb, P. E. McSharry, and B. A. Janz, “Model-based filtering, compression and classification of the ECG,” *Int. J. Bioelectromagnetis*, vol. 7, no. 1, pp. 158–161, 2005.

[16] M. Mneimneh, E. Yaz, M. Johnson, and R. Povinelli, “An adaptive Kalman filter for removing baseline wandering in ECG signals,” in *Proc. 33rd Annu. Int. Conf. Comput. Cardiol.*, 2006, pp. 253–256.

[17] R. Sameni,M. B. Shamsollahi, and C. Jutten, “Filtering electrocardiogram signals using the extended Kalman filter,” in *Proc. 27th Annu. Int. Conf. IEEE Eng. Med. Biol. Soc. (EMBS)*, Shanghai, China, Sep. 1–4, 2005, pp. 5639–5642.

[18] R. Sameni,M. B. Shamsollahi, C. Jutten, and M. Babaie-Zadeh, "Filtering noisy ECG signals using the extended Kalman filter based on a modified dynamic ECG model," in *Proc. 32nd Annu. Int. Conf. Comput. Cardiol.*, Lyon, France, Sep. 25–28, 2005, pp. 1017–1020.

[19] R. Sameni,M. B. Shamsollahi, C. Jutten, and G. D. Clifford, "A nonlinear bayesian filtering framework for ECG denoising," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 12, pp. 2172–2185, Dec. 2007.

[20] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

[21] P. E. McSharry and G. D. Clifford, "A statistical model of the sleep-wake dynamics of the cardiac rhythm," *Comput. Cardiol.*, vol. 32, pp. 591–594, Sep. 2005.

[22] G. D. Clifford and P. E. McSharry, "Method to filter ECGs and evaluate clinical parameter distortion using realistic ECGmodel parameter fitting," *Comput. Cardiol.*, vol. 32, pp. 735–738, Sep. 2005.

[23] G. D. Clifford, "A novel framework for signal representation and source separation: Applications to filtering and segmentation of biosignals," *J. Biol. Syst.*, vol. 14, no. 2, pp. 169–183, Jun. 2006.

[24] S. Haykin. Adaptive Filter Theory. Prentice-Hall, third edition, 1996.

[25] B.D.O. Anderson and J.B. Moore. Optimal Filtering. Prentice-Hall, Englewood Cliffs, NJ, 1979.

[26] B.M. Yu, K.V. Shenoy, and M. Sahani. Derivation of Kalman filtering and smoothing equations.2004.